# Geometric methods used in the construction of architectural forms

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ABSTRACT
All the architectural forms applied in modern construction use geometric shapes. Therefore, it is not good for a builder to start construction without mastering geometry. This article presents a brief overview on how to build some architectural shapes from geometric shapes.

**Keywords:** Arithmetic, geometry, architecture, section, proportion, circle, angle, triangle, rectangle, pattern.

It is known that in countries and peoples that have achieved culture in different historical periods, in addition to the simple proportions of small values, elementary geometric shapes: square, equilateral triangle and irrational, $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$, values of two squares, were used to establish proportions in this or that area.

Ibn al-Baghdadi (10-th to 11-th centuries) states in his treatise on dimensional and non-dimensional quantities is worth it.

Even Vitruvius in his time advocated the use of rectangles with sides equal to $\sqrt{2}$. Undoubtedly, this is why Palladio in 1570 included a rectangle with a side ratio of $\sqrt{2}$ in the list of seven shapes recommended for room planning.

Here are some necessary geometric concepts to make it easier to understand and use geometric operations later.

- If the difference between an arbitrary number and the next number does not change, the sequence of numbers $a_1, a_2, a_3, \ldots$ is called an arithmetic progression, ie:

$a_2 - a_1 = a_3 - a_2 = a_4 - a_3 = \ldots$ .

For example, the numbers 1, 3, 5, 7, 9 form an arithmetic progression. In any arithmetic progression, $a_1 + a_3 = 2a_2$, $a_2 + a_4 = 2a_3$, ..., that is, any number is equal to half the sum of its predecessors and successors. Each term of an arithmetic progression is called the arithmetic mean of adjacent numbers, and a rule is formed to find the arithmetic mean of two given numbers a and b:

***arithmetic mean = (**$a+b$**)/**$2$.

If, $a_1, a_2, a_3, \ldots$ the ratio of any term in the sequence of numbers to the next does not change, that is:

$a_2 / a_1 = a_3 / a_2 = a_4 / a_3 = \ldots$ .

such a sequence of numbers is called a geometric progression. In this case $a_1 a_2 = (a_2)^2$, $a_2\ a_4 = (a_3)^2, \ldots$ , so any term of the geometric progression is equal to the square root of the product of the adjacent numbers before and after it. The geometric mean of two given numbers a and b

***(geometric mean)²*** $= a \cdot b$

found by the formula

Finally, if $a_1, a_2, a_3, \ldots$ in a sequence of numbers, given

$1/a_1,\ 1/a_2,\ 1/a_3, \ldots$

if the numbers are inverse values and form an arithmetic progression, the harmony is called a progression. Any value of such a sequence is called the mean of the adjacent numbers before and after it. Therefore, to find the mean of the two corresponding numbers a and b, we first find the arithmetic mean of the numbers inverse of them, and then the inverse of this mean. And so,

***average match value*** *= 2 /(1/a+1/b)=2ab/(a+b).*

Obviously, the simplest of the matching sequences will look like *1, 1/2, 1/3, 1/4, ...*

Palladio (1518-1580) used arithmetic mean, geometric mean, and mean corresponding values to determine the heights of domed rooms.

It is important to note that the architectural ideas used for countries with relatively warm climates are unsuitable for countries with relatively cold climates. For example, it is impossible to heat the rooms of buildings with high ceilings to the extent necessary for living in cold climates. To determine the height of rooms with a flat ceiling, Palladio used a much simpler rule, that is, he strongly suggested that the height of the rooms should be equal to its width.

M. Bulatov in his book "Geometric harmony in the architecture of Central Asia in the IX - XV centuries", as a result of the study of the geometric harmony of architectural images of Central Asia in the Middle Ages, based on Plato's doctrine of proportion, $\sqrt{2}$, $\sqrt{3}$, $\sqrt{4}$, $\sqrt{5}$, concludes:

$\sqrt{2}$ – the average geometric size between one and two, i.e.

$$\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}.$$

$\sqrt{3}$ – the average geometric size between one and three, i.e.

$$\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}.$$

$\sqrt{4}$ – the average geometric size between one and four, i.e.

$$\frac{1}{\sqrt{4}} = \frac{\sqrt{4}}{4} \quad \text{or} \quad \frac{1}{2} = \frac{2}{4}.$$

$\sqrt{5}$ – the average geometric size between one and five, i.e.

$$\frac{1}{\sqrt{5}} = \frac{\sqrt{5}}{5}$$ *(Fig. 1).*

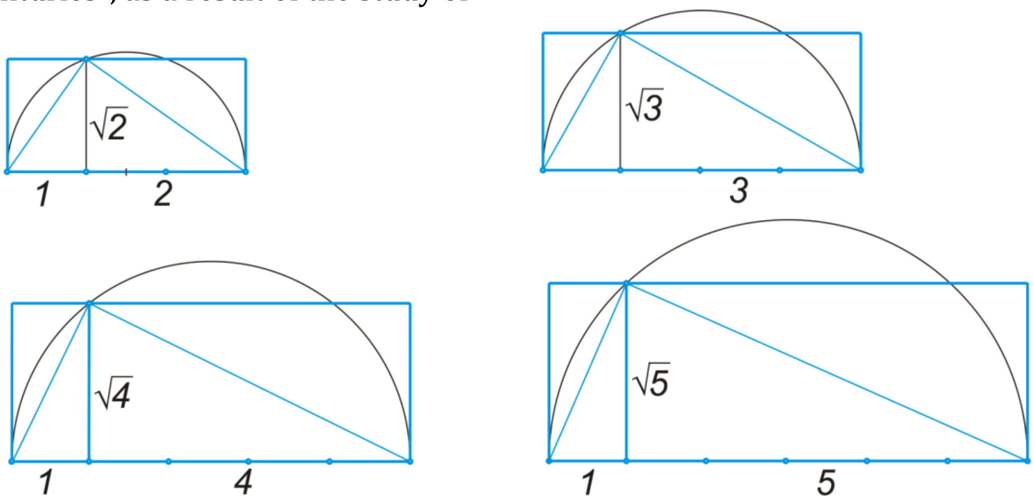


*Fig. 1.*

It is easy to see that the given proportions have the following general appearance:

$$\frac{a}{b} = \frac{b}{c},$$

that is, these proportions have been known since the time of the ancient Greek philosophers, and established a geometric size, a "proportion," and a harmonious connection between the extremes.

The secret of harmonized proportions is apparently hidden not in the individual forms, but in the proportions between them. Books about the golden section, which is a means of achieving harmony in architectural proportions, can be found in all libraries of architecture, and it is worthwhile to study some of them carefully.

One can find in them the. following consideration: in architecture, as well as in brush works, it all depends on the position of the observer, and if, in some respects, the proportions in the buildings seem to be the creators of the golden section, in other respects they are different has the appearance. The following simple geometric considerations reinforce this idea.

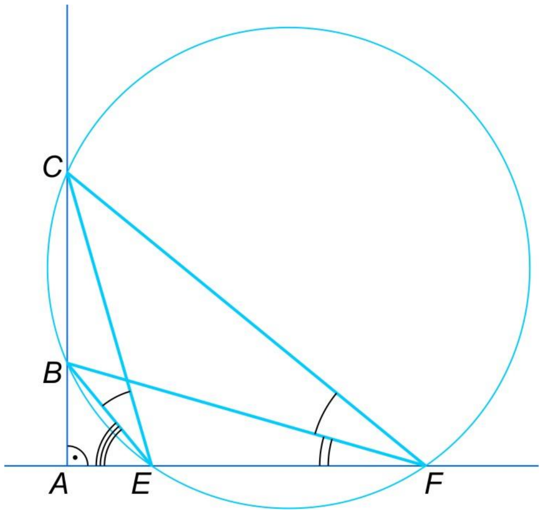


*Fig. 2.*

The façade of the building has a straight rectangular shape and it is divided by a horizontal straight line so that these pieces form a golden section, if a plane is drawn perpendicular to the façade, then $CB/BA= \mu$, Suppose that (Fig. 2). Also, the observer's eye moves along a horizontal straight line passing through point *A*, and the observer is embraced by a sharp aesthetic pleasure at point *F*. Because the dimensions seem to depend on the angle of view of the observer's eye, this

$\angle CFB/\angle BFA = \mu$ indicates.

*E* – *A* nuqtadan o'tkazilgan gorizontal to'g'ri chiziqning *C, F* va *B* nuqtalar orqali o'tuvchi aylana bilan kesishgan nuqtasi bo'lsin. Unda, $\angle CEB = \angle CFB$ aylanaga ichki chizilgan va bitta yoyga yopishgan burchaklar bo'ladi. Lekin *BEA* burchak *BFE* uchburchakning tashqi burchagi kabi *BFA* burchakdan kattadir. Shuning uchun, $\angle CEB/\angle BEA<\angle CFB/\angle BFA.$ Bu shuni bildiradiki, agar to'g'ri to'rtburchakni gorizontal to'g'ri chiziq bilan bo'linishi bir holatdan oltin kesim bo'lib ko'rinsa, qolgan hamma holatlarda u boshqacha ko'rinadi.

Let E be the point of intersection of a horizontal straight line passing through point *A* with a circle passing through points *C, F* and *B*. Then $\angle CEB = \angle CFB$ are the angles drawn inside the circle and attached to one arc. But the *BEA* angle is as large as the *BFA* angle, just like the outer angle of the *BFE* triangle. Therefore, $\angle CEB / \angle BEA < \angle CFB / \angle BFA$. This means that if the division of a rectangle by a horizontal straight line appears to be a golden section in one case, it will look different in all other cases.

But there are some things that can be said about the aesthetic appeal of certain proportions, including the golden cut. The comments are taken from Scolffield's Theory of Proportion in Architecture.

Fig, 3, a, depicts a rectangle bisected by a straight line parallel to both sides. We compare the shape of the first rectangle with the shapes of the two rectangles that make it up. In general, all rectangles have different shapes, but one of them can be excluded. In Fig. 3, *b*, one of the small rectangles is similar to the original rectangle. The sides of the first rectangle are $x^2$ and *x*, the ratio of the sides is *x*, and the straight line that bisects it is at a distance of *1* from its edge. The ratio of the sides of a small rectangle is also $x:1 = x$, so at any value of x it is similar to the original rectangle. It should be noted that the diagonals shown by the dotted line in Fig. 3, *b* are mutually perpendicular.

In Fig. 3, the sides of the first rectangle in c are $x^2 + 1$ and *x*, a straight line dividing it in two, passing through its edge again at a distance of *1*. Both of the small right quadrilaterals are similar to each other, the ratio of the sides of each being $x:1 = x$, but they are not necessarily similar to the first right quadrilateral. In Fig. 3, the sides of the first rectangle in *d* are *2x* and *x*, a straight line dividing it in two passes through the middle of the larger sides, so that both small vertices The rectangles are exactly the same.

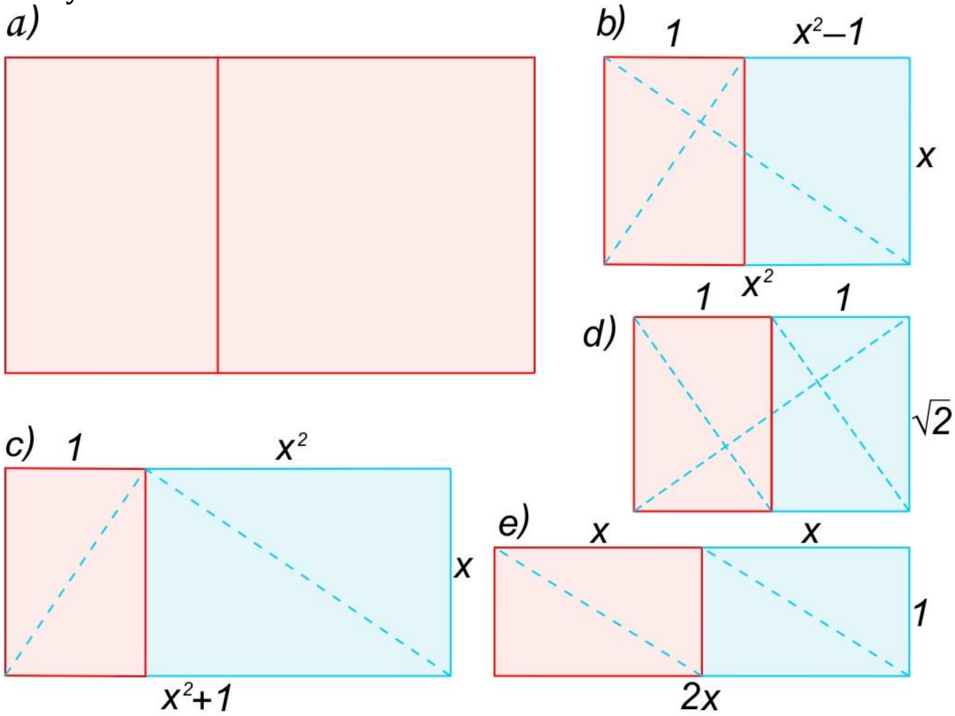


*Fig.3.*

If we return to Fig. 3, b, we can lose two of the three rectangles depicted in each Fig., and $1/x=(x^2-1)/x$

If we write the ratio, we get the equation $x^2 = 2$. If we put the value of x $=\sqrt{2}$, Fig. 3 turns from *b* to Fig. 3, d. Each of the three rectangles has the same shape, the diagonals of the small rectangle indicated by the dotted line are perpendicular to the diagonal of the first rectangle indicated by the dotted line.

If we try to make the shapes of all three rectangles forming the shape in Fig. 3 *d* the same shape, then we come to Fig. 3, *d* again. Everyone agrees that the images in Fig. 3, d, are consistent.

We will continue to study simple shapes and consider a rectangle divided by straight lines parallel to the small and large sides, as in Fig. 4, a. When we count, we notice that there are *9* rectangles in the picture that have different shapes. In Fig. 4, the living line from the dividing line *b* passes through the parallel sides of the initial rectangle, which reduces the number of different shapes from *9* to *6*. Continuing the problem of reducing the number of different shapes (Fig. 4, b), it becomes clear that there are three variants in which the rectangular shapes of the rectangles can be reduced to 3. These cases are illustrated in Fig. 4, c, d, and e. Fig. 4, e $\mu=(1+\sqrt{5})/2$, that is, a golden cut. When checking the similarity of the shapes, it is advisable to use the golden section satisfying the equation $1+\mu=\mu^2$ *(The geometric meaning of μ is given below)*.

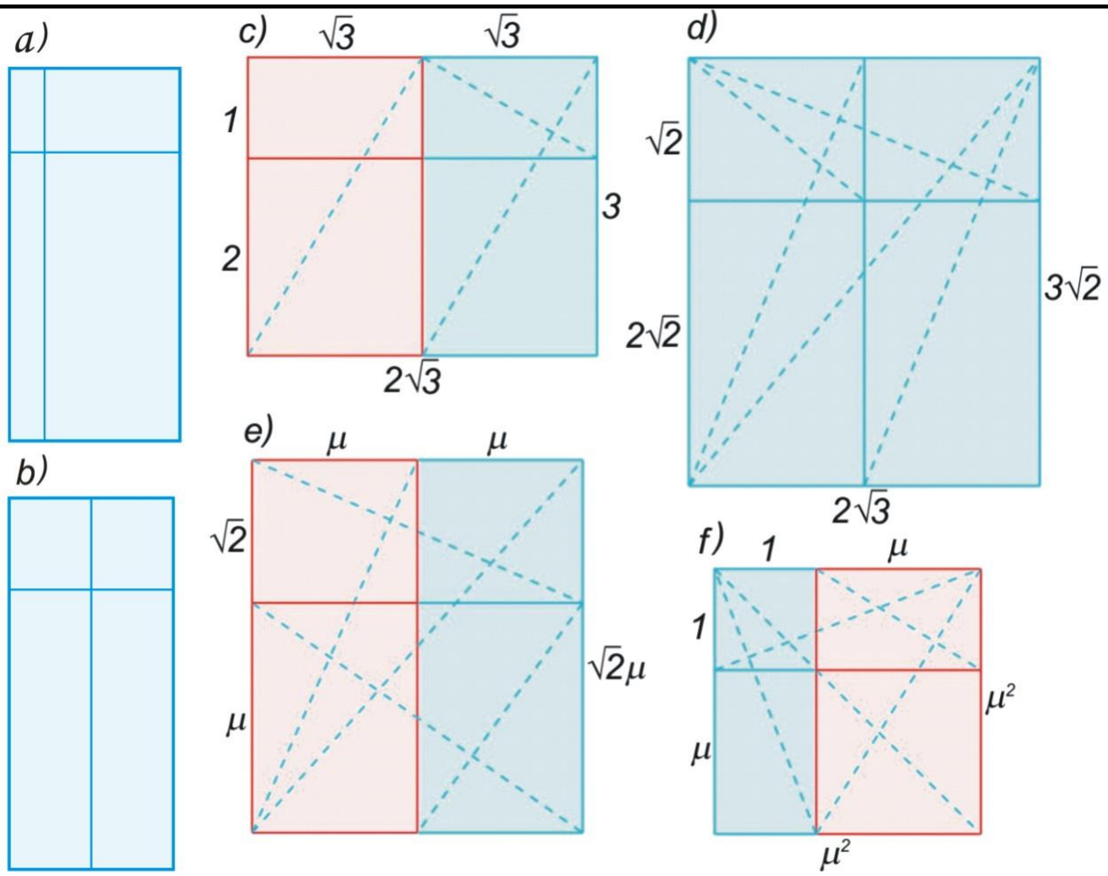


*Fig. 4.*

It is also possible that the situation described in Fig. 4, f, does not have axes of horizontal and vertical symmetry. There is only one axis of diagonal symmetry. The first rectangle is square. Fig. 4, f, shows 3 squares and 6 rectangles: The sides of 2 are in the ratio $1+\mu=\mu^2$ and the sides of the remaining 4 are in the ratio m; so here, too, we have a total of three different rectangles of different shapes.

Apparently, the gold cut really hides some charm. Therefore, we continue to study the mathematical properties of the number $\mu$. if,

$1, \mu, \mu^2, \mu^3, \mu^4, \mu^5, ..., \mu^n, ...,$

If we consider the geometric progression,

$\mu^3= \mu+ \mu^2= \mu+(1+ \mu)=2 \mu+1,$

$\mu^4=2 \mu^2+ \mu=3 \mu+2,$

$\mu^5=3 \mu^2+2 \mu=5 \mu+3$

etc. Since the ratio $\mu^n= \mu^{n-1}+ \mu^{n-2}$ is expressed by the degree $\mu^k$, the coefficients formed in $\mu$ are related to the previous ratio.

$u_n= u_{n-1}+ u_{n-2}$

The $u_n$ term of flour is a sequence of integers *1, 1, 2, 3, 5, 8, 13, ...* forms.

In fact, starting from the third term, 2 = 1 + 1, then 3 = 2 + 1, 5 = 3 + 2, 8 = 5 + 3, 13 = 8 + 5. The next limit of the row is 21 = 13 + 8 and so on.

This set of numbers has a long history and is known as Fibonacci numbers. Leonardo, a Pisa merchant nicknamed Fibonacci, came to this idea in 1202 because of the problem of breeding rabbits, and to this day the sequence of these numbers is named after the inventor.

According to Leonardo of Pisa, rabbits live indefinitely and a pair of rabbits give birth to two babies a month. The calculation of the number of rabbits begins with a pair of newborn rabbits, and we write the number 1, which corresponds to the first month. In the second month, the same pair of rabbits will remain, and we will write the number 1 again. By the third month, a pair of rabbits are born, so now we write the number 2. In the fourth month we have 3 pairs of rabbits, in the fifth month 5 pairs and so on.

$u_n= u_{n-1}+ u_{n-2}$

it is not difficult to understand where the equality came from; interestingly, this equation was first written not by Leonardo of Pisa, but by Kepler, who lived four centuries later.

Renaissance architects were familiar with the golden section created by Euclid's proposed pentagon. However, they have not been used effectively enough as a tool to create proportions from the gold cut. Still, there was a lot of interest in gold mining. For example, Pacholi called the golden cut a divine proportion. The term gold coin originated in Germany in the first half of the 19th century. It corresponds exactly to the ratio $\mu=(1+\sqrt{5})/2=1,618$.

The golden section is formed by a regular pentagon, so Fibonacci numbers also play a role in all of the regular pentagon-related bubble and star cases.

Fig. 5 shows just a few of the various proportions that belong to the gold cut that amazed Pacholi and the architects and painters who lived after him.

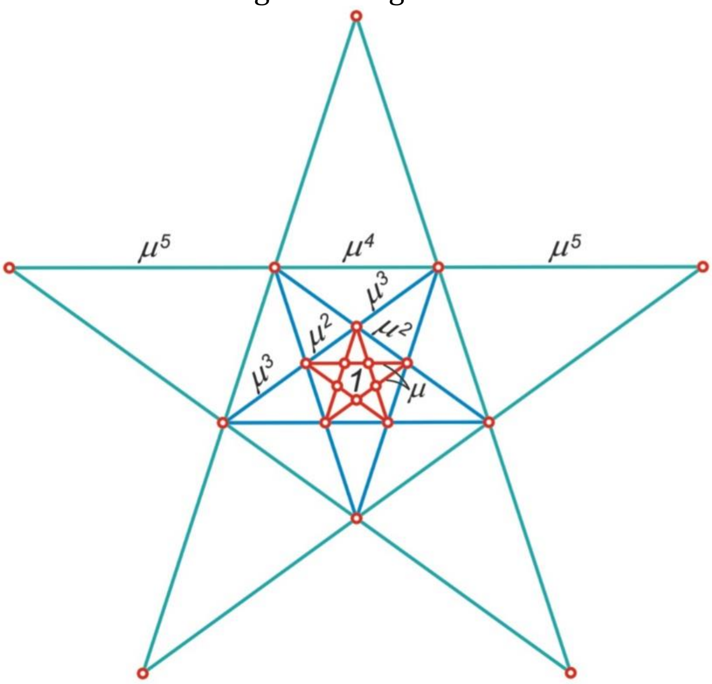


*Fig. 5.*

In the design of buildings, the additive properties of the gold cut are of particular importance. If the length of the new cut in the drawing is chosen to be equal to $\mu u_{n-1}$ each time ($u_{n-1}$ is the initial length), it is nice to recognize that the parts of the drawing correspond to each other, because $u_n= u_{n-1}+ u_{n-2}$.

Theodore Cook, in his 1914 book, The Curves of Life, published in London, examines Turner's sea view and cites the Universal Scale, based on the golden section. Scolfield rightly calls him the ancestor of the modulator proposed by Le Corbusier. The universal scale is based on simple geometric considerations (Fig. 6), which can be interpreted as follows:

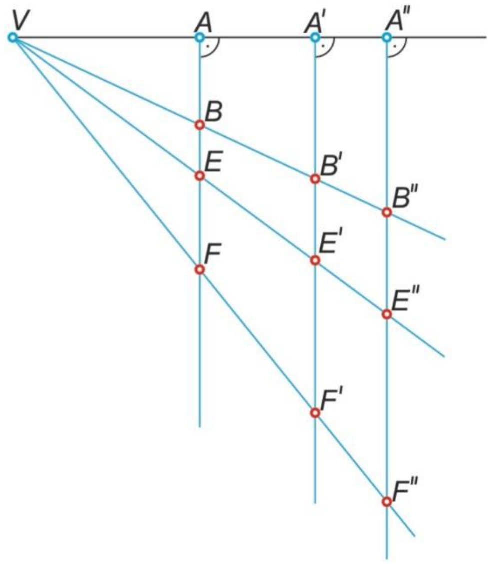


*Fig. 6.*

Let A, A ′, A ′ ′, ... lie on a straight line. AB, A′B ′, A′′B ′ ′, ... straight lines AA ′ perpendicular to the straight line, BB′B ′ ′, EE′E ′ ′, FF′F ′ ′ , ... all straight lines pass through a single V point of this straight line. According to the theorem on the similarity of triangles

$AB : AE : AF : ... = A'B' : A'E' : A'F' : ... = A''B'' : A''E'' : A''F'' : ... .$

Also AB, A′B ′, A′′B ′′, ... from the parallelism of straight lines

AA ′: AA ′ ′: ... = BB ′: BB ′ ′: ... = EE ′: EE ′ ′: ... arises.

Thus, we have a method of constructing any number scale based on the golden section, using a unit of length selected based on our constructions or inspections.

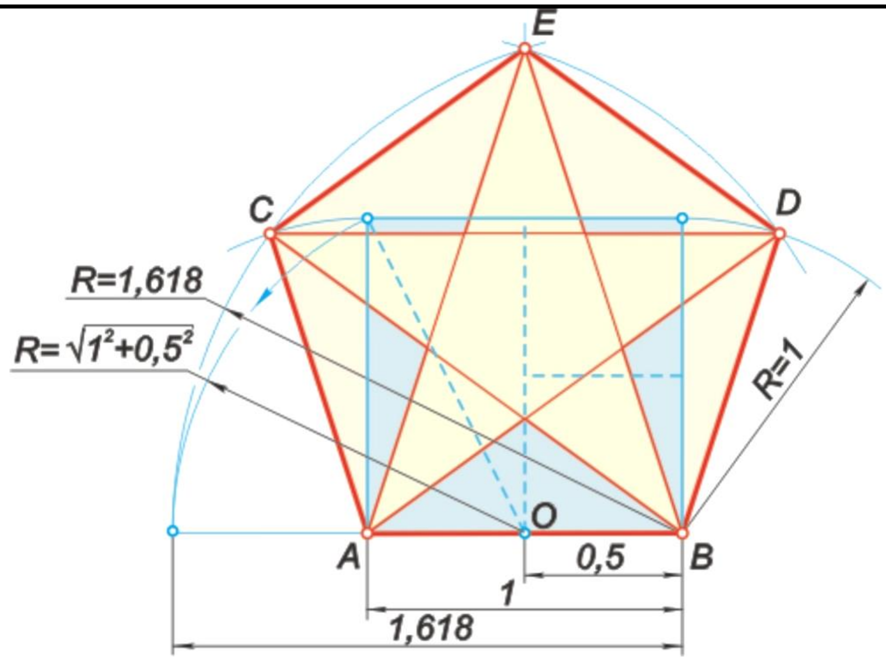


*Fig. 7.*

Suppose we combine two small rectangles consisting of two small squares, as shown in Fig. 7, to form a large square with sides equal to 1. Since the diagonal of the first rectangle is $\sqrt{(1^2+0{,}5^2)}$ *=1,118*, if we add to this value the base of the second rectangle, 0.5, the golden rectangle is larger. resulting in a value of *1,618,* which is equal to In a radius equal to the value obtained, we draw circular arcs first at point A and then at point B. These arcs intersect to form point E. Now, if we draw a circle arc from these points A and B with a radius of 1, that is, a radius equal to 1, these arcs will intersect with the previous drawn arcs to form the ends C and D of an equilateral pentagon.

If we draw the diagonals of the resulting equilateral pentagon, we get a five-pointed star. These five stars, in turn, belong to the golden ratio.

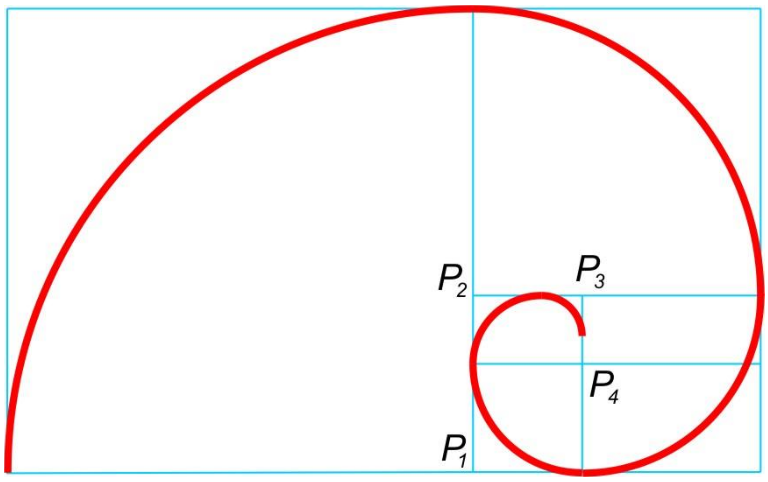


*Fig. 8.*

Fig. 8 shows an *AEFD* rectangle whose sides have an aspect ratio of gold. *ABCD* square is cut from it. The ratio of the sides of the remaining *BEFC* rectangle is also equal to the golden ratio. If we cut the *BEGH* square again, then the sides of the remaining *FGHC* rectangle will also be equal to the golden section. This process can take an infinitely long time.

If we place a quarter of the circle in each of the resulting squares as shown in the Fig., we get a very nice curve. The centers of the arcs of the series are located at points *C, H, I, K, ...*

Fig. 9 shows another modulator based on the ratio th $\theta=1+\sqrt{2}$ , which can be compared to the gold section. An example of such a scale is

Leonardo da Vinci's star-shaped octagon. This satisfies the equation th $\theta^2=2\theta+1$ and if,

$u_n= \theta u_{n-1}+ \theta^2 u_{n-2}$

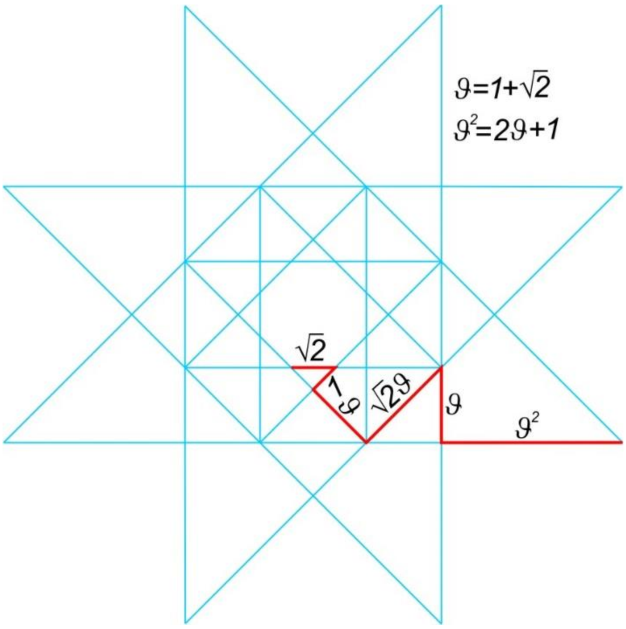


*Fig. 9.*

then $u_n= 2 u_{n-1}+ u_{n-2}$. This is the sequence of the Fibonacci series based on the gold cut

$u_n= u_{n-1}+ u_{n-2}$

comparing the ratios of the limits, it becomes clear that the simple additive property of the gold cut is unique and irreversible.

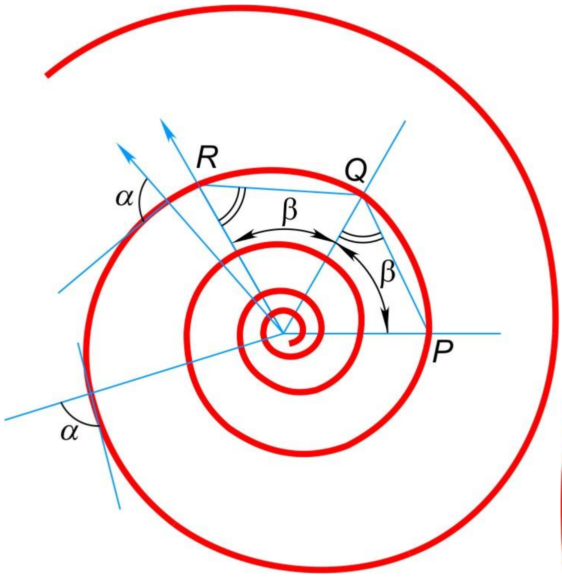


*Fig. 10.*

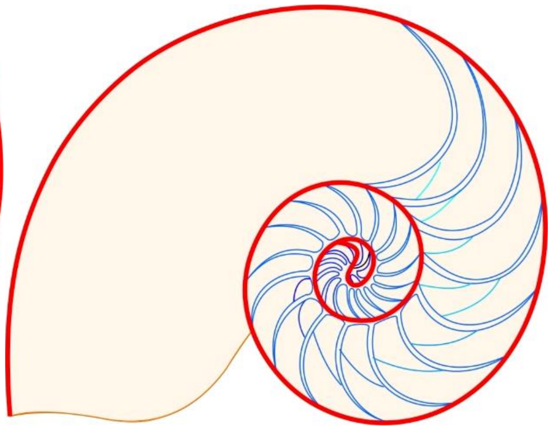

*Fig. 11.*

In Fig. 8, we looked at an artificial spiral made up of quarters of a circle. In it, we created a spiral based on dividing a rectangle whose sides are equal to a golden section, an infinite number of times squares, and a new gold rectangle. There is also a real spiral with a rectangle close enough to this spiral (Fig. 10). However, instead of trying to side the squares in a real spiral, they intersect at very small angles.

It should be noted that the shells of some mollusks are strikingly similar to a real rectangular spiral. Fig. 11 shows the cross-section of the shell for comparison with a spiral.

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