Stability study of some nonlinear time series models with application

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ABSTRACT

In this research, the stability of time series models, nine in general, were studied. The stability of some nonlinear time series models was also studied using Lagrange's method and the results compared.

The time series for seasonal allergies were studied. For this series, several non-linear mathematical models were built, including the seasonal model using transformation (natural logarithm), second-order exponential model and polynomial model. The stability of the above models was determined using the aforementioned method and through comparisons. The above models show that the combined seasonal model is the best-suggested model to represent the chain future value of a disease for one year, which were predicted using a seasonal model.

Keywords:

Introduction

Time series are usually built on the following assumptions: static, linear, natural. Most time series may not follow a normal distribution, and these characteristics are very important in the estimation and building of time-series models, and thus, the study of time-series includes these assumptions as well as how to treat unstable time series and determine the appropriate mathematical models for those series, such as whether they are linear or non-linear and whether they follow a normal distribution.

Our study in this research will focus on the study of the stationary of some models of nonlinear time series. B.M Priestley (1988) studied the instability of nonlinear models for time series[25]. H. Tong (1990) explored the dynamical system with stability in non-linear time series [28]. Bongeral N. and Picard (1992) investigated the stability of the GARCH model and some non-negative time series [11].

L-R Tweedie and. J.P. Brockwell (1992) explored the presence of stability in models and the threshold for the ARMA model [12]. E. Susko and. J. Liu (1992) conducted a study on the stability and annuals for nonlinear ARMA models [19], whereas Manncat (1992) studied the instabilities.

S.P. Meyn (1993) and R.L. Tweedie (1993) studied time-series models in Markov chains and stochastic stability [17], [20]. R. Manuca (1996) and R. Savit (1997) studied Markov chains and stochastic stability. Meanwhile, J. Husmeier and D. Taylor (1997) explored the conditions of probability and the process of stability in stochastic time series [16]. Schreiber (1997) analysed the instabilities of temporal chains [25]. Meanwhile, two scientists, B. Lerartgon and. S. Peterson (1997) studied the stability and robustness of hybrid systems by exploring the analysis and stability of the exponential function in non-linear models [24]. A.J. Kurths and A. Pikosvky

(1998) studied the time-series in stability cases [30].

2- Stationarity and Stability [1]

We encounter many physical and engineering problem processes that can be described being in an equilibrium statistical state. An equilibrium statistical state means that if we get observations of a process of this type and divide them into groups of periods, the different sections of these observations will look similar. More precisely, the statistical characteristics are fixed and do not change with time. The operations of the random processes that behave in this way are called stationary [1], which means the presence of growth or decay of time series data and the data are not spread around. It has a constant mean and variance. Hence, $x_1, x_2, x_3, \dots, x_t$ must have a density function is the same probability and is a real number k where $f(x_1, x_2, x_3, \dots, x_t) = f(x_{1+k}, x_{2+k}, \dots, x_{t+k})$.

The common probability distribution does not change with a change in period or when shifting with fixed numbers.

Theorem [1]: Stationary processes usually originate from a stable system that reaches a state steady after an appropriate time. Wei (1990) proposed a stable system in which a finite input always produces a finite output. A system is mathematically stable if the roots of the polynomial of the system equation are in the form. The operator lag is outside the circle unit or the roots of the discriminant equation all lie within the unit circle [1].

2-1 Stationarity of non-linear models [18], [23], [28]

The techniques developed for the study of stability include the stability of time series models' linearity, high dependence on the linear hypothesis and cannot easily expand to the state.

Therefore, we will explore special cases of the aforementioned nonlinear models and refer to several studies to find the stability of non-linear models, including finding the stability according to the method.

The direct (Libanov) depends on more than one variable [22] and Poisson's stability (not the same source) as well as the Lagrange stability, which is usually limited.

On the chain or several chains, and the chain must be specific i.e., $M \le X_i$, where the M constant is used to determine the stability of continuous-time or discontinuous time series according to the Ozaki method. Many studies have focused in this area (90 Tong) and but we will focus on the stability of the Lagrange and the stability of Ozaki and to increase the details note [5].

2.2 Stability according to the Lagrange Method

Several studies have focused on determining the stability of some non-linear models, including the exponential models (note [23]) the stability of the binary models (model bilinear) (note [29]) and finding the stability of the autoregressive threshold models was noted [28].

Most of these studies use a linear approximation for these nonlinear models, that is, it can be converted non-linear. Models are converted to a formula similar to that of linear models and with certain assumptions. For example, we take the following form:

$$X_t = a_1 X_{t-1} + a_2 X_{t-2} + P X_{t-1}^3 + Z_t \quad (1\text{–}1)$$

This model represents a polynomial model, which is a non-linear model. This model is written in the following form:

$$X_t = (a_1 + P X_{t-1}^2) X_{t-1} + a_2 X_{t-2} + Z_t \quad . \quad (1\text{–}2)$$

It is a second-order autoregressive model with certain constraints on the parameter that can find stability is similar to linear models.

The following are some of the basic acquaintances and proofs required to determine the Lagrange stability.

Theorem (1-1) [28]

Let $\{\in\}$ represent the following system:

$$\in = \{A \in_{t-1} \quad if \quad X_{t-2} > o \; B \in_{t-1} \quad if \quad X_{t-2} \le o \qquad (1\text{–}3)$$

where

$B = [f' \; o \; 1 \; o\,], A = [f_1 \; f_2 \; 1 \; o\,], \in = [X_t \; X_{t-1}\,]$

Let's say

$f_1^2 + 4f_2 < o$

The system is stable if one of the following conditions is met [21]:

1) $f_1' < o$, $f_1 \geq o$ and $f_1'(f_1 f_1' + f_2) \leq 1$
2) $f_1' < o$, $f_1 < o$, $f_1 f_1' + f_2 < o$ and $f_1'(f_1 f_1' + f_2) \leq 1$
3) $f_1' < o$, $f_1 < o$ and $f_1 f_1' + f_2 = o$
4) $f_1' < o$, $f_1 < o$ $f_1 f_1' + f_1$, and $f_1 f_1' + f_2 \leq 1$
5) $f_1' = o$
6) $1 \geq f_1' > o$, $f_1 \geq o$
7) $1 \geq f_1' > o$, $f_1 < o$, $(f_1^2 + f_2)f_1' + f_1 f_2 \leq o$
8) $1 \geq f_1' > o$, $f_1 < o$, $(f_1^2 + f_2)f_1' + f_1 f_2 > o$

and $(f_1^2 + f_2)f_1' + f_1 f_2 \leq 1$.

2.3 A singular point [23] [22]

Let

$$X_t = F(X_{t-1}, \dots, X_{t-p}) \quad . \qquad (1–4)$$

The single point X of Equation (1–4) is defined as that point to which the path of the equation approaches when $t \to \infty$ is called X a solitary stable point. If the point X is approached when $t \to \infty$, the point X is called an unstable solitary point.

2.4 Limit cycle [23]

Let

$X_t = F(X_{t-1}, \dots, X_{t-p})$.

The general form of the difference equation is defined in discontinuous time. The termination cycle of the above equation is defined as the isolated and closed paths.

$X_t, X_{t+1}, X_{t+2}, \dots, X_{t+q} = X_t,$

where q is a positive integer representing the period. An isolated path is a path that is very close to the end cycle when $t \to \infty$ or $t \to -\infty$.

When the end cycle approaches when $t \to \infty$, it is called a limit stable cycle and if the end cycle approaches when $t \to -\infty$, it is called a non-end cycle (cycle limit unstable), a closed path means the initial value is $(X_1, \dots, X_p)$ belongs to the end cycle:

$(X_{1+kq}, \dots, X_{p+kq}) = (X_1, \dots, X_p)$ for every positive integer q.

2.5 Limit cycle stability [23]

We will begin by explaining the stability of the limit cycle on a non-linear exponential model of the first order.

We have the following model:

$$X_t = \left(f_1 + \pi_1 e^{-X_{t-1}^2}\right) X_{t-1} + \in_t \quad , \qquad (1–5)$$

where $\{e^t\}$ represent the white noises and f_1 and P_1 are the real constants. Given terminator y_t near the point X_s on the path and close to limit cycle where

$y_t, y_{t+1}, \dots y_{t+q-1}, y_{t+q} = y_t$.

$$X_s = y_s + \in_s ,$$

where $|\in|$ is lowercase within the period $s = t, t-1$. The columns X_t, X_{t-1} on the path near its terminal cycle are as follows:

$X_{t-1} = y_{t-1} + \in_{t-1}$ and $X_t = y_t + \in_t$

Substituting Equation (1-5) and performing some simple algebraic operations, we obtain the following:

$$\in_1 = \left\{\emptyset_1 + \pi_1(1 - 2y_{t-1}^2)e^{-y_{t-1}^2}\right\} \in_{t-1} + o(\in_{t-1}) \quad . \qquad (1–6)$$

From Equation(1-6), we find that if the solution to the following equation is

$$\in_1 = \left\{\emptyset_1 + \pi_1(1 - 2y_{t-1}^2)e^{-y_{t-1}^2}\right\} \in_{t-1} \quad , \qquad (1–7)$$

it approaches zero when $t \to \infty$ the cycle limit is stable.

Equation (1-7) is a linear differential equation with periodic factors that are difficult to solve analytically, and thus, it is possible to test whether Equation (1-7) approaches zero using the relationship $\left|\in_{t+q}/\in_t\right|$ and approaches $\in_t$ to zero if this relationship is less than one.

From the Equation (1-7) we can obtain $\in_{t+q}$ as follows:

$$\in_{t+q}=\left\{\emptyset_1+\pi_1\left(1-2y_{t+q-1}^2\right)e^{-y_{t+q-1}^2}\right\}\in_{t+q-1}$$

$$=\left\{\emptyset_1+\pi_1\left(1-2y_{t+q-1}^2\right)e^{-y_{t+q-1}^2}\right\}\left\{\emptyset_1+\pi_1\left(1-2y_{t+q-2}^2\right)e^{-y_{t+q-2}^2}\right\}\in_{t+q-2}.$$

We continue in this way to obtain the following:

$$\in_{t+q}=\prod_{t=1}^{q}\left\{\emptyset_1+\pi_1(1-2y_{t+i-1}^2)e^{-y_{t+i-1}^2}\right\}\in_t.$$

Therefore, $\left|\in_{t+q}/\in_t\right|$ is given as follows:

$$\left|\frac{\in_{t+q}}{\in_t}\right|=\left|\prod_{i=1}^{q}\left\{\emptyset_1+\pi_1(1-2y_{t+i-1}^2)e^{-y_{t+i-1}^2}\right\}\right| . \qquad (1-8)$$

By using Equation (1–8) it is possible to benefit from the following theorem of stability.

Theorem (2): [23] [22]

Limit cycle period q in turn (1–5) of the model

Limit cycle period in turn q of model (1–5) is stable if

$\left|\in_{t+q}/\in_t\right|<1.$

Proof: note ([22])

Using the above method and the following theorem (3) it is possible to determine limit cycle stability for the exponential model of the following order P:

$$X_t=\left(\emptyset_1+\pi_1e^{-X_{t-1}^2}\right)X_{t-1}+\varepsilon_1 \quad . \qquad (1–9)$$

Theorem (3) [22] [23]

The limit cycle in turn [q] of the exponential model (1-9) is stable if the absolute value of the eigenvalues of the matrix is less than one in absolute terms:

$$A=A_q.A_{q-1}.\dots A_1 \quad , \qquad (1–10)$$

$$A_i=\begin{bmatrix} a_{11}^{(i)} & a_{12}^{(i)} & \cdots & a_{1P^{-1}}^{(i)} & a_{1.P}^{(i)} \\ 1 & o & \cdots & o & o \\ o & 1 & \cdots & \vdots & \vdots \\ \vdots & \ddots & \vdots & & \\ o & \cdots & o & 1 & o \end{bmatrix} \quad , \qquad (1–11)$$

and that

$$a_{11}^{(i)}=\emptyset_1+\pi_1-2\sum_{J=1}^{P}\left(\pi_j\bar{y}_{t+i-j}\right)\bar{y}_{t+j-i}\,e^{-\bar{y}_{t+i-1}^2}$$

$$a_{i,k}^{(i)}=\emptyset_k+\pi_k\,e^{-\bar{y}_{t+i-1}^2} \ , \ k=2,3,\dots,P .$$

Note [22]:

The stability of non–zero single points of the (1-9) model can be easily tested using the local linear approximation method near the fixed points as follows:

let X_t be close to the single non–zero point $\in$, which can be found in the relationship $X_t=\in+\in_t$, where $\in_t$ is too small.

Substituting for $\in+\in_{t-i}\longrightarrow X_{t-i}$ in the form (1-9), we can obtain the following:

$$\in_t=h_1\ \in_{t-1}+h_2\ \in_{t-2}+\cdots+h_P\ \in_{t-P} \ , \qquad (1–12)$$

where

$$h_1 = \frac{\pi_1 + \emptyset_1 \sum_{j=1}^{P} \pi_j - \pi_1 \sum_{j=1}^{P} \emptyset_j}{\sum_{j=1}^{P} \pi_j} + 2\left(1 - \sum_{j=1}^{P} \emptyset_j\right) L_n \left(\frac{1 - \sum_{j=1}^{P} \emptyset_j}{\sum_{j=1}^{P} \pi_j}\right) , \quad (1\text{– }13)$$

$$h_j = \frac{\pi_1 + \emptyset_1 \sum_{j=1}^{P} \pi_j - \pi_1 \sum_{j=1}^{P} \emptyset_j}{\sum_{j=1}^{P} \pi_j} \; ; i = 2,3, \dots , P \quad . \quad (1\text{– }14)$$

The necessary and sufficient condition $\in_t$ to approach zero is that the roots of the characteristic equation of equation (1–12) lie inside the unit circle.

Application Side:

In this research, we dealt with the data on one of the common diseases in the Middle East in general and in Iraq in particular. This disease is a seasonal allergy most prevalent during the spring.

Developments in the field of medicine has made it possible to prevent, combat, and even eliminate many diseases through the application of modern scientific methods in detailed studies of various types of infectious and non-communicable diseases by searching for their causes and methods to identify the methods of prevention, control, and prevention of its spread [15].

The series was modelled with linear models including ARMA, AR, MA, and SARIMA using the ready-made analogue program based on least square error MSE, we obtain the following combined seasonal model:

SARIMA (1, 0, 0) , (2, 0, 0)12 y_t,

where $y_t = l_n(X_t)$, $\{X_t\}$ is the original series. The autoregressive coefficients for the non-seasonal part are as follows:

$\emptyset_1 = 0.52273$ $\emptyset_2 = 0.213371$

Std . Err 0.05265 0.04477.

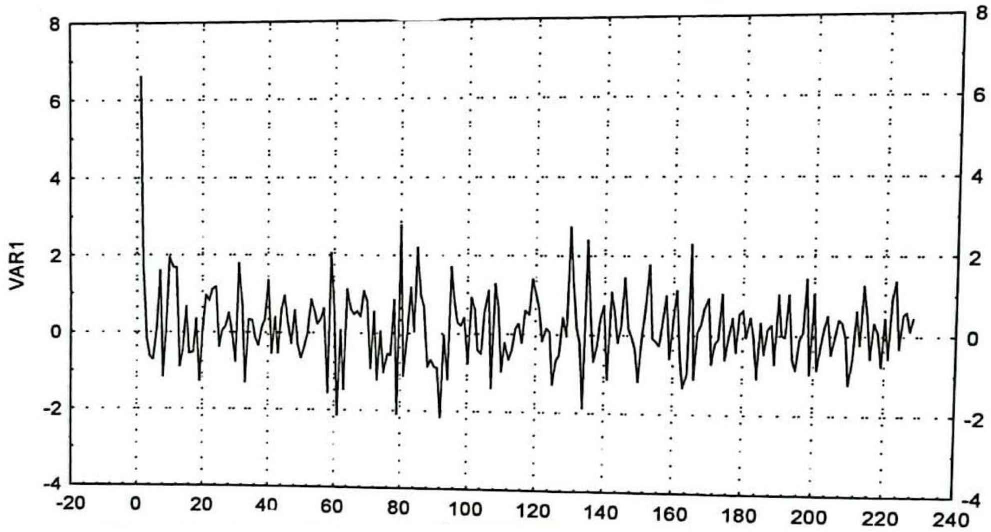
The autoregressive coefficients for the seasonal portion are as follows:

$\emptyset_1 = 0.42233$ $\emptyset_2 = 0.27893$ $\emptyset_3 = 0.12218$

Std. Err 0.6164 0.06054 0.05861.

Plot of variable: VAR1

In(X): ARIMA (1,0,0) (2,0,0) residuals:



Case Numbers

Figure (1–1) Plot of the seasonal model (1,0,0) (2,0,0) using the logarithm

Autocorrelation Function

VAR1: In(X); ARIMA (1,0,0) (2,0,0) residuals
(standard errors are white – noise estimates)

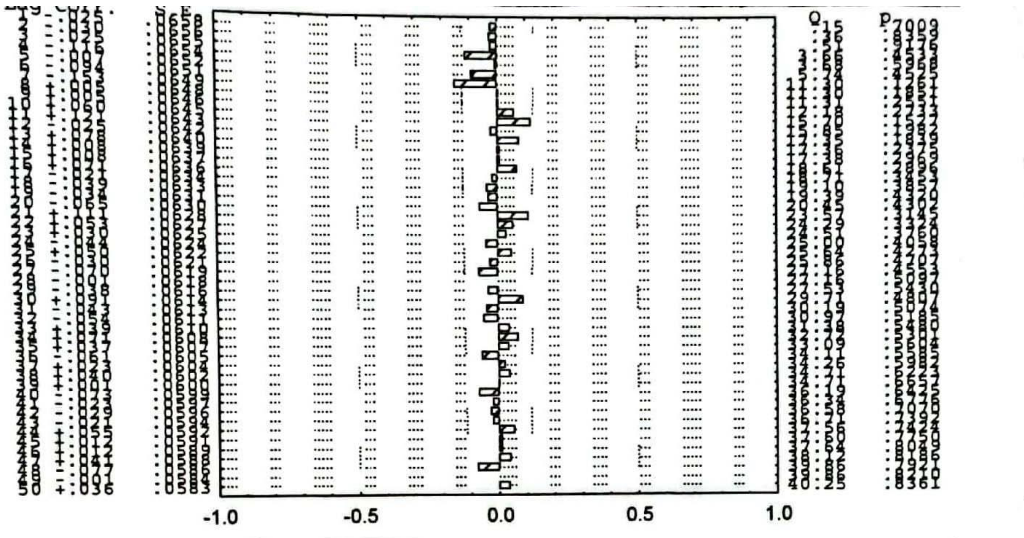


Figure (1–2) Plot of the seasonal autocorrelation function (1,0,0) (2,0,0) using the logarithm

Normal probability plot VAR1
$In(X)$; ARIMA (1,0,0) (2,0,0) residuals value

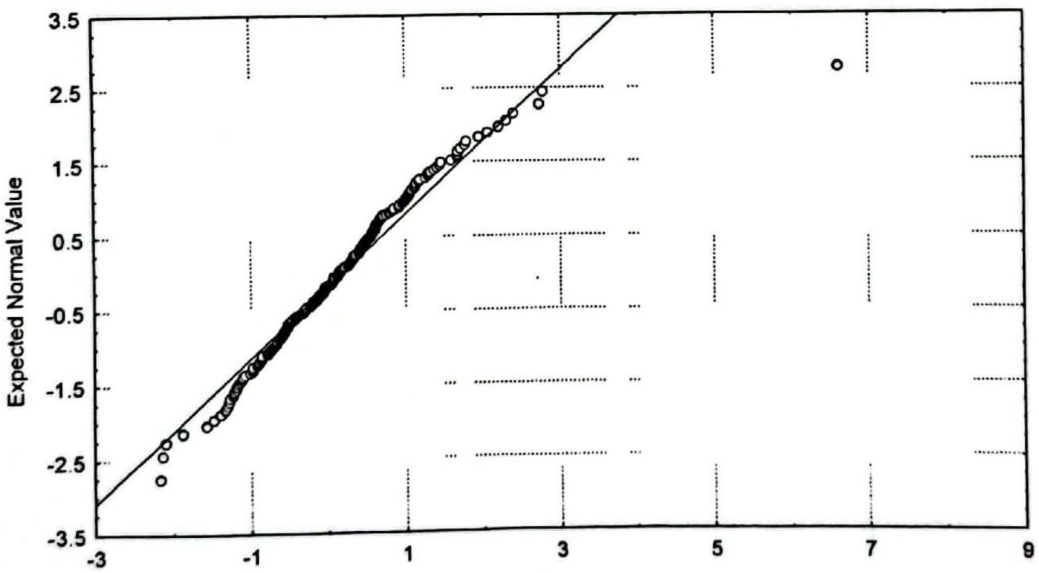


Figure (1–3) Normal probability plot of the seasonal model (1,0,0) (2,0,0) using the logarithm

Figure (1–1) shows the plot of a seasonal multiplicative model using the logarithm, while Figure (1–2) shows a drawing of the autocorrelation function (1,0,0) (2,0,0) of the seasonal multiplicative model using the logarithm. the figures show that the autocorrelation coefficients are within range (∓1.85 ES $P_k^1(2)$) and have a probability of 93%. The models are not correlated from Figure (1–3) and the residuals follow a normal distribution, thereby indicating the suitability of the model and that it is the best model among the models.

When studying the non-linear time series for the monthly incidence of seasonal allergies, it is necessary to study the stability of several non-linear models for this series. Several models were used, including the exponential autoregressive model and the polynomial model.

2.6 Stability of the second-order exponential autoregressive model [23]

$X_t = \left(\emptyset_1 + \pi_1 e^{-Y_{t-1}^2}\right)X_{t-1} + \left(\emptyset_2 + \pi_2 e^{-Y_{t-1}^2}\right)X_{t-2} + Z_t$

$$X_t = \left(\emptyset_1 + 0.02\, e^{-9_{t-1}^2}\right)X_{t-1} + \left(\emptyset_2 + 0.03\, e^{-20_{t-1}^2}\right)X_{t-2} + Z_t$$

Using the program Matlab, the values of $\emptyset_1$ and $\emptyset_2$ were obtained, and the model became as follows:
$X_t = \left(0.678 + 0.02\, e^{-9_{t-1}^2}\right)X_{t-1} + \left(0.0317 + 0.03\, e^{-20_{t-1}^2}\right)X_{t-2} + Z_t$.
For a stabilization study, the model has to find the values λ_1 , λ_2.
First: when $X_{t-1}^2 \longrightarrow 1$ then $e^{X_{t-1}^2} \longrightarrow o$,
hence, the equation becomes
$\lambda^2 - 0.639\,\lambda - 0.03 = 0$,
$\lambda_1 = -0.0412 \quad \lambda_2 = 0.6702$.

Second:
When $e^{X_{t-1}^2} \longrightarrow 0$ then $X_{t-1}^2 \longrightarrow \infty$.
$X_t = \left(0.678 + 0.02\, e^{-9_{t-1}^2}\right)X_{t-1} + \left(0.0317 + 0.03\, e^{-20_{t-1}^2}\right)X_{t-2} + Z_t$.
Using the characteristic equation, the equation becomes

$$\lambda^2 - 0.678 \quad \lambda - 0.0317 = 0$$

$\lambda_1 = -0.0156$
$\lambda_2 = 0.7147$.
To clarify whether the singularity fulfils the condition of stability, we use the aforementioned note because the following relationship must be fulfilled:
$\in_t = h_1 \in_{t-1} + h_2 \in_{t-2}$,
the characteristic equation is
$\lambda^2 - h_1\,\lambda - h_2 = 0$.
To clarify whether a single point fulfils the condition of stability, we use the aforementioned note because the following relationship must be fulfilled:
$\in_t = h_1 \in_{t-1} + h_2 \in_{t-2}$,
which has the following characteristic equation:
$\lambda^2 - h_1\,\lambda - h_2 = 0$.
Substituting the values of equations π_i and $\emptyset_i$ and their equivalents in equations (1–13) and (1–14), we obtain the following:
$h_1 = 0.2501$,
$h_2 = 0.018$.
We find the roots of the following equation:
$\lambda^2 - h_1\,\lambda - h_2 = 0$

$$\lambda_1 = \underline{0}.0355$$

$\lambda_2 = 0.3066$.
Thus, we conclude that the root is less than one, and thus, the second-order exponential model is stable.

2.7 Stability of the polynomial model:

The polynomial autoregressive model is known by the following formula:
$X_t = \emptyset_1 X_{t-1} + \cdots + \emptyset_P X_{t-P} + Q(X_{t-1}, \ldots, X_{t-P}) + Z_t$,
where $Q(X_{t-1}, \ldots, X_{t-P})$ is a polynomial $LX_{t-1} \ldots X_{t-P}$ representing the variables.

Using Matlab, we can obtain the following model:

$X_t = 0.768\ X_{t-1} - 0.000002\ X_{t-1}^3 + 0.008577\ X_{t-2} + Z_t$

or

$X_t = (0.768\ - 0.000002\ X_{t-1}^2)X_{t-1} + 0.008577\ X_{t-2} + Z_t$.

Suppose that

$(0.768 - 0.000002X_{t-1}^2) = (0.768 - 0.000001 - 0.000002\ e^{-X_{t-1}^2})$.

It becomes the model

$X_t = (0.768\ - 0.000002 - 0.000002\ e^{-X_{t-1}^2})X_{t-1} + 0.008577\ X_{t-2} + Z_t$.

It is an exponential model.

First case

When

$e^{-X_{t-1}^2} \longrightarrow 1$ then $X_{t-1}^2 \longrightarrow 0$

$X_t = (0.768\ )X_{t-1} + 0.008577\ X_{t-2} + Z_t$

$\lambda^2 - 0.768\ \lambda - 0.008577 = 0$

$$\lambda_1 = -0.0871$$

$\lambda_2 = 0.8772.$

Second case

When

$e^{-X_{t-1}^2} \longrightarrow 0$ then $X_{t-1}^2 \longrightarrow \infty$

The values of λ_1, λ_2 are similar to the first case.

When $X_{t-1}^2 \longrightarrow 0$ because the value of $\pi_1 = 0.000002$ is too small, which is

$$\lambda_1 = 0.7681$$

$\lambda_2 = 0.768$.

From the first and second cases, we conclude that the values of λ_1, λ_2 are less than one, and thus, the polynomial is stable.

The following table represents the models we obtained and their statistical characteristics (mean square error, standard deviation, and NBIC).

Table (1-1) Comparison table of the proposed models

Model / STAT	AR(6)	MA(7)	ARMA(7,7)	SAR1MA	EXP(AR(P))	Polynomial model
M.E.E	1308	13359	11632	1.0234	29361	29283
NM1C	4.8263	4.8543	4.8816	0.1300	1.4650	3.7506
S.D	115.718	117.385	103.765	0.8723	6.3010	6.6010

From Table (1-1) and from observing the autocorrelation function and drawing the natural probability of the rest of the above models, we note the superiority of the seasonal multiplicative model over the rest of the models because it has the lowest square error rate and the lowest NB1C, making it the best model to represent the series.

2.8 Stability of the proposed model

The proposed equation is as follows:

SARIMA (1,0,0)(2,0,0)12y_t,

whereas:

$\emptyset_1$ $\emptyset_2$ $\emptyset_{S1}$ $\emptyset_{S2}$ $\emptyset_{S3}$

$$0.62282 + 0.13372 + 0.42244 + 0.17882 + 0.22316$$

Sta Err 0.06246 0.05577 0.6164 0.06086 0.06861.
The model is stable if the roots of the characteristic equation for the non-seasonal and seasonal parts part lie within the unit circle.
The characteristic equation for the non-seasonal penalty is as follows:
$\lambda^2 - 0.6326\ \lambda - 0.133 = 0$

$$\lambda_1 = -0.1510$$

$\lambda_2 = 0.7856$.
The characteristic equation for the seasonal part is as follows:
$\lambda^2 - 0.2\,\lambda^2 - 0.18\ \lambda - 0.2 = 0$

$$\lambda_1 = 0.735$$

$$\lambda_2 = -0.156 + i0.3288$$

$$\lambda_3 = -0.156 - i0.3288$$

$$\theta_2 = tan^{-1}\frac{y}{X} = -57.582$$

$$r_2 = 0.412$$

$$\theta_3 = tan^{-1}\frac{y}{X} = 57.582$$

$r_3 = 0.4128$.
We note that $r^1 = 0.745, r_2 = 0.412$ and $r_3 = 0.4128$ are all less than one in absolute value along with the case with the roots of the characteristic equation for the non–seasonal part, thereby indicating that the proposed model is stable.
The following table represents the predicted values from the aforementioned model: Table (2–1)

Double seasonal pattern	Exponential form	Polynomial model	Real values
92	482	522	75
88	126	216	88
107	26	68	92
30	17	18	22
22	72	62	10
12	1	10	13
13	14	12	14
6	70	60	10
9	171	155	34
10	426	377	96
18	518	488	66

The above table shows that the predicted values for 12 months from the seasonal multiplicative model are the closest to the true values, which indicates the efficiency of the proposed model and represents the appropriate model for seasonal allergy data. Figure (1–4) represents the original series and the predicted values from the proposed model.
Forecasts; Model: (1,0,0) (2,0,0) seasonal lag: 12 Input: VAR1: In(X)
Start of origin: 1 End of origin: 228

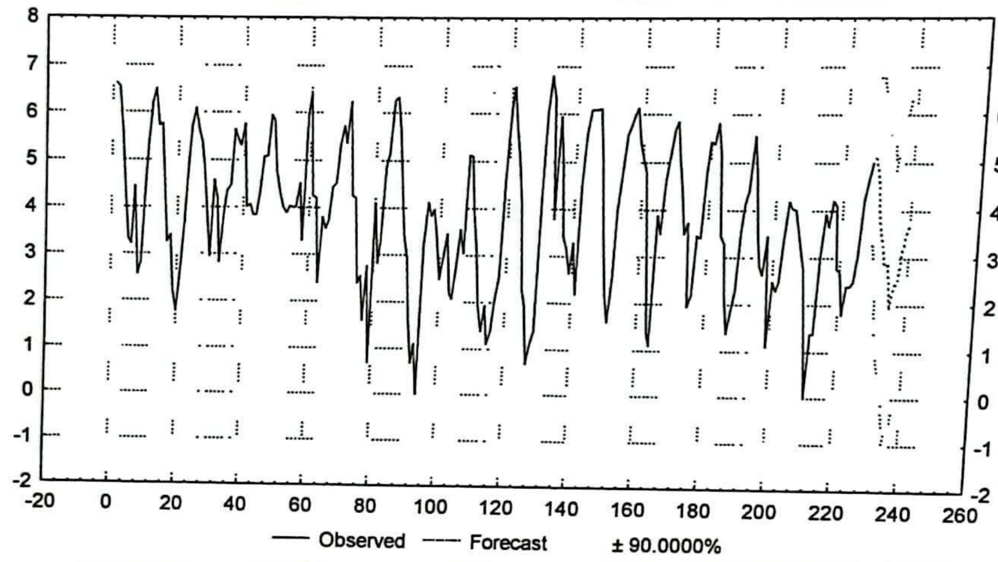


Figure (1–4) represents the prediction plot for the series of the model (1,0,0) (2,0,0) using the logarithm

Conclusions

1. The time series of seasonal allergy disease in Iraq is unstable, and no clear trend for the series was observed.
2. The seasonal multiplier model of rank (1,0,0) (2, 0,0) is the appropriate mode for the time series of seasonal allergy disease. It gave good estimates close to the actual values based on the comparison with the terms of prediction and choosing the residuals with the rest of the proposed models.
3. Linear models are stable if λ_1 and λ_2 are less than one, and in the case of non-linear models, stability is difficult to obtain.

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