Study of the Seismic Resistance of the Underground Spherical Shell

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ABSTRACT

The article presents the calculation of the subsurface structures of the spherical shell type using the seismodynamic theory. The results of the calculation were presented and compared with the results obtained in the classical form

Keywords: construction, dynamic theory of seismic resistance, interaction coefficients, shell, oscillations.

When applying the dynamic theory of seismic resistance, the interaction is taken into account by a system of coefficients that reflect the resistance of soils to the movement of a structure under the action of various loads. The repulse of the medium is reduced to an additional interaction force due to the relative displacement of the shell surface.

$$K_\tau(U-U_0)Rd\alpha_1 d\alpha_2,$$

$$K_n(W-W_o)Rd\alpha_1 d\alpha_2,$$

$$K_r(V-V_o)Rd\alpha_1 d\alpha_2.$$

где U,V,W - shell movement

U_0, V_0, W_0 - ground movements

K_τ, K_n, K_r - coefficients of interaction between the shell and the soil in the directions of the axes.

Seismic accelerations of the soil cause in the underground shell a time-varying inertial load from the attached mass of the soil, which oscillates together with the shell as a whole. Therefore, the components of the resulting inertia force corresponding to the relative displacements of the oscillating structure-soil system will be

$$m\frac{\partial^2 U}{\partial t^2}Rd\alpha_1 d\alpha_2, \qquad m\frac{\partial^2 V}{\partial t^2}Rd\alpha_1 d\alpha_2,$$

$$m\frac{\partial^2 W}{\partial t^2}Rd\alpha_1 d\alpha_2$$

Deriving the equations of oscillations of a spherical shell in the same way. As in the general theory of shells (2), external forces are replaced by interaction forces

$$\frac{2Eh}{(1-\nu^2)}\frac{1}{R^2}\left\{\frac{\partial^2 U}{\partial\alpha^2}-\frac{\partial W}{\partial\alpha}+\nu\left(-\frac{1}{\sin\alpha}U+ctq\alpha\frac{\partial U}{\partial\alpha}-\frac{\partial W}{\partial\alpha}\right)+ctq\alpha\left[(1-\nu)\left(\frac{\partial U}{\partial\alpha}-ctq\alpha U\right)\right]\right\}$$
$$-K_\tau(U-U_0)=2\rho h\frac{\partial^2 U}{\partial t^2}$$

$$\frac{2Eh}{(1-\nu^2)}\frac{1}{R^2}\left(\frac{\partial U}{\partial\alpha}-2W+ctq\alpha U\right)(1+\nu)-K_n(W-W_0)=2\rho h\frac{\partial^2 W}{\partial t^2}$$

(1)

Here, the third equation of the system is neglected, since for thin shells the cutting force is several orders of magnitude smaller than the others. This term is also neglected in the two remaining equations.

Let us introduce the following dimensionless variables (the prime means dimensionless variable)

$$t' = \frac{at}{R}, \qquad \frac{\partial}{\partial t} = \frac{a}{R}\frac{\partial}{dt},$$

$$\frac{\partial^2}{\partial t^2} = \frac{a^2}{R^2}\frac{\partial^2}{\partial t^2}, \qquad U' = \frac{1}{R}U, \qquad W' = \frac{1}{R}W,$$

$$K_\tau^{/} = \frac{K_\tau R^2}{2Eh}, \qquad K_n^{/} = \frac{K_n R^2}{2Eh}$$

$$\rho^{/} = \frac{\rho_{c\phi}}{\rho_{cp}}.$$

где $a = \sqrt{\frac{E}{\rho}}$ velocity of wave propagation in the rod. R is the radius of the middle surface of the shell.

To solve the system of equations (1), we use the method of incomplete separation of variables. Displacement components U and W, as well as stresses, are presented in the form of series in Legendre polynomials (in terms of the angular coordinate).

$$U = \sin\theta \sum_{n=0}^{\infty} a_n(\varpi) P_n^{/}(\cos\theta) e^{-i\varpi t}$$

$$\overline{\sigma_{rr}} = \sum_{n=0}^{\infty} \sigma_{rr}^{(n)} P_n(\cos\theta) e^{-i\varpi t}$$

$$W = \sum_{n=1}^{\infty} b_n(\varpi) P_n(\cos\theta) e^{-i\varpi t}$$

$$\overline{\sigma_{r\theta}} = -\sin\theta \sum_{n=1}^{\infty} \sigma_{r\theta}^{(n)} P_n^{/}(\cos\theta) e^{-i\varpi t} \qquad (2)$$

Also, to simplify the problem in spherical coordinates, we use the exponential Fourier transform in t (expansion in Legendre functions).

Then:

$$U_0 = U_\theta = \frac{\partial \varphi_0}{\partial \theta}\Big|_{r=1} = (2n+1) i^n j_n(\varpi) P_n^{/}(\cos\theta) e^{-i\varpi t}$$

$$W_0 = U_r = \frac{\partial \varphi_0}{\partial r}\Big|_{r=1} = (2n+1) i^n \left[\frac{n}{\varpi} j_n(\varpi) - j_{n+1}(\varpi)\right] P_n(\cos\theta) e^{-i\varpi t}$$

(3)

так как

$$\varphi_0 = \frac{i}{\alpha} A_0 e^{-i\varpi t} \sum_{n=0}^{\infty} (2n+1) i^n j_n(\varpi r) P_n(\cos\theta)$$

(4)

Здесь ϖ - conversion parameter

j_n - spherical Bessel functions of the nth order

P_n - Legendre polynomials

Expanding (1) in the form of series (2), taking into account (3), we obtain

$$(\varpi^2 + \alpha_{11})\bar{a}_n(\varpi) + \alpha_{12}\bar{b}_n(\varpi) = K_\tau (2n+1) i^n j_n(\varpi)$$

$$\alpha_{21}\bar{a}_n(\varpi) + (\varpi^2 + \alpha_{22})\bar{b}_n(\varpi) = K_n (2n+1) i^n \left(\frac{n}{\varpi} j_n(\varpi) - j_{n+1}(\varpi)\right)$$

(5)

As can be seen from the equations of soil displacement, they are determined by the parameters of the incident wave. The coefficients an, bn depend on the elastic characteristics of the medium and the shell and the geometric dimensions of the latter.

Formula (2) allows finding Fourier images of all parameters of the stress-strain state. For a complete solution of the problem, we sum the series (2). In this case, the question arises about the rate of convergence of these series. To improve the rate of convergence of these series, he resorts to various summation methods. In this case, the method of summation by a number of Cesaro means (1) was used. In this case, a formula was used that connects the subsequent and previous arithmetic averages

j_n и j_{n-1} $\qquad j_n = (nj_{n-1} + S_n) / (n+1)$

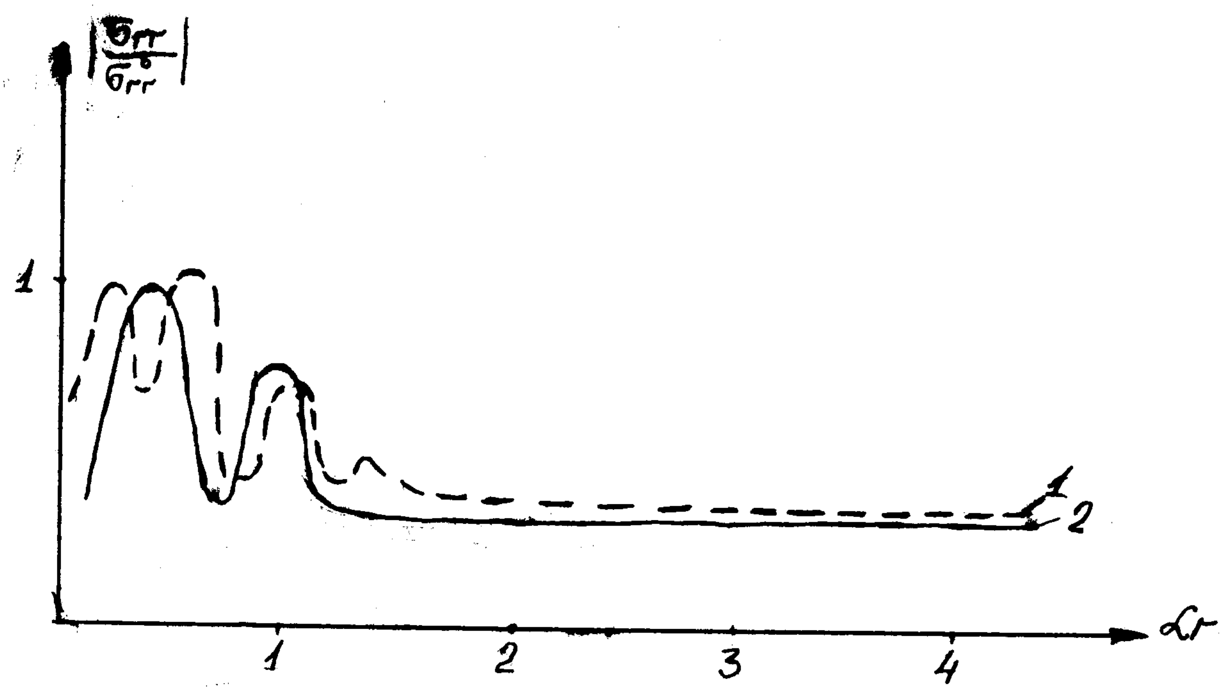
где S_n is the sequence of partial sums of the series

j_n – sequence of arithmetic averages.

Comparison of ordinary sums with Cesaro sums showed the advantages of Cesaro's method (2). In (2) it is shown that j n obtained using the Cesaro method for n=6:7 quite satisfactorily describes the function. Starting from these numbers (n=6, n=7), the subsequent values of the function differ little from the previous ones. The paper also found the number n, starting from which the value of the function changes little. This number for the above-described variants of the characteristics of the medium and the shell ranges from 6 to 8. Therefore, the number of terms in the series is limited to these numbers in the work.

As a result of the calculations, the dependences of the ratios of stress modules on α r were obtained, where α is the wave number, R is the radius. The figure shows this dependence for various ratios of velocities a/c1 and densities ρ/ at θ = 0 (at the frontal point). It can be seen that there are some resonant frequencies at which there is a significant increase in the voltage modulus.

In the case of a steel shell in soil 1 (a/c1=2.8, ρ'=3.4) and a reinforced concrete shell in soil 2 (a/c1=1.2, ρ'=1.6), these stress peaks belong to the region αr<1. It can be seen that in the first case, the stresses reach their maximum at αr=0.5, and in the second case, at αr=0.4. For values of αr>1, such significant stress surges are not observed. Both of these cases are in good agreement with the previously obtained solutions to the problems of diffraction of a nonstationary wave by spherical elastic shells.



References:

1. Рашидов Г.Р. Динамическая теория сложных систем подземных сооружений. Ташкент, Фан. 1973.
2. Власов В.З. Общая теория оболочек и её приложение в технике М.Гостехиздат, 1948.
3. Горшков А.Г., Тарлаковский Д.В. Нестационарная аэрогидроупругость тел сферической формы. М. Наука, 1990.
4. Мубораков Я.Н. Сейсмодинамика подземных сооружений типа оболочек. Ташкент, Фан. 1987.
5. Алексич Г. Проблемы сходимости ортогональных рядов – М.: ИЛ, 1963