Main Experience Solutions of the Problem of Free Vibration of a Viscoelastic Rectangular Plate Including and Excluding Cut-Out

Asatullaev A.A. TASI
Jamolidinov A.N TASI

ABSTRACT

The paper shows the place and importance of the computer in the calculation of problems in mechanics and mathematical modeling. Basic information on the use of differential and integral models of the relationship between stress and strain in problems of the theory of viscoelasticity is given. Using the example of free vibrations of viscoelastic plates with and without taking into account the notch, the main stages of solving this problem are given

Keywords: Viscoelasticity, mathematical model, stress, strain, plate with hole, integro-differential equation, Bubnov-Galerkin method

In the modern scientific world, it is difficult to imagine a researcher who would not use a personal computer. It can be argued that many professions have arisen precisely thanks to the computer. Computers are used to carry out the most complex calculations, the modern researcher is highly dependent on computers, which is not so surprising when you consider when and in what cases they are used. If we consider the profession of an engineer - a person who is engaged in various research, design and conduct a huge amount of calculations - then the computer is used primarily to facilitate work. And also for tasks that are practically impossible without a computer, for example, for very complex calculations, which, even when using the capabilities of computer technology, last for hours and days, and without its use, they would either be solved for many months, or could not be implemented in principle. .

An engineer needs a computer to simulate complex physical and other processes using various applications, in order to know in advance what will happen under certain conditions with the object in question. This may be a matter of convenience and ease of work, for example, the standard problem of the strength of materials is easier to solve in the program than to put real experience, as well as experiments.

It should be noted that personal computers are used in various fields of human activity. Historically, their first use was in scientific and technical calculations and the study of natural phenomena. It is difficult to imagine progress in many branches of science and technology without the widespread use of the computer. One of the most important stages in the study of processes occurring in nature and for their cognition, methods of mathematical modeling are used, which allow studying the various properties of an object. In most cases, in mathematical modeling of problems in mechanics, in particular, the above problem, we obtain integro-differential equations that are solved on the basis of the method of computational mathematics.

The process of solving a problem in various fields, in particular, in the mechanics of a deformable solid body on a computer, consists of the following stages: problem statement; construction of mathematical models; application of numerical methods for solving the obtained equations or systems of equations in mathematical modeling; development of a computational algorithm for solving the problem; compiling a program in one of the algorithmic languages; making calculations. those. carrying out a series of computational experiments and finally, analysis of the results with the corresponding conclusions.

When constructing mathematical models of phenomena in nature or processes in a particular area of scientific research, it is necessary to use the experience and results obtained at the initial stage. This process of consistent development and refinement of the model occurs repeatedly. Let us consider the construction of mathematical models of viscoelastic rectangular plates with and without cuts in a linear formulation.

Modeling of viscoelastic materials is based on the linear theory of viscoelasticity of materials with "memory". The linear theory of viscoelasticity was formed on the basis of ideas about such properties of solids as plasticity, creep, and relaxation [1–3]. The literature describes approaches to assessing the damping properties of materials used for these purposes, and their various models [2–4].

To describe the processes of deformation of viscoelastic materials, various differential models of the hereditary theory of viscoelasticity are used, which establish relationships between stresses and deformations of a body: the Maxwell, Voigt and Voigt-Kelvin model [5, 6].

These models can only be used to describe individual phenomena in viscoelastic media. For example, the Maxwell model describes the stress relaxation process, while the Voigt model describes the creep process. The disadvantage of these models is that they do not take into account the time factor associated with strain creep and stress relaxation. The Voigt model in systems with a finite number of degrees of freedom greater than one leads to incorrect results, since for most materials the internal friction is actually independent or at least weakly dependent on the oscillation velocity over a fairly wide frequency range. Simple differential models have the disadvantage of poor approximation of the relaxation and creep processes at the initial stage of deformation, which turns out to be significant in problems of dynamic type.

Integral models allow you to create a mathematical model of the theory of viscoelasticity in a general setting and characterize the law of change between stress and strain, it best represents internal friction and the time factor.

In the general case, the optimal one should be understood as a mathematical model that has the algorithmic efficiency of an adequate description of the real process, the minimum complexity of implementing algorithms, the discretization of the continuum problem, and the solution of the resulting equations described by a discrete model on a computer.

Since the boundary and initial-boundary dynamic problems of the hereditary theory are described by integro-differential equations in partial derivatives, the solution of which is associated with significant mathematical difficulties, the natural way to solve such problems is to discretize in space variables at each moment of time and obtain ordinary integro-differential equations with initial conditions. This is achieved by choosing a finite number of coordinate functions according to the Bubnov-Galerkin variational method [7]. There is no universal approach to constructing a discrete model, and difficulties are overcome in solving each specific problem. The main problem is how to discretize with respect to spatial variables in order to most effectively solve the integro-differential equations of the discrete model.

The main final step in solving the problems posed is the numerical solution, in the case of a linear formulation, of a system of ordinary integro-differential equations of the form

$$A\ddot{\mathrm{x}} + \mathrm{B}\dot{\mathrm{x}} + C\left(1 - R^*\right)\mathrm{x} = \mathrm{F}(t), \qquad (1)$$
$$\boldsymbol{x}(0) = \boldsymbol{x}_0, \ \dot{\boldsymbol{x}}(0) = \tilde{\boldsymbol{x}}_0.$$

describing discrete models, where A, B and C are known square matrices; - vector function; R* is an integral operator with a relaxation kernel R(t) of the form:

$$R * \boldsymbol{x} = \int_0^t R(t-\tau)\boldsymbol{x}(\tau)d\tau.$$

Equations (1) are integro-differential equations of the Volterra type.

The presence of a notch in the plate is taken into account by a method similar to that used in monograph [8], using the Heaviside unit function and the filtering properties of impulsive functions.

The exact solution of equations (1) in the presence of weakly singular Koltunov-Rzhanitsyn kernels is associated with significant mathematical difficulties. Therefore, approximate-analytical and numerical methods [9] for solving equations (1) are natural methods.

In further studies, we will consider numerical solutions to the problem of free oscillation of viscoelastic plates with and without taking into account the notch according to the method described above and analyze the results obtained.

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