Methodology Of Teaching The Topic Of Rational Inequalities In School Mathematics Courses

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ABSTRACT

The main goal of this work is to develop effective methods for teaching the topic of rational inequalities in school mathematics courses and to demonstrate their practical application. Rational inequalities are an important part of the algebra course, and mastering them helps develop students' mathematical thinking abilities. The article discusses the definition, types, and basic methods of solving rational inequalities, as well as the difficulties encountered in solving them and ways to overcome these difficulties.

Keywords: rational inequalities, teaching methodology, algebra, mathematical thinking, interval method.

Introduction

In teaching mathematics at school, in addition to general educational goals, there are specific goals related to the unique characteristics of the subject. One of these goals is the formation and development of mathematical thinking. This helps identify and effectively develop students' mathematical abilities, prepares them for creative activities, and deepens their life perspectives.

In general, children's intellectual development can be accelerated in three directions: through observation, verbal intelligence, and actions.

Strengthening knowledge cannot be achieved without developing thinking abilities, which is one of the main tasks of modern school education.

I believe it is appropriate to focus on two main problems of mathematics didactics: modernizing the content of school mathematics education and improving the structure of the course.

The rapid growth of scientific knowledge, the limited duration of school education, and the impossibility of reducing the volume of foundational subjects studied in school complicate the process of modernizing school education. Therefore, reforms must be prepared carefully, over a longer period, and on a solid scientific basis.

There are successful experiences in modernizing the primary school curriculum and studying the basics of algebra, which provide a strong foundation for algebra in grades I-V. This allows for faster learning of subjects and the transfer of certain topics from higher grades to middle grades, as well as the inclusion of higher mathematics elements in the upper-grade curriculum. Thus, improving the course system is possible independently of modernization, i.e., during the period between reforms.

We do not address these issues in detail because we work in a narrower direction and, at this stage, analyze the methodology of teaching the topic "Rational Inequalities" in the general education course.

Let us consider an inequality of the form $f(x) > g(x)$. The domain of this inequality is understood as the set of values of X for which both the expression $f(x)$ and the expression $g(x)$ are defined. In other words, the domain of the inequality $f(x) > g(x)$ is the intersection of the domains of the expressions $f(x)$ and $g(x)$.

A particular solution of the inequality $f(x) > g(x)$ is any value of X that satisfies the inequality. The set of all particular solutions is called the solution of the inequality.

Given two inequalities $f_1(x) > g_1(x)$ and $f_2(x) > g_2(x)$ if the solutions of the first inequality are also solutions of the second (and vice versa), then these inequalities are said to be *equivalent*.

It is known that adding an expression defined in the domain of the inequality to both sides of the inequality preserves the domain, i.e., $f(x) > g(x)$ and $f(x) + \varphi(x) > g(x) + \varphi(x)$ are equivalent.

Similarly, multiplying or dividing both sides of the inequality by an expression that is always positive in the domain of the inequality preserves the inequality's sign, i.e., x $\varphi(x)$ and $f(x)\varphi(x) > g(x)\varphi(x)$ or $\frac{f(x)}{\varphi(x)} > \frac{g(x)}{\varphi(x)}$ are equivalent.

From this, it follows that multiplying or dividing both sides of an inequality by the same positive number does not change the inequality's sign.

If we multiply or divide both sides of an inequality by any expression $\varphi(x)$ that takes only negative values for all acceptable values of X, then the inequality reverses its sign, meaning for $\varphi(x)$<0, $f(x) > g(x)$, and $f(x)\varphi(x) < g(x)\varphi(x)$ (or $\frac{f(x)}{\varphi(x)} < \frac{g(x)}{\varphi(x)}$ are equivalent).

From this, it follows that if we multiply or divide both sides of an inequality by the same negative number, the inequality sign reverses.

If an inequality is given for some $f(x) > g(x)$ (when $(x) > 0$), then both sides of it can be raised to the same natural power, i.e., $(f(x))^n > (g(x))^n$, and the inequality will preserve its sign.

We use the interval method, which is the most convenient way to solve rational inequalities. For this, we first transform the given inequality into the form $F(x) \geq 0$ or $(F(x) \leq 0)$, and then factor $f(x)$ into linear factors. Each resulting factor expression should be brought to the form $(ax \pm b)$ or $(ax^2 + bx + c)$. If $(b - ax)$ occurs, we take the minus sign out of the parentheses and multiply both sides of the inequality by "-1". After that, we find the values of X that make each parenthesis zero and construct the intervals.

If the rational expression is in the form $\frac{(P(x))}{Q(x)}$, then we use the following theorem and proceed to solve it.

Theorem: If $\frac{P(x)}{Q(x)} > 0$ (or $\frac{P(x)}{Q(x)} < 0$) holds, then $P(x)Q(x) > 0$ (or $P(x)Q(x) < 0$) will be true.

Now, let's look at a few examples.

Example. Let's solve the inequality $(2x-1)(x+3)(\frac{1}{2}x - 5) > 0$.

Solution: For this, we find the values of $x = \frac{1}{2}, x = -3, x = 10$, as these values make the parentheses above equal to zero.

Now, we construct the following intervals:

$$(-\infty; -3), \left(-3; \frac{1}{2}\right), \left(\frac{1}{2}; 10\right), (10; \infty)$$

If the numbers that make a polynomial $F(x)$ equal to zero are $k_1, k_2, \ldots, k_n$, and the largest of these numbers is k_n, then the polynomial $F(x)$ will always be positive in the interval $(k_n; +\infty)$, i.e., $F(x) > 0$.

Based on this reasoning, we place the numbers on the number line and consider that the expression on the left side of the inequality is positive in the interval $(10; +\infty)$.

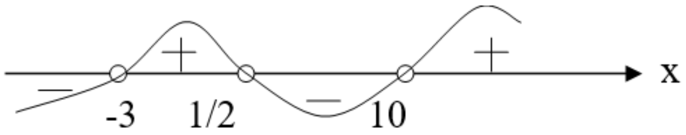

Figure 1

On the number line, we always draw the line from top to bottom, from right to left. (Figure 1) Thus, the expression on the left side of the inequality is positive in the interval $(10; +\infty)$, negative in the interval $(1/2; 10)$, positive in the interval $(-3; 1/2)$, and finally, negative in the interval $(-\infty; -3)$.

Since the given inequality is greater than zero, we take the intervals with a positive sign, i.e., $(-3; 1/2)$ and $(10; +\infty)$. The solutions to the inequality are the numbers in these intervals.

The answer can be written in two ways:

1. $(-3; \frac{1}{2}) \cup (10; +\infty)$;

2. $-3 < x < \frac{1}{2}; \quad x > 10$

The solutions can be represented on the number line as follows (Figure 2).

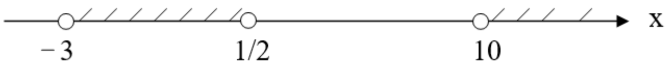

Figure 2

Note: The endpoints are not included in the interval, so we use open circles for them, and we fill in the circles for the points that are included in the interval.

Example 2: Let's solve the inequality $(\frac{2}{5} - x)(3x - 2)(x - 4)(x + 1) \geq 0$.

Solution: The difference between this example and the previous one is that the first parenthesis is in the form $(b - ax)$. As mentioned earlier, we take the minus sign out of the parenthesis and multiply both sides of the inequality by "-1", which reverses the inequality sign, resulting in:

$$(x - \frac{2}{5})(3x - 2)(x - 4)(x + 1) \leq 0$$

Now, we solve this inequality as shown in the previous example and obtain the following solutions:

$$\left[-1; \frac{2}{5}\right] \cup [\frac{2}{3}; 4]$$

We represent these solutions on the number line as follows (Figure 3).

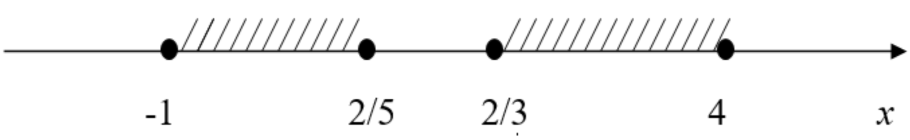

Figure 3

Example 3: Let's solve the inequality $\frac{x^2(x-2)^5(x+3)^7}{(x-4)^3} > 0$.

Solution: We write this inequality in the following form based on the theorem mentioned above:

$$x^2(x-2)^5(x+3)^7(x-4)^3 > 0$$

This is because this inequality is equivalent to the previous one.

Here, the degree of x is even, while the degrees of the other parentheses are odd. The points $x = 0, x = 2$ and $x = -3$ make the left side of the inequality zero, and at $x = 4$, there is a discontinuity. Therefore, $x = 2, x = -3$ and $x = 4$ are called simple points, while $x = 0$ is called a double point.

For this reason, to find the solutions, we represent the number line differently from the previous examples, as shown below (Figure 4):

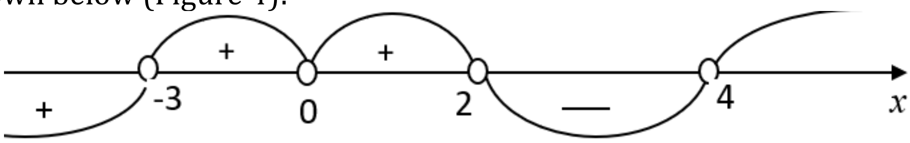

Figure 4

Thus, the solution to the given inequality is $(-3; 0) \cup (0; 2) \cup (4; +\infty)$. This solution is represented on the number line as follows (Figure 5):

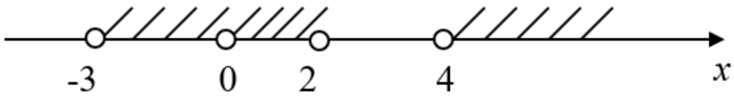

Figure 5

Example 4: Let's solve the inequality $2x^3 - 5x^2 + 2x \leq 0$.

Solution: We factor the expression on the left side of the inequality into linear factors: $x\,(2x^2 - 5x + 2) \leq 0$ or $2x(x-2)(x-\frac{1}{2}) \leq 0$

We find the solution as follows (Figure 6):

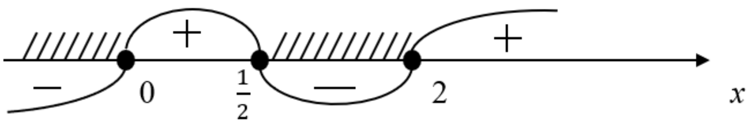

Figure 6

The values of x that satisfy the final inequality or the given inequality lie in the following intervals:

$$(-\infty; 0] \cup [\frac{1}{2}; 2]$$

Example 4: Let's solve the inequality $\frac{x^2-2x-15}{13x-x^2-40} \geq 0$.

Solution: Multiply both sides of the inequality by "-1" to obtain:

$\frac{x^2-2x-15}{x^2-13x+40} \leq 0$ or $\frac{(x-5)(x+3)}{(x-5)(x-8)} \leq 0$

We simplify the fraction on the left side by canceling $(x-5) \neq 0$, resulting in:

$$\begin{cases} \frac{x+3}{x-8} = 0 \\ x \neq 5 \end{cases}$$

using the interval method (Figure 7), we obtain the interval [−3;8).

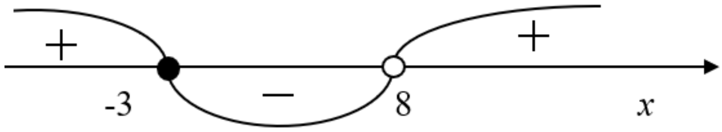

Figure 7

However, $x = 5$ is not in the domain of the given inequality. Therefore, the solution is: [-3;5) ∪ (5;8).

We represent this solution on the number line as follows (Figure 8):

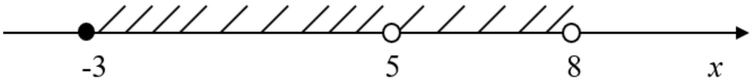

Figure 8

Problems for independent practice: solve the following inequalities::

1. $\frac{(x-3)(x+2)}{x^2-1} < 1$
2. $\frac{(x-1)^3(x+2)^4(x-3)^5(x+6)}{x^2(x-7)^3} \le 0$
3. $\frac{x^2-2x+7}{5x+9} < 0$
4. $\frac{x^3-x^2+x-1}{x+8} \le 0$
5. $\frac{x^4-2x^2-8}{x^2+x-1} < 0$

The topic of rational inequalities holds an important place in the school mathematics curriculum as it strengthens algebraic knowledge and develops students' logical thinking abilities. Teaching this topic using a differentiated approach, which takes into account the varying levels of students' knowledge, leads to effective results. The use of visual materials, graphical methods, and interactive software helps students better understand the topic.

Teachers can make the topic more engaging by incorporating practical examples and real-life situations. Organizing independent work and teaching students to solve complex problems step-by-step is of great importance.

The use of modern pedagogical technologies in teaching rational inequalities makes the learning process more effective. Assessing students' knowledge through various formats such as tests, projects, and practical exercises allows for a more accurate determination of their level of understanding. Teachers' creative approaches and encouragement of students contribute to their success in mastering the topic.

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