Application Of Functional Analysis Methods To Find Solutions Of Non-Linear Equations

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ABSTRACT

The different expressions of nonlinearity require a separate approach to the analysis of each nonlinear problem, the application of methods for finding its solution, and the practical interpretation of the obtained results.

Keywords: Quasi-linear, differential equation, boundary and initial conditions, Green's function, nonlinear integral equation, function space, compressive reflection principle, sufficiency condition.

Linear functional problems have been studied for a long time, and now it can be said that the theoretical and practical foundations of their analysis and finding solutions are almost completely created. Equations and inequalities involving an unknown function or its derivative are nonlinear in many cases, providing an opportunity to adequately represent the models of the process and objects being studied [1-8].

This paper investigates the solution of problems related to finding the sufficiency condition of applying the compressive reflection theorem to finding solutions of nonlinear functional equations in Banach spaces.

It is known that the norm is any given in the defined space

$\{x_1, x_2, \ldots, x_n, \ldots\} = \{x_i\}_{i=1}^{\infty} \subset X$ fundamental, that is $\lim\limits_{\substack{n\to\infty \\ m\to\infty}} \|x_n - x_m\|_X = 0$ there is a sequence that satisfies the condition when approaching an element, such a normalized space is called a fully normalized space or Banach space [9].

Consider the following linear ordinary differential equation

$$Lu = a_0(x)\frac{d^n u}{dx^n} + a_1(x)\frac{d^{n-1}u}{dx^{n-1}} + \ldots + a_{n-1}(x)\frac{du}{dx} + a_n(x)u(x) = f(x), \qquad (1)$$

here $a_i(x),\, i = 1,\ldots n$ representing the coefficients and the right-hand side of the equation $f(x)$ function $D \subset R$ is continuous in the field. This equation

$$Lu = a_0(x)\frac{d^n u}{dx^n} + a_1(x)\frac{d^{n-1}u}{dx^{n-1}} + \ldots + a_{n-1}(x)\frac{du}{dx} + a_n(x)u(x) = 0 \qquad (2)$$

is called a homogeneous equation corresponding to the given equation (1).

In most cases, the particular solution of equation (1) is the Green's function $G(x,\xi)$ through

$$u(x)=\int_a^b G(x,\xi)f(\xi)d\xi, \qquad (5)$$

and the general solution

$$u(x)=\int_a^b G(x,\xi)f(\xi)d\xi + A_1u_1(x)+A_2u_2(x)+...+A_nu_n(x), \qquad (6)$$

is represented here $u_1(x),u_2(x),...,u_n(x)$ functions (2) are linearly independent solutions of the equation, $A_k,(k=1,2,...,n;)$ the constant coefficients are found from the initial or boundary conditions.

Green's function (2) is not linearly related to Eq can be represented by solutions

$$G(x,\xi)=\frac{1}{f(\xi)}[c_1'(x)u_1(x)+c_2'(x)u_2(x)+...+c_n'(x)u_n(x)]U(x-\xi),$$

Had $c_k'(x), \quad (k=1,2,...,n;)$ (4) solution of the system of equations, function $U(x-\xi)$ this

$$U(x-\xi)=\begin{cases}0, \text{агар } x \prec \xi \text{ бўлса};\\ 1, \text{агар } x \succ \xi \text{ бўлса}.\end{cases}$$

satisfies the condition.

If according to the initial or boundary conditions in expression (6). $A_k,(k=1,2,...,n;)$ if the coefficients are equal to zero, then the general solution of the equation (1) is expressed in the form (5) [1,10]. This conclusion is used in the analysis of problems related to the solution of quasi-linear differential equations. Differential equations that are linear with respect to their higher-order derivative or derivatives are called quasilinear [1].

For example:

$$L_{n,0}u=a_0(x)\frac{d^nu}{dx^n}+a_1(x)\frac{d^{n-1}u}{dx^{n-1}}+...+a_{n-1}(x)\frac{du}{dx}=f_1(x,u(x)); \qquad (7)$$

$$L_{n,m}u=a_0(x)\frac{d^nu}{dx^n}+a_1(x)\frac{d^{n-1}u}{dx^{n-1}}+...+a_{n-1}(x)\frac{d^{n-m-1}u}{dx^{n-m-1}}=f_2\left(x,u(x),u^{(1)}(x),u^{(2)},...,u^{(m)}(x)\right); \quad (8)$$

1) equations are examples of quasi-linear differential equations of practical importance.

2) If the homogeneous equations obtained by setting the linear parts of these equations to zero do not have non-zero solutions that satisfy the initial and boundary conditions, then finding their solutions can be reduced to finding solutions of nonlinear integral and integro-differential equations using appropriate Green's functions [10,11]:

3) 1) From equation (7) above

$$u(x)=\int_a^b G_{n,0}(x,\xi)f_1(\xi,u(\xi))d\xi; \qquad (9)$$

4) Next from equation (8).

5)

$$u(x)=\int_a^b G_{n,m}(x,\xi)f_2\left(\xi,u(\xi),u^{(1)}(\xi),u^{(2)}(\xi),...,u^{(m)}(\xi)\right)d\xi; \qquad (10)$$

it is possible to pass to the corresponding integro-differential equations such as

This

$$u(x)=Au(x) \qquad (11)$$

satisfying Eq $[a,b]=D\subset R$ unknown identified in the field $u(x)\in X$ be required to find a function, where the space of functions is related to the problem statement $u(x)$ determined based on the properties of the function.

According to the principle of compression reflection normalized full space self-reflection

$A: X \to X$ to be $\forall u_1, u_2 \in X$ for functions $0 < \alpha < 1$ the following relation satisfying the condition

$$\|Au_1(x) - Au_2(x)\|_X < \alpha\|u_1(x) - u_2(x)\|_X$$

if appropriate, equation (11) is unique $u_0(x) = Au_0(x)$ will have a solution This solution is also called a fixed point of reflection [9].

Since the generated equation (9) is actually derived from equation (7) and the corresponding initial and boundary conditions, the function space to which the unknown function in this integral equation belongs is determined by the conditions associated with equation (7). Thus, the unknown function in equation (7).

$u(x) \in C^{(n-1)}[a,b] \subset C[a,b] \subset L_p[a,b]$ being this $\|L_p[a,b]\| \le \|\cdot\|_{C[a,b]} \le \|\cdot\|_{C^{(n-1)}[a,b]}$ attitude is appropriate.

In the derived equation (9).

$$B(\cdot) = \int_a^b G_{n,0}(x,\xi)(\cdot)d\xi$$

ва $F_1(\cdot) = f_1(\xi, \cdot)$ (12)

it can be expressed in operator form by making notations

$$u = BF_1 u \,, \tag{13}$$

where B linear integral operator , F_1 – without a line. Then this is a superposition of operators A – by specifying the operator

$$u = BF_1 u = Au \quad \text{ёки} \quad u = Au \tag{14}$$

equalities are obtained. If $u(x) \in L_p[a,b]$ is a non-linear operator $F_1 : L_p[a,b] \to L_q[a,b]$ and the linear operator $B: L_q[a,b] \to L_p[a,b]$ is an operator consisting of their superposition $A: L_p[a,b] \to L_p[a,b]$ will be After that, the condition that the operator has compressive reflection property is found.

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