Application of vectors to some complex problems

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ABSTRACT

This article gives you some easy ways to solve common trigonometric inequalities.

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As you know, in the course of elementary mathematics, the science of geometry is conventionally divided into two parts: planimetry and stereometry. In stereometry, shapes are studied in three dimensions. The study of stereometry requires the student to apply the knowledge learned in the department of planimetry and to have a deeper understanding of geometric shapes. Nowadays, stereometric questions are among the questions in mathematics when entering higher education institutions. In solving these problems, it is important to find the largest and smallest values of the forms. Finding the largest and smallest values is found using the product. However, not all students can apply it. To solve some problems in geometry, such as finding the largest or smallest quantities, depends in many respects on the experience of solving them and the degree of mastery of the methods of solving them. Therefore, it is important to apply and prove any inequality based on the given. Now let's look at some of the issues below.Today, high school students find it difficult to prove more inequalities, mainly in math. This is because there are almost no uniform proof methods for inequalities. In this paper, we prove some inequalities that fall into a certain rule for trigonometric inequalities with the help of vectors. In particular, we will try to prove inequalities in simpler ways by substituting auxiliary vectors. Problem 1. A party wants to make a roofless box out of a square with side 6a. The box, cut from the corner of the square and erected vertically, had the largest volume. What length square should I cut for this?

Solution: We denote the side of the truncated square by x. The bottom of the box is 6a-2x. Let the height be x.

Its volume is V = (6a-2x) (6a-2x) x = 4x (3a-x) (3a-x). However, the maximum of V depends on the following function y = 4x (3a-x) (3a-x). This function reaches a maximum at x = a.

To us $\bar{a}$; $\bar{b}$; $\bar{c}$ vectors be given:

1. $(\bar{a},\bar{b}) = |\bar{a}| * |\bar{b}| * \cos(\bar{a}\hat{}\bar{b})$
2. $(\bar{a}+\bar{b},\bar{c}) = (\bar{a},\bar{c}) + (\bar{b},\bar{c})$

We now introduce the following condition $|\bar{a}| = |\bar{b}| = |\bar{c}| = 1$

$x+y+z=2\pi$

$(\bar{a}+\bar{b}+\bar{c}\ ,\bar{a}+\bar{b}+\bar{c}\ )=\left|\bar{a}+\bar{b}+\bar{c}\ \right|^2 \geq 0$

$(\ \bar{a}+\bar{b}+\bar{c}\ ,\bar{a}+\bar{b}+\bar{c}\ ) = |\bar{a}|^2 + |\bar{b}|^2+|\bar{c}|^2 +2\ |\bar{a}| * |\bar{b}| \cos z +2|\bar{a}| * |\bar{c}| \cos y+2|\bar{b}| * |\bar{c}| \cos x$

This is from inequality $\cos x + \cos y + \cos z \geq -\frac{3}{2}$,[1] $x+y+z=2\pi$ is equal to.

In a triangle $\alpha+\beta+\gamma=\pi$ is equal to. So we get the following substitution $x=2\alpha$, $y=2\beta$, $z=2\gamma$ This substitution gives the inequality $\cos 2\alpha+\cos 2\beta+\cos 2\gamma \geq -\frac{3}{2}$. Now let's look at how $2\alpha, 2\beta, 2\gamma$ to move from α, β, γ

$\cos 2\alpha = 2\cos\alpha^2 - 1$, $\cos 2\beta = 2\cos\beta^2 - 1$, $\cos 2\gamma = 2\cos\gamma^2 - 1$

$\cos 2\alpha + \cos 2\beta + \cos 2\gamma = 2(\cos\alpha^2+\cos\beta^2+\cos\gamma^2) - 3$ is equal to.

$\cos 2\alpha+\cos 2\beta+\cos 2\gamma=2(\cos\alpha^2+\cos\beta^2+\cos\gamma^2) - 3 \geq -\frac{3}{2}$ hence

$\cos\alpha^2+\cos\beta^2+\cos\gamma^2 \geq \frac{3}{4}$ There is an inequality.

$\cos\alpha^2=1-\sin\alpha^2$ $\cos\beta^2=1-\sin\beta^2$ $\cos\gamma^2=1-\sin\gamma^2$ instead of the expression

$\sin\alpha^2+\sin\beta^2+\sin\gamma^2 \leq \frac{9}{4}$ There is an inequality.

Using the Koshi-Bunyakasky inequality $(1+1+1)(a^2+b^2+c^2) \geq (a+b+c)^2$

$(1+1+1)(\ \sin\alpha^2 + \sin\beta^2 + \sin\gamma^2\ ) \geq (\sin\alpha + \sin\beta + \sin\gamma)^2 \rightarrow$

$\sin\alpha + \sin\beta + \sin\gamma \leq \frac{3\sqrt{3}}{2}$ There is an inequality.

We will now prove the following using the thyroganometric equation.

$\sin\alpha + \sin\beta + \sin\gamma = 4\cos\frac{\alpha}{2}\cos\frac{\beta}{2}\cos\frac{\gamma}{2}$

$4\cos\frac{\alpha}{2}\cos\frac{\beta}{2}\cos\frac{\gamma}{2} \leq \frac{3\sqrt{3}}{2}$

$\cos\frac{\alpha}{2}\cos\frac{\beta}{2}\cos\frac{\gamma}{2} \leq \frac{3\sqrt{3}}{8}$ Now $\cos 2\alpha + \cos 2\beta + \cos 2\gamma = -1-4\cos\alpha\cos\beta\cos\gamma$ hence

$$\cos\alpha\cos\beta\ \cos\gamma \leq \frac{1}{8}$$

$\cos\alpha+\cos\beta+\cos\gamma$ create an inequality for. for this $x=\pi-\alpha$, $y=\pi-\beta$, $z=\pi-\gamma$ we choose $x+y+z=3\pi-(\alpha+\beta+\gamma)$ is equal to.

The following is from this replacement $\cos\alpha+\cos\beta+\cos\gamma \leq \frac{3}{2}$ we come to inequality.

$\cos\frac{\alpha}{2}^2 + \cos\frac{\beta}{2}^2 + \cos\frac{\gamma}{2}^2 \leq \frac{9}{4}$

We will now use the Cauchy inequality to prove the inequality we need. $\frac{a+b+c}{3} \geq \sqrt[3]{abc}$

$\frac{3\sqrt{3}}{2} \geq \cos\frac{\alpha}{2} + \cos\frac{\beta}{2} + \cos\frac{\gamma}{2} \geq 3\sqrt[3]{\cos\frac{\alpha}{2}\cos\frac{\beta}{2}\cos\frac{\gamma}{2}}$

$\rightarrow \cos\frac{\alpha}{2}\cos\frac{\beta}{2}\cos\frac{\gamma}{2} \leq \frac{3\sqrt{3}}{8}$

There is an inequality. We have now used many trigonometric equations to prove the above inequalities. These are the following equations.

1. $\sin\alpha + \sin\beta + \sin\gamma = 4\cos\frac{\alpha}{2}\cos\frac{\beta}{2}\cos\frac{\gamma}{2}$
2. $\cos\alpha+\cos\beta+\cos\gamma=1+4\sin\frac{\alpha}{2}\sin\frac{\beta}{2}\sin\frac{\gamma}{2}$
3. $\sin 2\alpha + \sin 2\beta + \sin 2\gamma = 4\sin\alpha\sin\beta\sin\gamma$
4. $\cos 2\alpha+\cos 2\beta+\cos 2\gamma = -1-4\cos\alpha\cos\beta\cos\gamma$
5. $\tan\frac{\alpha}{2}\tan\frac{\beta}{2}+\tan\frac{\beta}{2}\tan\frac{\gamma}{2}+\tan\frac{\gamma}{2}\tan\frac{\alpha}{2}=1$
6. $\cot\frac{\alpha}{2}\cot\frac{\beta}{2}+\cot\frac{\beta}{2}\cot\frac{\gamma}{2}+\cot\frac{\gamma}{2}\cot\frac{\alpha}{2}=1$
7. $\sin\frac{\alpha}{2}^2+\sin\frac{\beta}{2}^2+\sin\frac{\gamma}{2}^2+2\sin\frac{\alpha}{2}\sin\frac{\beta}{2}\sin\frac{\gamma}{2}=1$
8. $\cos\alpha^2+\cos\beta^2+\cos\gamma^2+2\cos\alpha\cos\beta\cos\gamma=1$

Proof of these equations is an independent task for students.

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