Dynamic Stresses Near the Working Surface from A Plane Wave

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ABSTRACT

The paper considers the reaction of a cylindrical layer located in a viscoelastic medium, when exposed to a transverse (longitudinal) seismic wave. It has been established that the maximum dynamic stress is 10-15% higher than the static one, and the wave numbers at which the maximum value is reached.

Keywords: spherical shell; spherical protective domes; hesitation; cubic equation

We assume that the harmonic wave is plane and that the wave front is parallel to the axis of the cylindrical layer (Fig. 1). The basic equations of the theory of viscoelasticity for this plane strain problem are reduced to the following equation

$$\Delta\varphi - \int_{-\infty}^{t} \left[R_\lambda(t-\tau)+2R_\mu(t-\tau)\right]\Delta\varphi d\tau = \frac{1}{a^2}\frac{\partial^2\varphi}{\partial t^2}; \quad \Delta\vec{\psi} - \int_{-\infty}^{t} R_\mu(t-\tau)\Delta\vec{\psi} d\tau = \frac{1}{b^2}\frac{\partial^2\vec{\psi}}{\partial t^2} \quad (1)$$

where $\alpha^2 = (\lambda_0+2\mu_0)/\rho$; $b^2 = \mu_0/\rho$; φ и ψ – movement potentials. $R_\lambda(t-\tau)$ and $R_\mu(t-\tau)$ – relaxation core; ν - Poisson's ratio, which we consider to be a non-relaxing quantity [1]. The incident plane wave is considered to propagate in the positive direction of the x-axis and is represented as follows:

$$\begin{cases} \varphi^{(i)} = \varphi_0 e^{i(\alpha x-\omega t)}, \varphi^{(i)} = 0 - \text{when exposed to prolonged pain} \\ \psi^{(i)} = \psi_0 e^{i(\beta x-\omega t)}, \psi^{(i)} = 0 - \text{when exposed to shear waves} \end{cases} \quad (2)$$

φ_0 и ψ_0 – amplitude values; ω - circular frequency; α and β are wave numbers, which must be complex numbers $\alpha = \alpha_R + i\alpha_I$; $\beta = \beta_R + i\beta_I$ $\alpha_I < 0$ and $\beta_I < 0$ denote the attenuation coefficients; α R and βR denote the wave numbers of longitudinal waves and shear waves, respectively. The solution of equation (1) can be sought in the form:

$$\varphi(r,\theta,t) = \sum_{k=1}^{\infty} \varphi_k(r\theta)e^{i\omega t}; \quad \psi(r,\theta,t) = \sum_{k=1}^{\infty} \psi_k(r\theta)e^{i\omega t}, \quad (3)$$

где $\varphi_к(r,\theta)$ и $\Psi_к(r,\theta)$ – real functions satisfying the equations

$$\Delta\Phi_\kappa + \frac{\alpha_k}{1-L_\kappa}\Phi_\kappa = 0\,; \Delta\psi_\kappa + \frac{\beta_K}{1-M_\kappa}\psi_\kappa = 0\ ; \tag{4}$$

$$\text{где}\ \ L_k = \int_0^\infty \left[R_\lambda(\xi) + 2R_\mu(\xi)\right]\exp(-i\omega\xi)d\xi\ \ , M_k = \int_0^\infty R_\mu(\xi)\exp(-i\omega\xi)d\xi\,.$$

To describe the viscoelastic properties of the material, the Boltzmann-Voltaire theory with the Rzhanitsyn-Koltunov relaxation kernel in the form $R(t) = Ae^{-\beta t}t^{\alpha-1}$. In this case, the sine Γ(s) and cosine Γ(c) of the Fourier samples, the relaxation kernel R(t) is expressed in terms of the Γ(α)-Gamma function

$$\Gamma^S = \frac{A\Gamma(\alpha)}{(\omega^2+\beta^2)^{\alpha/2}}Sin(\alpha arctg\frac{\omega}{\beta}),\quad \Gamma^C = \frac{A\Gamma(\alpha)}{(\omega^2+\beta^2)^{\alpha/2}}Cos(\alpha arctg\frac{\omega}{\beta}).$$

The solution of equation (4) is expressed in terms of the Hankel functions of the 1st and 2nd kind of the nth order:

$$\begin{aligned}\varphi &= \sum_{n=0}^{\infty}\left[A_n H_n^{(1)}(\alpha^* r) + A_n' H_n^{(2)}(\alpha^* r)\right]\cos n\theta e^{-i\omega t}\\ \psi &= \sum_{n=0}^{\infty}\left[B_n H_n^{(1)}(\beta^* r) + B_n' H_n^{(2)}(\beta^* r)\right]\sin n\theta e^{-i\omega t}\end{aligned}, \tag{5}$$

$$\alpha_{k\pi}^{*2} = \frac{\alpha_{k\pi}}{1-L_k}\,; \beta_k^{*2} = \frac{\beta_k}{1-M_k}\,,$$

where A_n, A'_n, B_n, B'_n – expansion coefficients, which are determined by the corresponding boundary conditions ; $H_n^{(1)}(\alpha^* r)$ and $H_n^{(2)}(\alpha^* r)$ – Hankel function of the 1st and 2nd kind of the nth order, respectively $H_n^{(2)}(\alpha r) = I_n(\alpha r) - iN_n(\alpha r)$. Solution (5) satisfies at infinity $r \to \infty$ to the Sommerfeld radiation condition [1]:

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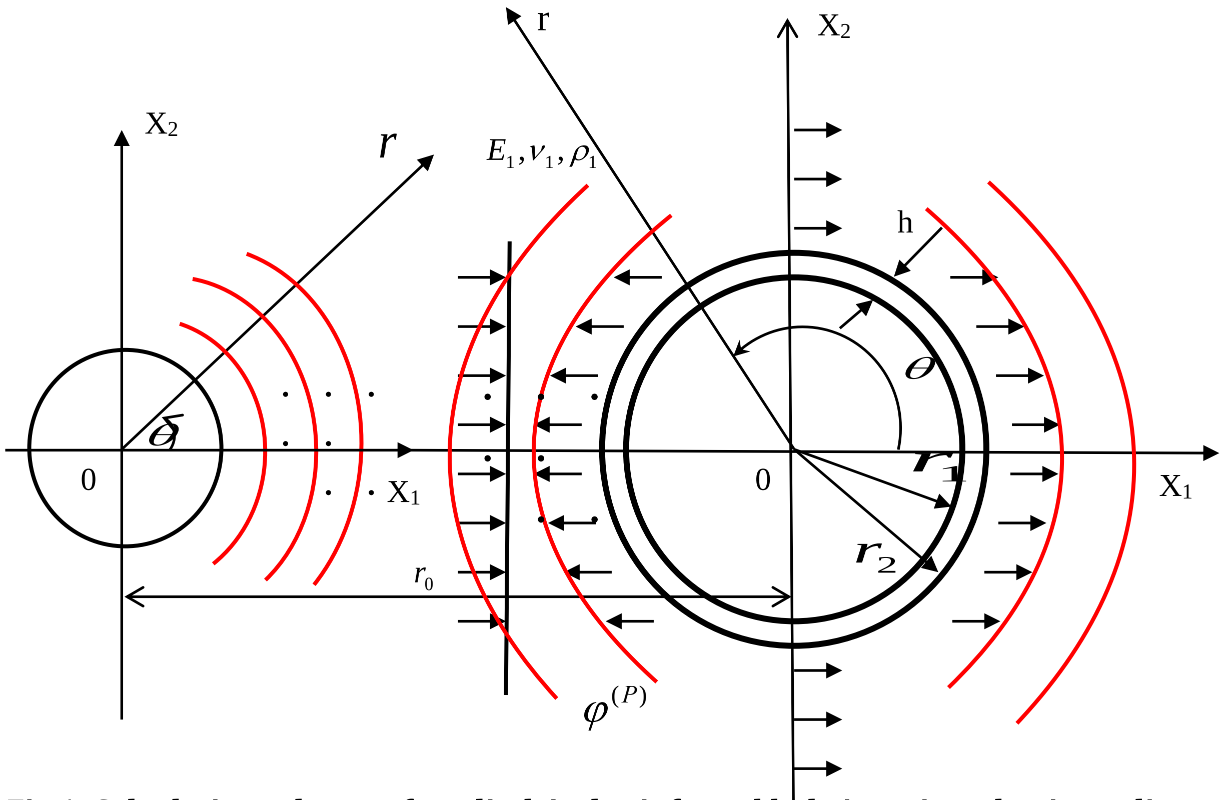

Fig.1. Calculation scheme of a cylindrical reinforced hole in a viscoelastic medium

$$\lim_{r\to\infty}\varphi = 0, \qquad \lim_{r\to\infty}(\sqrt{r})^{\kappa}\left(\frac{\partial\varphi}{\partial r}+i\alpha\varphi\right)=0 ,$$

$$\lim_{r\to\infty}\psi = 0, \qquad \lim_{r\to\infty}(\sqrt{r})^{\kappa}\left(\frac{\partial\psi}{\partial r}+i\beta\,\psi\right)=0 .$$

For this, $A'_n = B'_n = 0$ must be present. The solution of equation (5) is represented as:

$$\varphi^{(r)} = \sum_{n=0}^{\infty} A_n H_n^{(1)}(\alpha^* r)\cos n\theta)e^{-i\omega t}; \quad (6)$$

$$\psi^{(r)} = \sum_{n=0}^{\infty} B_n H_n^{(1)}(\beta^* r)\sin(n\theta)e^{-i\omega t}.$$

The total potential can be determined by superimposing the potentials of the incident and reflected waves. Thus, the displacement potentials will be [1]:

$$\varphi = \varphi^{(i)} + \varphi^{(k)}, \varphi^{(p)} = \varphi(r,\theta,t), \psi = \psi^{(i)} + \psi^{(k)}, \psi^{(p)} = \psi(r,\theta,t) \quad (7)$$

It follows that stresses and displacements can easily be expressed in terms of displacement potentials

$$u_r = \frac{\partial\varphi}{\partial r} + \frac{1}{r}\frac{\partial\psi}{\partial\theta}; \quad u_\theta = \frac{1}{r}\frac{\partial\varphi}{\partial\theta} - \frac{\partial\psi}{\partial r},$$

$$\varepsilon_{rr} = \frac{\partial u_r}{\partial r}; \quad \varepsilon_{\theta\theta} = \frac{1}{r}\frac{\partial u_\theta}{\partial\theta} + \frac{u_r}{r}; \quad \varepsilon_{r\theta} = \frac{1}{2}\left(\frac{1}{r}\frac{\partial u_r}{\partial\theta} + \frac{\partial u_\theta}{\partial r} + \frac{u_\theta}{r}\right),$$

$$\sigma_{rr} = \tilde{\lambda}\nabla^2\varphi + 2\tilde{\mu}\left[\frac{\partial^2\varphi}{\partial r^2} + \frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial\varphi}{\partial\theta}\right)\right];$$

$$\sigma_{\theta\theta} = \tilde{\lambda}\nabla^2\varphi + 2\tilde{\mu}\left[\frac{1}{r}\left(\frac{\partial\varphi}{\partial r} + \frac{1}{r}\frac{\partial^2\varphi}{\partial\theta^2}\right) + \frac{1}{r}\left(\frac{1}{r}\frac{\partial\psi}{\partial\theta} - \frac{\partial^2\psi}{\partial r\partial\theta}\right)\right]; \quad (8)$$

$$\sigma_{zz} = \tilde{\lambda}\nabla^2\varphi; \quad \sigma_{r\theta} = 2\tilde{\mu}2\left(\frac{1}{r}\frac{\partial^2\varphi}{\partial\theta\partial r} - \frac{1}{r^2}\frac{\partial\varphi}{\partial\theta}\right),$$

where $\{(\tilde{\lambda},\tilde{\mu})\ f(t)\} = (\lambda_0,\mu_0)\left[f(t) - \int_{-\infty}^{t} R_{\lambda,\mu}(t-\tau)f(\tau)d\tau\right],$

$f(t)$ – some function; ur - radial displacement; uθ is the tangential displacement; εrr, ε θ θ ,ε r θ - strain tensor elements; σ rr,σ rθ , σ θθ , σ zz - stress tensor elements. As mentioned above, the coefficients An and Bn are determined from the corresponding boundary conditions.

The boundary conditions for r = a, and is the radius of the cylindrical discontinuity surface will be: σrr = 0 ; σrθ = 0 .

The coefficients An and Bn are determined from the corresponding boundary conditions for each value of n. Thus, the stress concentration in the flow under the influence of a shear wave (2) takes the following value[1]

$$\sigma^*_{\theta\theta} = -\frac{8}{\pi}(1-\frac{1}{K_*^2})\sum_{n=1}^{\infty} i^{n+1}\frac{n(n^2-1-\frac{\Omega_2}{2})H_n(\Omega_1)}{\delta_n} Sinn\theta;$$

$$\delta_n = \Omega_2^2(n^2+n-\frac{\Omega_2^2}{4})H_n(\Omega_1)H_n(\Omega_2)+\Omega_1\Omega_2(n^2-1)*H_{n-1}(\Omega_1)H_{n-1}(\Omega_2)+$$

$$+(n^2-n^2-\frac{\Omega_2^2}{2})[\Omega_2 H_n(\Omega_1)H_{n-1}(\Omega_2)+\Omega_1 H_{n-1}(\Omega_1)H_n(\Omega_2)];$$

$$\Omega_1 = \alpha_1 a \quad ; \quad \Omega_2 = \beta_1 a \quad ; \quad K_*^2 = \frac{\beta_*^2}{\alpha_*^2} = \frac{C''_{\alpha*}}{C^2_{\beta*}} = \frac{2(1-\nu)\Gamma_1^*}{(1-2\nu)\Gamma_2^*}$$

Γ_1^* и Γ_2^* - describe the viscoelastic properties of the material.

In the case of an elastic cylindrical body in a viscoelastic medium, rigid contact conditions are set at the boundary r = a, under which stresses and displacements are continuous at the boundary:

σ rr 1 = σ rr 2 ; σ rθ 1 = σ rθ 2;

u r 1 = u r 2 ; u θ 1 = u θ 2

Where, σ rr 1 , σ rθ 1 - stresses in a viscoelastic environment; σ rr 2 and σ rθ2 - viscoelastic inclusion stresses; ur1, uθ1 - radial and tangential displacements of the environment; ur2, uθ2 - radial and tangential displacements of the elastic inclusion. If there is no friction at the contact boundary, then σ rr 1 = σ rr 2 ; σ rθ 1 = σ rθ 2 = 0 ; u r 1 = u r 2 .

Determination of stresses at r = a. Near the cylindrical cavity, the contour stresses σθθ at r = a express the stress concentration. The contour stress under the influence of a longitudinal harmonic wave has the form:

$$\sigma^*_{\theta\theta} = \Big\{ (\overline{\alpha}a)H_{n-1}(\overline{\alpha}a)\Big[(n^2-1)\overline{\beta}aH_{n-1}(\overline{\beta}a)-(n^3-n+\frac{1}{2}\overline{\beta}^{-2}a^2)H_n(\overline{\beta}a)\Big]-$$
$$-H^{(1)}{}_n(\overline{\alpha}a)\Big[(n^3-n+\frac{1}{2}\overline{\beta}^{--2}a^2)\overline{\beta}aH_{n-1}(\overline{\beta}a)-n^2+n-\frac{1}{4}\overline{\beta}^2a^2\Big]\Big\}$$

The stress concentration under the influence of a shear wave (or transverse waves) has the following form

$$\sigma^*_{\theta\theta} = \frac{\sigma_{\theta\theta}}{\sigma_0}\Big|_{r=a} = \frac{y}{\pi}\left(1-\frac{1}{\chi^2}\right)\sum_{n=0}^{\infty}\in_n i^{n+1}S_n^*\cos n\theta\, e^{i\omega t}$$

Where

$$S^*{}_n = \Big\{(n^2-1)\overline{\beta}aH_{n-1}(\overline{\beta}a)-(n^3-n+\frac{1}{2}\overline{\beta}^{-2}a^2)H_n(\overline{\beta}a)\Big\}.$$

$$\sigma^*{}_{\theta\theta} = -\frac{8}{\pi}(1-\frac{1}{\chi^2})\sum_{m=0}^{\infty} i^{n+1}\frac{n(n^2-1-\frac{\overline{\beta}^2\alpha^2}{2})H_n^1(\overline{\alpha}a)}{\Delta_n}\sin n\theta\; e^{-i\omega t};$$

$$\Delta_n = \overline{\beta}^2\alpha^2(n^2+n-\frac{\overline{\beta}^2\alpha^2}{2})H_n^1(\overline{\alpha}a)H_n^{(1)}(\overline{\beta}a)+\overline{\alpha}\overline{\beta}a^2(n^2-1)H_{n-1}(\overline{\beta}a)+$$

$$+(n-n^2-\frac{\overline{\beta}^2\alpha^2}{2})\Big[\overline{\beta}\alpha\; H_n(\overline{\alpha}a)H_{n-1}(\overline{\beta}a)+\overline{\alpha}aH_{n-1}(\overline{\alpha}a)H_n(\overline{\beta}a)\Big]$$

Stress at the boundary of a rigid inclusion under the influence of a shear wave (r = a). in dimensionless form has the form:

$$\sigma_{rr}^{*}=\frac{4}{\pi}\left\{-\frac{(1-\eta)H_1^{(1)}(\overline{\alpha}a)}{\delta_1}\sin\theta+\sum^{\infty}\frac{i^{n+1}H_n(\overline{a}\ a)}{\Delta n}\sin\ n\theta\ \right\}e^{-i\omega t}$$

$$\sigma_{r\theta}^{*}=\frac{2}{\pi}\left\{-\frac{i\ \overline{\beta}a^2}{\overline{\beta}^3a^3H_1^{(1)}(\overline{\beta}a)+8\eta\left(\frac{\overline{\beta}^2a^2}{2}H^{(1)}{}_0(\beta a)-\overline{\beta}aH_1(\beta a)\right)}-\right.$$

$$-\frac{2}{\delta_1}\left[(1+\eta)H_1\ (\overline{\alpha}\ a)-\overline{\alpha}\ aH_0(\overline{\alpha}\ a)cos\theta\right]-$$

$$\left.-2\sum_{n=2}^{\infty}\frac{i^{n+1}\left[-nH_n^{(1)}(\overline{\alpha}\ a)+(\overline{\alpha}\ a)H_{n-11}(\overline{\alpha}\ a)\right]}{\Delta n}\cos n\theta\ \right\}e^{i\omega t}$$

$$\delta_1=-4\eta H_1^{(1)}(\overline{\alpha}\ a)H_1\ (\overline{\beta}\ a)+(1+\eta)\overline{\alpha}\ aH_0\ (\overline{\alpha}\ a)+H_0\ (\overline{\beta}\ a),$$

$$\Delta n=n\overline{\alpha}\ aH_{n-1}(\overline{\alpha}\ a)H_{n1}(\overline{\beta}\ a)+n(\overline{\beta}\ a)H_{n-1}(\overline{\beta}\ a)H_n\ (\overline{\alpha}\ a)-$$

$$-\alpha\overline{\beta}a^2H_{n-1}^{(1)}(\overline{\alpha}\ a)H_{n-1}^{(1)}(\overline{\beta}\ a).$$

Here η = ρ/ρ1 is the ratio of the ambient density to the inclusion density. Under the influence of longitudinal waves in a rigid inclusion, the stress tensor components σ*rr and σ*rθ take the form:

$$\sigma_{rr}^{*}=-\frac{2}{\pi}\left\{i\left[\ (\overline{\alpha})H_1^{(1)}(\overline{\alpha}a)\ \right]^{-1}-2\left[(1+\eta)H_1^{(1)}(1-\eta)H_1^{(1)}(\overline{\beta}a)-\overline{\beta}aH_0(\overline{\beta}a)\ \right]\cos(\theta/\Delta_1)\right)$$

$$\left.+2\sum_{n=2}^{\infty}i^{n+1}\left[-\overline{\beta}aH_0(\overline{\beta}a)/\Delta_{r1}\right]\cos n\theta\ \right\}e^{-i\omega t};$$

$$\sigma^{*}_{r\theta}=-\left(\ \frac{2}{\pi}\ \right)\left\{2(1-\eta)-H_1^{(1)}(\overline{\beta}\sin\theta/\Delta_1+2\sum_{n=2}^{\infty}i^{n-1}\left[nH_n^{(1)}(\overline{\beta}\ a)/\Delta_n\right]\sin\theta\ \right\}e^{-i\omega t}$$

$$\sigma_{\theta\theta}^{*}=(1-\frac{1}{\chi^2})\sigma_{rr}^{*};\ \overline{\chi}^2=\frac{\overline{c}_{\alpha}^2}{\overline{c}_{\beta}^2}.$$

The stress tensor components σ*rr and σ* θθ of an elastic cylindrical inclusion at r = a under the action of longitudinal waves take the following form:

$$\sigma^{*}{}_{rr}=-\frac{\eta}{\pi}\sum_{m=0}^{\infty}\frac{\in_n i^{n+1}I_n(\overline{\alpha}_2\ a)\overline{\beta}_1^{\,2}a^2}{\Delta_n}\times\left\{\beta_1 aH_{n-1}^{(1)}(\overline{\beta}_1 a)-(n^2+n-\frac{\overline{\beta}_1^{\,2}a^2}{2}H_n^{(1)}(\overline{\beta}_1 a)\ \right\}\cos\ ne^{-i\omega t}\beta;$$

$$\sigma^{*}{}_{r\theta} = -\frac{\eta}{\pi}\sum_{n=0}^{\infty}\frac{\in_n i^{n+1}}{\Delta n}\left\{\left[\begin{array}{l}(\frac{1}{\overline{\chi}^2}-1)(\frac{n\overline{\beta}_1^2a^2}{2}(n-1)-n^2(n^2-1)-\eta\frac{\overline{\beta}_1^4a^4}{4})+\\ +\frac{\overline{\beta}_1^2a^2}{4}\eta(n(n+1)-\frac{1`}{2}\overline{\beta}_1^2a^2)\end{array}\right]*\right.$$

$$* I_n(\alpha_2 a)+\left(\frac{1}{\chi^2}-1\right)\left(n^3-n+\frac{1}{2}\overline{\beta}_1^2a^2\right)(\alpha_2 a)I_{n-1}(\alpha_2 a)H_n^{(1)}(\overline{\beta}_1 a)+$$

$$\left.+\left(\frac{1}{\chi^2}-1\right)(1-n^2)(\alpha_2\overline{\beta}a^2)I_{n-1}(\alpha_2 a)H_{n-1}(\overline{\beta}_1 a)\right\}\cos n\theta e^{-i\omega t},$$

$$где\ \eta = \rho_2/\rho_1 \qquad \overline{\chi}^2 = \frac{\overline{\beta}_1^2}{\overline{\alpha}_1^2}.$$

The Bessel and Hankel functions of the real argument are tabulated in the same way as the trigonometric functions of the logarithm and are known functions of their argument. Since these are real functions, they describe steady waves, in fact, a steady cylindrical wave is a superposition of two traveling waves: one diverges from the axis of the cylinder, and the other converges with it. Since at r=0 the phases of these two waves are opposite, they cancel each other, and therefore the amplitude of the steady wave remains finite at r=0 [1]. This statement is used in solving the problem of wave propagation in viscoelastic media. Cylindrical wave I0 (kr) corresponds to a plane steady wave cos(kx-π/4); the amplitudes of successive maxima, due to the distribution of energy on ever larger cylindrical surfaces, are not constant, but decrease with distance. Cylindrical wave N0(kr) asymptotically corresponds to a plane wave sin (kx-π/4). On the axis of the cylinder (r=0) grad u of the incident and reflected waves are the same, and the amplitude of the reflected wave becomes infinitely large, in contrast to the case of a plane wave; therefore, the function N0 on the axis of the cylinder (r=0) has a pole, i.e. represents the source.

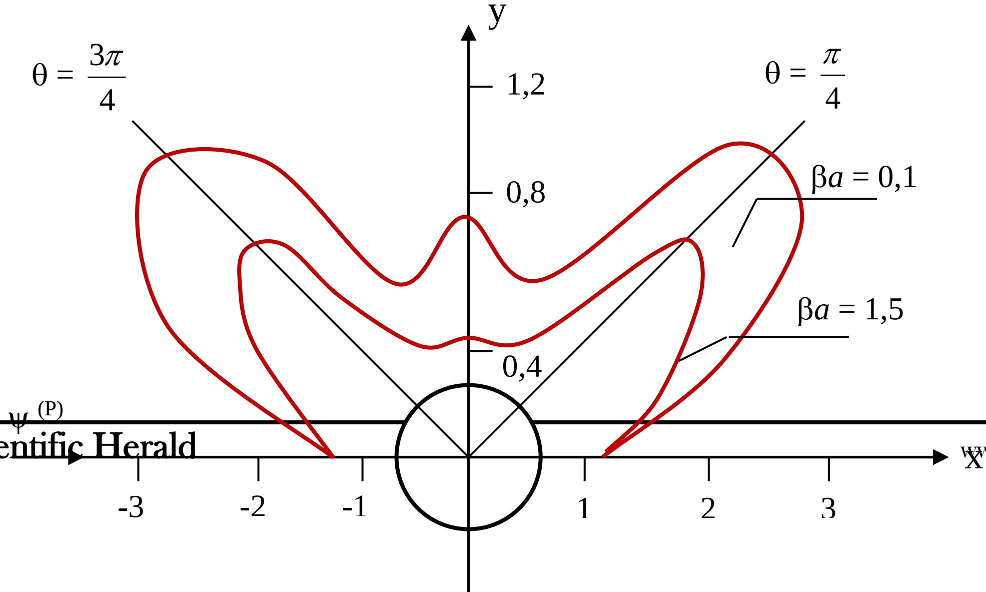

Fig.2. Distributions |⍰⍰⍰| at different values of ⍰a under the influence of shear waves.

1) A=0.01; $\beta = 0.05$; $\alpha = 0.1$; A=0.05; $\beta = 0.1$; $\alpha = 0.1$.

and for dimensionless wave numbers in the interval $0.01 \le \alpha^* a \le 3.0$

The results of distribution calculations for different values of the wave numbers are shown in Fig.2. It should be noted that at and the stress distribution is almost the same as in the static case, while at higher wave numbers the stress distribution differs significantly from the static case. The maximum dynamic stress is 10-15% higher than the static one, and the wave numbers at which the maximum value is reached lie between 0.25-0.75.

References

1. V.K.Kabulov. Algorithmization in the theory of elasticity. - T: FAN.1968.-394c.
2. N.A.Strelchuk, S.K.Slavin, V.N.Shaposhnikov. Investigation of the dynamic stress state of tunnel lining under the influence of blast waves. // Izv.Universities. Building and architecture, 1971, No. 9, -S. 129-136.
3. A.A.Chekalkin, A.A.Pankov. Lectures on structural mechanics from composite materials / Perm. state tech. Univ., Perm, 1999.- 150 p.
4. I.I.Safarov. Oscillations and waves in dissipative inhomogeneous media and Designs - Tashkent. Fan. 1992 - 250 s.
5. I.I.Safarov, U.T.Edgorov, T.O.Zhuraev, Z.F.Dzhumaev. About steady-state oscillations of three-layer cylindrical bodies // Mechanics problem. 2000.№.1, p.31-34.
6. Sh.R.Akhmedov, T.O.Zhuraev, D.Agzamova. On resonance damping vibrations of tubular structures. - Bukhoro, 1998.-c. 44-45
7. KMK 11-7-81 Construction in seismic areas. Design Standards. - M.,1981.-49 p.
8. A.M.Demin. Patterns of manifestations of slope deformations in quarries. - M: Science. 1981. - 144 p.
9. V.Z.Vlasov.Structural mechanics of thin-walled spatial systems. -M.:Stroyizdat, 1949, -250 p.
10. N.I. Muskhelishvili. Some basic tasks of the mathematical theory of elasticity. M: Nauka, 1966.- 478 p.
11. T.R.Rashidov. The dynamic theory of earthquake resistance of complex systems of underground structures. – Tashkent: Fan. 1973. - 180 p.
12. Y.H.Pac, C.C.Mow. The Diffraction of Elastic Wufes and Dunamic stress Constration-N.J. Crane Russah and 1973. - 675.
13. S.Okamoto Seismic resistance of engineering structures. - M.: Stroyizdat, 1980 - 344 p.
14. K.S.Sultanov. Interaction of an extended underground structure with soil under dynamic loading // Sat. scientific labor. Dynamics of heterogeneous media and the interaction of waves with structural elements.-Novosibirsk.-1987.-S.150-157
15. Ya.N.Muborakov.Earthquake resistance of underground structures such as cylindrical shells. - Tashkent: Fan, 1991, - 218.

16. T.Sh.Shirinkulov, Yu.K.Zaretsky. Creep and soil consolidation. - Tashkent: FAN, UzSSR, 1986. - 391 p.