Analytical Study of Cylindrical Channels in Dense Layers

Saparov Bobur Jumaboevich
Tashkent Institute of Chemical Technology
Chief specialist of the education quality control department
Saparov.boburbek@mail.ru

Tavbaev Jumaboy Saparovich
Tashkent Institute of Chemical Technology
Professor acting Department "Fundamentals of mechanics - machines and equipment of the food industry"

ABSTRACT

The paper presents the formulation of the boundary value problem of a thin channel in a boiling distance near the free surface. The modified boundary layer method is used to obtain an analytical solution of the boundary value problem for measuring the fraction and determining the crosslinking.

Keywords: Fluidized bed, catalyst, reactor, reagent, fluctuations, quasi-stationary, filtration, boundary layer, packed bed, crosslinking parameter

One of the main problems that often arise during the operation of chemical reactors in a fluidized bed is the formation of through thin channels in the bed, through which the reactants pass, bypassing the solid particles of the catalyst. Channel formation leads to a decrease in efficiency. reactor, to its output to off-design mode.

This raises the question of the laws of development of long and narrow channels of a liquid or gaseous reagent in dense and fluidized beds of solid catalyst particles. It is natural to try to answer this question by first constructing an effective solution of the hydrodynamic problem of flow in a given channel of finite length, and then, by analyzing the solution, draw conclusions about the nature of the change in length depending on the reagent flow rate, as well as on the physical and geometric parameters of the system. The initial channels of a sufficiently small size are always present in a dense layer of solid particles due to uneven stacking. In a fluidized bed, such initial channels can be considered as some inevitable fluctuations in the random mutual arrangement of particles.

In the quasi-stationary approximation, this problem is effectively solved below by the method proposed earlier in [1,2,3].

1. Statement of the problem. Let a porous layer of solid particles occupy a half-space $z<0$ and let there be a channel in the layer in the form of a right circular cylinder of radius $r0$ and length l. Let us denote the cylindrical coordinates by $0rz$: the Z axis coincides with the axis of the cylinder, the origin of coordinates coincides with the beginning of the channel on the layer surface (Fig. 1). The length of the channel $r<r0$, $0<z<l$ will be considered large compared to its radius, i.e. $l>>r0$. Therefore, the flow of a liquid or gaseous reagent in a channel can be considered one-dimensional.

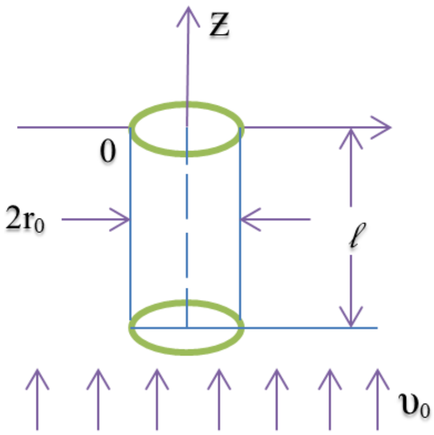

Fig.1

Outside the channel, the flow of the reagent between the solid particles is described differently for the case of a fluidized bed. Away from the channel, the flow can be considered undisturbed; here, the volumetric flow rate of the reagent moving in the direction of the Z axis is assumed to be given and equal to V_0. The pressure of the environment at z=0 will be denoted by P_0.

It is required to determine the flow field and, in particular, the outlet flow rate of the liquid (through the channel section at z=0).

We present the basic equations.

Packed Layer:

Equation of axisymmetric fluid filtration

$$\frac{\partial^2 p}{\partial r^2}+\frac{1}{r}\frac{\partial p}{\partial r}+\frac{\partial^2 p}{\partial z^2}=0$$

(1)

Darcy's Law

$$V_r=-\frac{K_0}{\mu}\frac{\partial p}{\partial r},\ V_z=-\frac{K_0}{\mu}\left[\frac{\partial p}{\partial r}+pg\right]$$

(2)

Equation of motion in a channel

$$Q=-\frac{\pi r_0^4}{8\mu}\left[\frac{\partial p}{\partial r}+pg\right]$$

(3)

Mass Conservation Equation

$$\frac{dQ}{dz}=q\ (q=-2\pi r V_r \text{ at } r=r_0)$$

(4.)

Here: ($p(r,z)$ is the liquid pressure in the layer, $P(z)$ is the pressure in the channel, V_r and V_z are the components of the filtration rate, $Q(z)$ is the volumetric flow rate of the liquid in the channel, (with a plus sign at movement in the direction of the z axis) $q(z)$ is the volumetric inflow of liquid into the channel (per unit length of the channel), K_0 is the permeability of the packed layer, μ is the dynamic viscosity of the liquid, ρ is the density of the liquid, g is the acceleration of gravity directed opposite to the axis z The flow in the channel is considered to be Poiseuille according to (3) Fluidized bed [4]:

Fluid motion equation

$$\frac{\partial p}{\partial r}=-\frac{\mu}{K(\varepsilon)}V_r\left(1+\lambda(\varepsilon)V\right)$$

(5.)

$$\frac{\partial p}{\partial r}+pg=-\frac{\mu}{K(\varepsilon)}V_z\left(1+\lambda(\varepsilon)V\right)$$

$$\left(K(\varepsilon)=a_1\frac{d^2\varepsilon^2}{(1-\varepsilon)^2},\lambda(\varepsilon)=a_2\frac{d\varepsilon\rho}{\mu(1-\varepsilon)}\right)$$

(6)

($V^2=V_r^2+V_z^2$)

The law of conservation of mass of liquid

$$\frac{1}{r}\frac{\partial}{\partial r}(rV_r)+\frac{\partial V_z}{\partial z}=0$$

(7)

(a_1 and a_2 are numerical components)

In addition, equations (3) and (4) will be valid for the flow in the channel. The value of $K(\varepsilon)$ is the effective permeability of the fluidized bed. Layer porosity ε in this case is a function of V_z implicitly determined by the following equation [4].

$$\frac{\mu}{K(\varepsilon)}V_z\left(1+\lambda(\varepsilon)V_z\right)=(\rho_S-\rho)(1-\varepsilon)g$$

(8)

(V_r<<V_z)

(ρ_S – solids density).

Equations (1)-(2) are valid for $V_z<V_*$, and equations (5)-(8) are valid for $V_z>V_*$. Here V_* is the critical flow rate of the start of fluidization, determined by equation (8) at $\varepsilon=\varepsilon_0$, where ε_0 is the porosity of the packed bed. We also give the boundary conditions:

at $z=0$ $P=P_0$ (9)
at $z=l$ $Q=0$ (10)
at $z=0$ $q=0$ (11)
at $z^2+r^2\to\infty$, $V_z=V_0$, $V_r=0$ (12)

Condition (10) is satisfied asymptotically for $l >> r_0$. It means that the influx of liquid through the end of the cylinder is neglected in comparison with the inflow through the side surface of the channel. Boundary condition (11) physically means that the liquid inflow into the channel near the free surface is negligibly small due to the predominant liquid outflow to the surface.

It is required to determine the functions p, Vr, Vz, P, Q, g. The formulated boundary value problems belong to the class of singular boundary value problems [1, 2]: they have two independent small parameters

$$\delta = \frac{K}{r_0^2} << 1, \lambda = \frac{r_0}{l} << 1$$

(13)

By virtue of the condition δ <<1, the pressure graph p in various sections $z=const$ has a form resembling a velocity profile in a boundary layer in a viscous fluid. Therefore, in some neighborhood of the channel for $r_0<r<r_*$ (we will call it the zone of influence of the channel), it is natural to accept the following assumptions

$$\frac{\partial p}{\partial r} << \frac{\partial p}{\partial z}, \frac{1}{r}\frac{\partial}{\partial r}(rV_r) >> \frac{\partial V_z}{\partial z},$$

($V_r>>V_z$) (14)

(approximation of the boundary layer, r* is the radius of the zone of influence)

Under condition (14), equations (1), (2) take the form

$$\frac{\partial^2 p}{\partial r^2} + \frac{1}{r}\frac{\partial p}{\partial r} = 0, \quad V_r = -\frac{K_0}{\mu}\frac{\partial p}{\partial r}$$

(15)

Let us write down the solution of equations (15) in the zone of influence $r_0<r<r_*$, $0<z<l$, which satisfies the natural conditions $p=P$ and $q=-2\pi r V_r$ at $r=r_0$:

$$p = P + \frac{\mu q}{2\pi K_0}\ln\frac{r}{r_0}\left(V_r = -\frac{q}{2\pi r}\right)$$

(16)

(packed layer).

At the boundary of the zone of influence at $r=r_*$, this solution should be "matched" with the unperturbed solution. Since according to (2) and (3) it will be

$$V_z = -\frac{K_0}{\mu}\left(\frac{\partial p}{\partial z} + pg\right) - \frac{1}{2\pi}\frac{\partial q}{\partial z}\ln\frac{r}{r_0}$$

(17)

then, according to the boundary condition (12), we obtain

$$V_0 = -\frac{K_0}{\mu}\left(\frac{\partial p}{\partial z} + pg\right) - \frac{1}{2\pi}\frac{\partial q}{\partial z}\ln\frac{r_*}{r_0}$$

(18)

Equations (18), (3) and (4) constitute a closed system of ordinary differential equations with respect to the functions P, q and Q.

Eliminating q and Q successively using (3) and (4), we arrive at the following equation

$$\frac{1}{\Delta^2}\frac{d^3 p}{dz^3} - \frac{dp}{dz} = \rho g + \frac{\mu V_0}{K_0}\left(\Delta_2 = \frac{16K_0}{r_0^4 \ln(r_*/r_0)}\right)$$

(19)

The general solution of this equation has the form

$$P = C_1 + C_2 l^{\Delta z} + C_3 l^{-\Delta z} - z\left(\rho g + \frac{\mu V_0}{K_0}\right)$$

(20)

where C_1, C_2, C_3 are arbitrary constants.

Three boundary conditions (9)–(11) serve to determine them. From here we find with the help of (3), (4) and (20)

$C_1=p_0$, $C_2 = \frac{\mu V_0}{2\Delta K_0 ch\Delta l}$, $C_3=-C_2$ (21)

As a result, we get the following solution

$$P = p_0 - \rho g z - \frac{\mu V_0}{K_0} z\left(1 - \frac{sh\Delta z}{\Delta z ch\Delta l}\right)$$

(22)

$$Q = -\frac{\pi V_0 r_0^4}{8K_0}\left(\frac{ch\Delta z}{ch\Delta l} - 1\right)$$

$$q = -\frac{\pi V_0 r_0^4}{8K_0}\frac{sh\Delta z}{ch\Delta l}$$

Note the following formulas:

$$Q = -\frac{\pi V_0 r_0^4}{8K_0}\left(1 - \frac{1}{ch\Delta l}\right)$$ (Q_0=Q at z=0) (23)

(total debit channel):

$$P = p_0 + \rho g z + \frac{\mu V_0 l}{K_0}\left(1 - \frac{tg\Delta z}{\Delta l}\right)$$ (P_0=P at z=-l) (24)

(channel bottom pressure;)

$$\Delta P = \frac{\mu V_0}{K_0} th\Delta l$$

(25)

(differential pressure):

$$q_0 = +\frac{\pi V_0 r_0^4}{8K_0} th\Delta l$$

(26)

(maximum inflow of liquid into the channel near and its bottom).

The last expression determines the absolute value of the pressure difference at the channel bottom and at the nearby undisturbed point of the layer. We also note the following limiting cases of formulas (21)–(23):

at Δl<<1

$$Q_0 = \frac{\pi V_0 r_0^4}{16K_0}(\Delta l)^2, \quad P_0 = p_0 + \rho g l,$$

$$\Delta l = \frac{\mu V_0 l}{K_0} \qquad (27)$$

at Δl>>1

$$Q_0 = \frac{\pi V_0 r_0^4}{16K_0}, \quad P_0 = p_0 + \rho g l + \frac{\mu V_0 l}{K_0},$$

$$\Delta P = \frac{\mu V_0}{\Delta K_0} \qquad (28)$$

It is convenient to introduce the dimensionless matching parameter a as follows.

$$\frac{r_*}{r_0} = (l / r_0)^{\alpha}$$

(29)

To determine this parameter, one can use the data of the numerical calculation of the original problem in any one particular case or some limiting case in which an analytical solution can be found by another method.

To this end, we study the following limiting case ourselves:

$\delta \to 0$, $\lambda \to 0$, $\Delta l \to 0$, (p_0=0, g=0).

(30)

Recall that according to (19), (13) and (29) we have:

$$\Delta l = \frac{4\sqrt{\delta}}{\lambda\sqrt{\alpha \ln(1/\lambda)}}$$

(31)

According to (22), the pressure P in the channel increases monotonically with an increase in $|Z|$ from p_0 to P_0, see (24). Therefore, in the limiting case (30), based on (24) and (22), we have

P=0, $$q = -\frac{\pi V_0 r_0^4}{8K_0} z,$$

$$Q = -\frac{\pi V_0 r_0^4}{8K_0}(l^2 - z^2)$$

(32)

On the other hand, the limiting case (29) is a classical problem of potential theory: to determine the harmonic function p (vanishing on the surface of the cylinder $r<r_0$, $-l<z<0$, and also on the surface of the half-space z=0) and tending to linear function $-\mu V_0 z/K_0$ at infinity (Benjamin Franklin problem).

According to [5], a thin cylinder ξ $r<r_0$, $-l<z<0$, in this problem can be replaced by a spheroid

$$\frac{z^2}{l^2 + \xi} + \frac{r^2}{r_0^2 + \xi} = 1(\xi \ge 0)$$

(33)

at ξ =0. To prove this fact, a direct numerical calculation was made on a computer using the finite element method of the original problem for a thin cylinder. On fig. Table 2 shows one of the calculation results: the ratio of q for a cylinder and a spheroid $(q_{цил})/(q_{сфер})$ depending on the coordinate z/l. As you can see, these two channels (at l=50r_0).

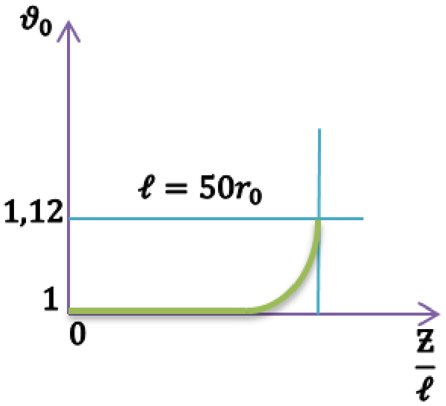

Fig.2

The exact solution of the Benjamin Franklin problem for the spheroid (33) is easy to find.

It has the form (for $l>r_0$)

$$P = -\frac{\mu V_0}{K_0} z \left\{ 1 - \frac{\frac{2}{\sqrt{\xi + l^2}} + \frac{1}{\sqrt{l^2 - r_0^2}} \ln \frac{\sqrt{\xi + l^2} - \sqrt{l^2 - r_0^2}}{\sqrt{\xi + l^2} + \sqrt{l^2 - r_0^2}}}{\frac{2}{l} + \frac{1}{\sqrt{l^2 - r_0^2}} \ln \frac{l - \sqrt{l^2 - r_0^2}}{l + \sqrt{l^2 - r_0^2}}} \right\}$$

(34)

Here $\xi=\xi(r,z)$ is defined by formula (33). According to (34) we have:

$$V_n = \frac{2V_0 z}{r_0^2} \cdot \frac{\left(\frac{z^2}{l^4} + \frac{r^2}{r_0^4} \right)^{-1/2}}{2 + \frac{1}{\sqrt{l^2 - r_0^2}} \ln \frac{l - \sqrt{l^2 - r_0^2}}{l + \sqrt{l^2 - r_0^2}}}$$

(35)

Here V_n is the filtration rate on the surface of the spheroidal cavity ξ=0.

In the limiting case of a thin prolate spheroid, when $\lambda=r_0/l<<1$, by formulas (20) and (33) we find

$$V_n = \frac{V_0 z}{r \ln(1/\lambda)}, q = -2\pi r V_n = \frac{-2\pi V_0 z}{\ln(1/\lambda)}$$

(36)

Comparing the value of q obtained by approximate and exact methods - see formulas (32) and (36) - using (19) and (29) we find the value of the matching parameter

$$a=1 \qquad \left(l\Delta = \frac{4\sqrt{\delta}}{\lambda \sqrt{\ln(1/\lambda)}} \right)$$

(37)

Formulas (16), (22) and (37) give the complete solution of the original problem for a packed layer of solid particles.

References

1. Cherepanov G.P. Fracture mechanics of composite materials. – M.: Science. 1983-296 p.
2. Cherepanov G.P. On the opening of oil and gas wells / / Dokl. Academy of Sciences of Russia -1985-vol.284, №4-p.816-820
3. Cherepanov G. P. Mechanics of Brittle Fracture. New York: Mc Graw Hill. 1979.
4. Mamasaidov M.T., Ergashov M., Tavbaev Zh.S. Strength of flexible elements and pipelines of drilling rigs. Bishkek. Ilim. 2001. 251 p.
5. Ergashov M., Tavbaev Zh.S. Strength of pipelines of drilling rigs. Tashkent. Fan. 2002. 119 p.