The Enemy boundary value problem for a class of third-order equations

Abulov M.O.

Institute of Military Aviation of the Republic of Uzbekistan, Karshi, Uzbekistan
Abulov1959@mail.ru

ABSTRACT

The Vrag boundary value problem for a third-order equation of a mixed-compound type in a quadrilateral domain is considered. Using the Galerkin method under certain conditions for the coefficients and the right side of the equation, the existence of a weakly generalized solution in Sobolev space has been proved. Under the same conditions, the uniqueness of the generalized solution has been proved.

Keywords: *Vragov boundary value problem, mixed-compound type, generalized solution, third-order equation.*

Introduction

It is known that the study of boundary value problems for nonclassical equations of mathematical physics is of considerable mathematical interest due to the importance of their applications in various branches of mechanics, physics, and engineering. Quite a lot of works are devoted to the formulation of boundary value problems for equations of mixed type of the second and high order, of the mixed-composite type, the study of their generalized and fredholm solvability in various spaces [1 – 13].

This paper investigates the generalized solvability of the Vragov boundary value problem [4,5,12] for a third-order equation of a mixed-compound type.

In this area, $Q=\{(x,t):-1\le x\le 1,\ 0\le t\le T\}$ consider the equation

$$Lu\equiv k(x,t)u_{ttt}+\mu(x)u_{xxx}+a(x,t)u_{xx}+b(x,t)u_{tt}=f(x,t) \qquad (1)$$

where $x\mu(x)>0$ at the $x\neq 0,\ \mu(0)=0.$

Put

$$P_0^+=\{(x,t):k(x,0)>0,\ x\in[-1,1]\},\ P_0^-=\{(x,t):k(x,0)<0,\ x\in[-1,1]\},$$

$$P_T^+=\{(x,t):k(x,T)>0,\ x\in[-1,1]\},\ P_T^-=\{(x,t):k(x,T)<0,\ x\in[-1,1]\}.$$

The problem of the enemy. Find the Q solution of equation (1) in the domain that satisfies the boundary conditions

$$u\big|_{\partial Q}=0,\quad u_t\big|_{\bar{P}_0^+}=0,\quad u_t\big|_{\bar{P}_T^-}=0. \qquad (2)$$

For the sake of simplicity, we will assume that the coefficients of equation (1) are infinitely differentiable functions.

Definition 1. Let us denote in terms $H(Q)$ of the space of functions obtained by closing functions from $C^{\infty}(Q)$, satisfying conditions (2) according to the norm

$$\|u\|_{H(Q)} = \int_Q (u_{xx}^2 + u_{xt}^2 + u_x^2 + u_t^2 + u^2)dQ$$

Definition 2. A function $u \in H(Q)$ will be called a weak generalized solution of problem (1), (2), if $v \in C_0^{\infty}(Q)$ the identity is fulfilled for all

$$\int_Q (ku_t v_{tt} + 2k_t u_t v_t + k_{tt} u_t v - \mu u_{xx} v_x - \mu_x u_{xx} v + au_{xx} v - bu_t v_t - b_t u_t v)dQ = \int_Q fvdQ \quad (3)$$

Theorem. Let the conditions be fulfilled

$$a(x,t) - \frac{3}{2}|\mu_x| \geq \delta > 0, \quad b(x,t) - \frac{3}{2}|k_t| \geq \delta_1 > 0, \quad (4)$$

then, for any function $f(x,t)$ such that $f \in L_2(Q)$, there is a single solution to the problem (1), (2) of $H(Q)$.

Proof. The solution to problem (1) and (2) will be searched by the Galerkin method

$$u_m(x,t) = \sum_{i=1}^{m} j_i(t)\varphi_i(x)$$

where functions $\varphi_i(x)$ are solutions to a problem

$$\varphi_i'' = -\lambda_i \varphi_i, \quad \varphi_i(-1) = \varphi_i(1) = 0.$$

And the coefficients $j_i(t)$ are derived from the solution of the system of ordinary differential equations

$$(ku_{mttt}, \varphi_i)_0 + (\mu u_{mxxx}, \varphi_i)_0 + (au_{mxx}, \varphi_i)_0 + (bu_{mtt}, \varphi_i)_0 = (f, \varphi_i)_0 \quad (5)$$

$$j_i(0) = j_i(T) = j_{it}(0)\Big|_{\overline{P}_0^+} = 0 = j_{it}(T)\Big|_{\overline{P}_T^-} = 0 \qquad i = 1, 2, \ldots, m \quad (6)$$

The solvability of problem (5) and (6) with a fixed one m follows from the general theory by ordinary differential equations.

By virtue of the boundary conditions (2), it is not difficult to see that the correct estimate for the solution is

$$\int_Q u_m^2 dQ \leq C \int_Q u_{mx}^2 dQ \quad (7)$$

Let's get uniform estimates m for Galerkin approximations. To do this, multiply (5) by $-j_i(t)$ and, summing up by i, we get

$$(ku_{mttt}, -u_m)_0 + (\mu u_{mxxx}, -u_m)_0 + (au_{mxx}, -u_m)_0 + (bu_{mtt}, -u_m)_0 = (f, -u_m) \quad (8)$$

Hence, by integrating and t, integrating in parts, in force (2), (4), (7), after some transformations, we arrive at inequality

$$\int_Q (u_{mt}^2 + u_{mx}^2 + u_m^2)dQ \leq C \quad (9)$$

Next, consider the following equations

$$(ku_{mttt}, u_{mxx})_0 + (\mu u_{mxxx}, u_{mxx})_0 + (au_{mxx}, u_{mxx})_0 + (bu_{mtt}, u_{mxx})_0 = (f_m, u_{mxx})_0 \quad (10)$$

From identity (10), in virtue of (2), (4) and evaluation (9), integrating and integrating in t, parts, after simple transformations, the following evaluation follows

$$\int_Q (u_{mxx}^2 + u_{mxt}^2)dQ \leq C. \quad (11)$$

From the estimates (9), (11) follows the limitation of the sequence of approximate solutions $\{u_m(x,t)\}$ in space $H(Q)$, we can select the subsequences $\{u_{m_k}(x,t)\}$ and proceed to the limit of the $m_k \to \infty$ system (5). It is not difficult to verify that the limit function belongs to space $H(Q)$ and satisfies the identity (3). Since the system $\{\varphi_i(x)\}$ is dense in $L_2(-1,1)$.

Let us prove that the solution of problem (1) and (2) is unique.

If $u, v -$ two solutions to the problem (1), (2), then $w = u - v$ satisfies the equation

$$kw_{ttt} + \mu(x)w_{xxx} + a(x,t)w_{xx} + b(x,t)w_{tt} = 0$$

Consider the integral

$$\int_Q (kw_{ttt} + \mu(x)w_{xxx} + a(x,t)w_{xx} + b(x,t)wdQ = 0$$

and integrating in parts, in force (2) we get

$$\int_Q (w_t^2 + w_x^2 + w^2)dQ \le 0$$

Hence it follows that $w = 0$ in the Q.

Theorem proved.

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