Development of Creative Skills of Students in Solution of Some Problems of Vectoral, Mixed and Double Multiplications of Vectors

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ABSTRACT

This article considers some aspects of the development of creative activities of students in solving some problems related to vectorial and mixed multiplication, and gives concrete examples of the use of each studied method in the teaching of analytical geometry.

Keywords: Vector multiplication, mixed multiplication, volume of tetrahedron, double vector [1]multiplication, properties.

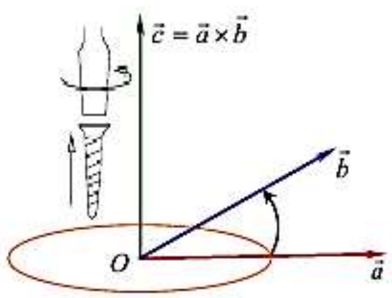
Definition: $\vec{a}$ expressed as the $\bar{c}$ vector product of vectors $\vec{a}\times\vec{b}$ and , $\vec{b}$ it is said that the vector satisfies the following three conditions [2]:

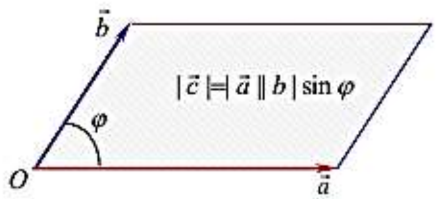
1^0. $\vec{a}\times\vec{b} = |\bar{c}| = |\vec{a}||\vec{b}|\sin(\vec{a}\wedge\vec{b})$

2^0. $\bar{c}\perp\vec{a}, \bar{c}\perp\vec{b}$

3^0. $\vec{a}$, $\vec{b}$, $\bar{c}$ vectors to the common head, and $\bar{c}$ from the end $\vec{a}$ of , $\vec{b}$ when viewed in the plane where the vectors lie, let the shortest path turn from the $\vec{a}$ vector $\vec{b}$ in the direction of the vector be counter-clockwise. Vector multiplication is $\vec{a}$ $\vec{b}$ denoted as [[3]], $\vec{a}\times\vec{b}$ or $\bar{c}$ =[$\vec{a}$ $\vec{b}$].

According to property 1 in the given definition, the $\bar{c}$ length of a vector is equal to the face of a parallelogram consisting of sides $\vec{a}$ and $\vec{b}$ vectors. Property 2 means that it is a vector product (that is, a $\bar{c}$ vector) $\vec{a}$ and $\vec{b}$ is perpendicular to the plane on which the vectors lie.





Vector multiplication has the following properties.

1^0. [$\vec{a}$ $\vec{b}$]=0 if at least one of the multiplier vectors is a zero vector or $\vec{a}$ // $\vec{b}$.

Proof: If indeed $\vec{a} // \vec{b}$ then [$\vec{a}\ \vec{b}$]=0. If they are parallel, the angle between them is 0 °or 180 °, so sin($\vec{a}$ ^ $\vec{b}$)=0, and according to condition 1 is perpendicular to the plane, but in the product [$\vec{a}\ \vec{b}$] $\vec{a}$, the $\vec{b}$ right $\vec{c}$ vector product is the zero vector .

2 ⁰. If the positions of the multipliers of the vector multiplication are interchanged, the sign of the vector multiplication changes: [$\vec{a}\ \vec{b}$]=-[$\vec{b}\ \vec{a}$]

Proof: In fact, according to clauses 1 and 2 of the definition of vector multiplication, vectors [$\vec{a}\ \vec{b}$] and [$\vec{b}\ \vec{a}$] have equal lengths and both are perpendicular to one plane, but [$\vec{a}\ \vec{b}$] in multiplication $\vec{a}$ Since forms the $\vec{b}$ right triple and [$\vec{b}\ \vec{a}$] forms the left triple, we create a vector [$\vec{a}\ \vec{b}$] opposite to the direction [$\vec{b}\ \vec{a}$] .

3^0. These relations hold for any real number λ

λ[$\vec{a}\ \vec{b}$]=[λ$\vec{a}\ \vec{b}$]= $\vec{a}$ [λ$\vec{b}$]

4^0. The distributive law holds true for vector multiplication.

[$\vec{a}$ ($\vec{b}$ + $\vec{c}$)]=[$\vec{a}\ \vec{b}$]+[$\vec{a}\ \vec{c}$]

1. Vector products of unit vectors are as follows.

   [$\vec{i}\ \vec{j}$]= – [$\vec{j}\ \vec{i}$]= $\vec{k}$; [$\vec{i}\ \vec{i}$]=0

   [$\vec{k}\ \vec{i}$]= –[$\vec{i}\ \vec{k}$]= $\vec{j}$; [$\vec{j}\ \vec{j}$]=0

   [$\vec{j}\ \vec{k}$]= –[$\vec{k}\ \vec{j}$]= $\vec{i}$; [$\vec{k}\ \vec{k}$]=0

If in the Cartesian coordinate system $\vec{a}$ and $\vec{b}$ given by vector coordinates, i.e

$\vec{a} = a_x\vec{i} + a_y\vec{j} + a_z\vec{k}$ $\vec{b} = b_x\vec{i} + b_y\vec{j} + b_z\vec{k}$, then [$\vec{a}\ \vec{b}$]=($a_x\vec{i} + a_y\vec{j} + a_z\vec{k}$)($b_x\vec{i} + b_y\vec{j} + b_z\vec{k}$)=($a_y b_z - a_z b_y$) $\vec{i}$ -($a_x b_z - a_z b_x$) $\vec{j}$ +($a_x b_y - a_y b_x$) $\vec{k}$ =

$$= \begin{vmatrix} a_y & a_z \\ b_y & b_z \end{vmatrix}\vec{i} - \begin{vmatrix} a_x & a_z \\ b_x & b_z \end{vmatrix}\vec{j} + \begin{vmatrix} a_x & a_y \\ b_x & b_y \end{vmatrix}\vec{k} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix} \quad or\ [\vec{a}\ \vec{b}] = \left\{ \begin{vmatrix} a_y & a_z \\ b_y & b_z \end{vmatrix}; -\begin{vmatrix} a_x & a_z \\ b_x & b_z \end{vmatrix}; \begin{vmatrix} a_x & a_y \\ b_x & b_y \end{vmatrix} \right\}$$

a formula for calculating the area of a triangle can be derived using vector multiplication. Let ABC be given by the coordinates of the vertices of the triangle;

A (x_1,y_1,z_1,) , B(x_2,y_2,z_2) , C(x_3,y_3,z_3) formed according to the definition of vector multiplication the modulus of the vector is equal to the area of the parallelogram. And half of it gives the face of the triangle;

$$S_{ABC} = \frac{1}{2}\left|\left[\overrightarrow{AB} \cdot \overrightarrow{AC}\right]\right|$$

three $\vec{a}$, $\vec{b}$, $\vec{c}$ vectors be given.

Definition: $\vec{a}$, $\vec{b}$ and $\vec{c}$ is a mixed product of vectors (according to the specified order of vectors) $\vec{a}$ and $\vec{b}$ is the number obtained by scalar multiplication of a vector by a vector equal to the vector product of vectors . $\vec{c}$

A mixed product is specified as [$\vec{a}\ \vec{b}$] or ($\vec{a}\ \vec{b}\ \vec{c}$).

A mixed product has the following geometric meaning. $\vec{a}$, $\vec{b}$, $\vec{c}$ vectors placed at a point O and form a non-coplanar right triad. We make a parallelepiped whose edges consist of these vectors. $\left|[\bar{a}\bar{b}]\right|$ we see that the quantity represents the face of this parallelepiped. According to the definition of scalar multiplication: $\left|[\bar{a}\bar{b}]\right| \cdot \vec{c} = \left|[\bar{a}\bar{b}]\right| \cdot \left|\vec{c}\right| \cos\varphi$.

[4]Being | $\vec{c}$ | cosφ here $\varphi = (([\vec{a}\vec{b}])\hat{}\,\vec{c})$ is equal to the straight line projection of the vector in the direction and is the height of the parallelepiped | $\vec{c}$ | cosφ = $\vec{c}$ h

So [$\vec{a}\ \vec{b}$] $\vec{c}$ =S_{base} h=V. Here V is the volume of the parallelepiped. So the volume of the parallelepiped: V= |[$\vec{a}\ \vec{b}$] $\vec{c}$ | [5]

={ a_x, a_y, a_z} $\vec{b}$ ={b_x,b_y,b_z} and $\vec{c}$ ={c_x,c_y,c_z} are given in the coordinate system $\vec{a}$

R={o, $\vec{i}$, $\vec{j}$,} $\vec{k}$. Let's find the mixed product of three vectors given by their coordinates.

$\vec{a}$ and $\vec{b}$ the vector product of vectors is:

$$[\vec{a}\,\vec{b}]=\begin{vmatrix} a_y & a_z \\ b_y & b_z \end{vmatrix}\vec{i}-\begin{vmatrix} a_x & a_z \\ b_x & b_z \end{vmatrix}\vec{j}+\begin{vmatrix} a_x & a_y \\ b_x & b_y \end{vmatrix}\vec{k}$$

Now $\vec{c}$ we multiply the resulting vector by $\vec{a}$ vector scalar:

$$[\vec{a}\,\vec{b}]\,\vec{c}= c_x\begin{vmatrix} a_y & a_z \\ b_y & b_z \end{vmatrix}-c_y\begin{vmatrix} a_x & a_z \\ b_x & b_z \end{vmatrix}+c_z\begin{vmatrix} a_x & a_y \\ b_x & b_y \end{vmatrix}=\begin{vmatrix} c_x & c_y & c_z \\ b_x & b_y & b_z \\ a_x & a_y & a_z \end{vmatrix}$$

We write this determinate in the following form:

$$[\vec{a}\,\vec{b}]\,\vec{c}=\begin{vmatrix} a_x & a_y & a_z \\ b_x & b_y & b_z \\ c_x & c_y & c_z \end{vmatrix}$$

an application of this formula, let's derive the formula for calculating the tetrahedron volume based on the coordinates of its vertices *Points A* (x_1,y_1,z_1), B(x_2,y_2,z_2), C(x_3,y_3,z_3) and D(x_4,y_4,z_4) of the tetrahedron be the ends.

$\overline{AB}=\{x_2-x_1,\ y_2-y_1, z_2-z_1\}$, $\overline{AC}=\{x_3-x_1, y_3-y_1, z_3-z_1\}$ $\overline{AD}=\{x_4-x_1, y_4-y_1, z_4-z_1\}$

Since the volume of a tetrahedron is equal to 1/6 of the volume of a parallelepiped built on three edges from one end of the tetrahedron

$$V_{tetr}=\frac{1}{6}|(\overline{AB}\,\overline{AC}\,\overline{AD})|=\frac{1}{6}\bmod\begin{vmatrix} x_2-x_1 & x_3-x_1 & x_4-x_1 \\ y_2-y_1 & y_3-y_1 & y_4-y_1 \\ z_2-z_1 & z_3-z_1 & z_4-z_1 \end{vmatrix}$$

A mixed product has the following properties.

1^0. $(\vec{a}\ \vec{b}\ \vec{c})=(\vec{b}\ \vec{c}\ \vec{a})=(\vec{c}\ \vec{a}\ \vec{b})$ Indeed, the absolute values of the volumes of the parallelepiped constructed from these three vectors are equal.

2^0. $(\vec{a}\ \vec{b}\ \vec{c})=-(\vec{b}\ \vec{a}\ \vec{c})$, because $(\vec{a}\ \vec{b}\ \vec{c})=[\vec{a}\ \vec{b}]\ \vec{c}=-[\vec{b}\ \vec{a}]\ \vec{c}=-(\vec{b}\ \vec{a}\ \vec{c})$ means $(\vec{a}\ \vec{b}\ \vec{c})=-(\vec{b}\ \vec{a}\ \vec{c})$, $(\vec{b}\ \vec{c}\ \vec{a})=-(\vec{c}\ \vec{b}\ \vec{a})$, $(\vec{c}\ \vec{a}\ \vec{b})=-(\vec{a}\ \vec{c}\ \vec{b})$

3^0. - $((\vec{a}+\vec{b})\ \vec{c}\ \vec{d})=(\vec{a}\ \vec{c}\ \vec{d})+(\vec{b}\ \vec{c}\ \vec{d})$ because $((\vec{a}+\vec{b})\ \vec{c}\ \vec{d})=[\vec{a}+\vec{b}, \vec{c}]\ \vec{d}=([\vec{a}\ \vec{c}]+[\vec{b}\ \vec{c}])\ \vec{d}$ $=[\vec{a}\ \vec{c}]\ \vec{d}+[\vec{b}\ \vec{c}]\ \vec{d}=(\vec{a}\ \vec{c}\ \vec{d})+(\vec{b}\ \vec{c}\ \vec{d})$

4^0.$\forall\ \lambda\in$for R $(\lambda\vec{a}\ \vec{b}\ \vec{c})=\lambda(\vec{a}\ \vec{b}\ \vec{c})$ because $(\lambda\vec{a}\ \vec{b}\ \vec{c})=[\lambda\vec{a}\ \vec{b}]\ \vec{c}=\lambda[\vec{a}\ \vec{b}]\ \vec{c}=\lambda(\vec{a}\ \vec{b}\ \vec{c})$

5^0 . - Coplanar $\vec{a}$, $\vec{b}$ and $\vec{c}$ vectors the mixed product is equal to 0 , because the parallelepiped constructed from these vectors is in the plane, its height is equal to zero, and vice versa $(\vec{a}\ \vec{b}\ \vec{c})=0$ from which $\vec{a}$, $\vec{b}$, $\vec{c}$ vectors are coplanar.

$\forall$ Let $\vec{a}$, $\vec{b}$, $\vec{c}$ be a given vector. For them $\vec{a}\cdot(\vec{b}\cdot\vec{c})$ vector is called double vector product . We show the simplest rule for finding double vector multiplication by the following theorem.

Theorem 1: for $\forall$ three $\vec{a}$, $\vec{b}$, $\vec{c}$ vectors this equality $\vec{a}\,(\vec{b}\cdot\vec{c})=(\vec{a}, \vec{c})\ \vec{b}-(\vec{a}, \vec{b})\vec{c}$ holds.

Proof : Let arbitrary vectors be $\vec{a}$ of $\vec{b}$,$\vec{c}$ the form $\vec{a}=a_1\vec{i}+a_2\vec{j}+a_3\vec{k}$, $\vec{b}=b_1\vec{i}+b_2\vec{j}+b_3\vec{k}$ $\vec{c}=c_1\vec{i}+c_2\vec{j}+c_3\vec{k}$. Then the $\vec{b}$ vector product of $\vec{c}$

$$\vec{b}\cdot\vec{c}=\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}=\begin{vmatrix} b_2 b_3 \\ c_2 c_3 \end{vmatrix}\vec{i}-\begin{vmatrix} b_1 b_3 \\ c_1 c_3 \end{vmatrix}\vec{j}+\begin{vmatrix} b_1 b_2 \\ c_1 c_2 \end{vmatrix}\vec{k}$$

gives the vector. Now $\vec{a}$ we vectorially multiply the vector ( ) by the vector: $\vec{b}\ \vec{c}$

$$\vec{a}\cdot(\vec{b}\cdot\vec{c})=\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a_1 & a_2 & a_3 \\ \begin{vmatrix} b_2 & b_3 \\ c_2 & c_3 \end{vmatrix} & -\begin{vmatrix} b_1 & b_3 \\ c_1 & c_3 \end{vmatrix} & \begin{vmatrix} b_1 & b_2 \\ c_1 & c_2 \end{vmatrix} \end{vmatrix}=$$

$$[\left(a_2b_1c_2-a_2b_2c_1+a_3b_1c_3-a_3b_3c_1\right)\vec{i}+\left(a_1b_2c_1-a_1b_1c_2+\ +a_3b_2c_3-\ a_3b_3c_2\right)\vec{j}+$$

$$+\left(a_1b_3c_1-a_1b_1c_3+a_2b_3c_2-a_2b_2c_3\right)\vec{k}]=\left\{b_1\left(a_1c_1+a_2c_2+a_3c_3\right)-c_1(a_1b_1+a_2b_2+a_3b_3\right\}\vec{i}+$$

$$+\left\{b_2\left(a_1c_1+a_2c_2+a_3c_3\right)-c_2\left(a_1b_1+a_2b_2+a_3b_3\right)\right\}\vec{j}+\left\{b_3\left(a_1c_1+a_2c_2+a_3c_3\right)-c_3\left(a_1b_1+a_2b_2+a_3b_3\right)\right\}\vec{k}=$$

$$=(\vec{a}\,\vec{c})\,\vec{b}-(\vec{a}\,\vec{b})\,\vec{c}$$

Theorem 2 : This equality holds for arbitrary three $\vec{a}$, $\vec{b}$, $\vec{c}$ vectors
$\vec{a}$ x($\vec{b}$ x $\vec{c}$)+ $\vec{b}$ x ($\vec{c}$ x $\vec{a}$)+ $\vec{c}$ x ($\vec{a}$ x $\vec{b}$)=0

Proof : according to theorem 1 $\vec{a}$ x($\vec{b}$ x $\vec{c}$)=($\vec{a}\cdot\vec{c}$) $\vec{b}$ -($\vec{a}\cdot\vec{b}$) $\vec{c}$, $\vec{b}$ x ($\vec{c}$ x $\vec{a}$)=($\vec{b}\cdot\vec{a}$) $\cdot\vec{c}$ -($\vec{b}\cdot\vec{c}$)$\cdot\vec{a}$
$\vec{c}$ x ($\vec{a}$ x $\vec{b}$)=($\vec{c}\cdot\vec{b}$) $\cdot\vec{a}$ -($\vec{c}\cdot\vec{a}$)$\cdot\vec{b}$

Adding these equations and using the symmetry of scalar [6]multiplication gives the above equation.

Example 1 : Calculate the sine of the angle between the diagonals of the parallelogram made of the following vectors. $\vec{a}=2\vec{m}+\vec{n}-\vec{p}$ *va* $\vec{b}=\vec{m}-3\vec{n}+\vec{p}$, *in this* $\vec{m}$, $\vec{n}$, $\vec{p}$ are mutually perpendicular sides. Calculate the length of this vector. [7]

Solution :
$\vec{a}(2;1;-1)$ *and* $\vec{b}(1;-3;1)$ $\vec{m},\vec{n},\vec{p}$ *since they are unit vectors and reciprocals* $\vec{m}\cdot\vec{n}=\vec{m}\cdot\vec{p}=\vec{n}\cdot\vec{p}=0$ will be. $\vec{a}+\vec{b}=(3;-2;0)$ *and* $\vec{a}-\vec{b}=(1;4;-2)$, from this

$$\cos\alpha=\frac{(\vec{a}+\vec{b})\cdot(\vec{a}-\vec{b})}{\left|\vec{a}+\vec{b}\right|\cdot\left|\vec{a}-\vec{b}\right|}=\frac{3-8+0}{\sqrt{13}\cdot\sqrt{21}}=\frac{-5}{\sqrt{273}},\sin\alpha=\sqrt{1-\cos^2\alpha}\ \textit{from being}\ \sin\alpha=\sqrt{1-\frac{25}{273}}=\sqrt{\frac{248}{273}}$$

$$\vec{c}=[\vec{a}+\vec{b};\vec{a}-\vec{b}]=\begin{vmatrix} \vec{m} & \vec{n} & \vec{p} \\ 3 & -2 & 0 \\ 1 & 4 & -2 \end{vmatrix}=4\vec{m}+12\vec{p}+2\vec{p}+6\vec{n}=4\vec{m}+6\vec{n}+14\vec{p}$$

$$[\vec{a}+\vec{b};\vec{a}-\vec{b}]=\vec{c}(4;6;14),\left|\vec{c}\right|=\sqrt{16+36+144}=\sqrt{248}$$

Example 2 : $\vec{a}(3;-1;-2)$ *and* $\vec{b}(0;-2;4)$ *if* , $(\vec{a}+2\vec{b})\times(2\vec{a}-3\vec{b})$ find the product.

Solving . First $\vec{m}=\vec{a}+2\vec{b}$ *and* $\vec{n}=2\vec{a}-3\vec{b}$ we find the coordinates of the vectors.

this $\vec{m}$ ={1·3+2·0;1·(-1)+2·(-2);1(-2)+2·4}={3;-5;6}, $\vec{n}$ ={2·3 -3·0;2·(-1)-3·(-2);2(-2)-3·4}={6;-8;-16}

$$\vec{m}\times\vec{n}=\begin{vmatrix} -5 & 6 \\ -8 & -16 \end{vmatrix}\vec{i}-\begin{vmatrix} 3 & 6 \\ 6 & -16 \end{vmatrix}\vec{j}+\begin{vmatrix} 3 & -5 \\ 6 & -8 \end{vmatrix}\vec{k}=128\vec{i}+84\vec{j}+6\vec{k}$$

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