Methods Of Solving Equations Related to Whole and Fractional Part of a Number

N.Rakhimov, Uzbek-Finnish Pedagogical Institute
B.Mamasidikov Uzbek-Finnish Pedagogical Institute

ABSTRACT

This article is devoted to equations related to integer and fractional parts of numbers, and methods for their solution. The article is aimed at developing students' knowledge and skills related to the subject.

Keywords: Integer part of a number, fractional part, equation, definition, property, problem, solution, integral

It is known that the subject of whole numbers and fractions is not included in the school mathematics course, and very few hours are allocated in academic lyceums. But there are many questions related to this topic in mathematics Olympiads and various examinations. Finding information on the subject in the literature is a bit more difficult. Therefore, in this article, we have presented some information on the topic and some recommendations on methods of solving equations related to the whole and fractional parts of the number.

Definition. The largest integer n that does not exceed the real number x is called the integer part of the number x and it is defined as [x].

Definition. A fractional part of a number is a part of it that is not less than zero, but less than one, and is denoted as {x}. The fractional part of the number consists of the interval 0≤{x}<1.

Properties of integer and fractional parts of a number:

1. Any real number x can be written as *x=[x]+{x}.*

2. The fractional part of a number is found using the formula *{x}=x-[x].*

3. For $n \in N$ and *[x]=n*, then $n \le x < n+1$ the inequality holds. If *[x]=-n*, then $-n-1 < x \le -n$ the inequality holds.

4. For the entire part of the number, the equality *[x+1]=[x]+1* is appropriate. In general, the relation *[x+n]=[x]+n* holds for any integer *n*.

5. For the fractional part of the number, the equality *{x+1}={x}* is appropriate. The relation *{x+n}={x}* is also fulfilled for an arbitrary integer *n*.

7. If the equality *{x}={y}* holds, then the difference *x-y* is an integer.

8. For any real number x, the equality *{x}+{-x}=1* is valid.

9. If $x, y \in Z$ so, the equality *[x+y]=[x]+[y]* is valid. If $x, y \in R$ so, the $[x+y] \ge [x]+[y]$ attitude will be appropriate. [3]

Here are some ways to solve problems related to whole and fractional parts of a number.

Issue 1. Solve the equation. *[x]+[3x]+[5x]=4*

Solution: From the equation, we can see that the problem at $x<0$ and $x>1$ has no solution. Now we will consider all possible cases.

1) $[x]=0\ \ 0\le x<1$

$[3x]=2\ \frac{2}{3}\le x<1$

$[5x]=2\ \frac{2}{5}\le x<\frac{3}{5}$ from this $\frac{2}{5}\le x<\frac{3}{5}$

$[x]=0\quad[3x]=1\quad[5x]=2\quad 0+1+2=3\ne 4\ \emptyset$

2) $[x]=0\ \ 0\le x<1$

$[3x]=1\ \frac{1}{3}\le x<\frac{2}{3}$

$[5x]=3\ \frac{3}{5}\le x<\frac{4}{5}$ from this $\frac{3}{5}\le x<\frac{4}{5}$

$[x]=0\quad[3x]=1\quad[5x]=3$ 0+1+3=4. Hence the answer. $\frac{3}{5}\le x<\frac{4}{5}$

Issue 2. Solve the equation. *[x]+[2x]+[3x]=3* [2]

Solution: From the equation, we can see that the problem at $x<0$ and $x>1$ has no solution. Now we will consider all possible cases.

$[x]=0\ \ 0\le x<1$

$[2x]=1\ \ \frac{1}{2}\le x<1$

$[3x]=2\ \ \frac{2}{3}\le x<1$ from this

$\frac{2}{3}\le x<1\quad[x]=0\quad[2x]=1\quad[3x]=2$ 0+1+2=3. Answer. $\frac{2}{3}\le x<1$

Issue 3. $[sinx]\{sinx\}=sinx$ solve the equation.

Solution: $[x]+\{x\}=x\quad[sinx](sinx-[sinx])=sinx$

1) $-1<sinx<0$

$[sinx]=-1,\quad sinx=\frac{1}{2},\quad x=\pm\frac{\pi}{6}+\pi n$

2) $0<sinx<1$

$[sinx]=0,\quad sinx=0,\quad x=\pi n$

3) $sinx-1=sinx\ \ \emptyset$

Issue 4. Solve the equation. $[cosx]\{cosx\}=cosx$.

Solution: $[x]+\{x\}=x\quad[cosx](cosx-[cosx])=cosx$

1) -1 $<cosx<0\qquad[cosx]=0>-1$

$cosx=-\frac{1}{2}\quad x\in\mp\frac{\pi}{3}+2\pi n$

2) $0<cosx<1\ \ [cosx]=0\ \ cosx=0\ \ x=\frac{\pi}{2}+\pi n$

3) $cosx=1,\ \ cosx-1=cosx,\quad\emptyset$

Issue 5. $\int_1^3[e^x]dx$ calculate the integral.

Solution: In order to calculate this integral, we need to know what powers of *e* are integers.

$e^1=2.71\ldots$

$e^{a_0}=3\qquad a_0=ln3$

$e^{a_1}=4\qquad a_1=ln4$

$e^{a_2}=5\qquad a_2=ln5$

……………………………

$e^{a_{16}}=19\qquad a_{16}=ln19$

$e^{a_{17}}=20\qquad a_{17}=ln20$

And from that, $\int_1^3[e^x]dx=\int_1^{a_0}[e^x]\,dx+\int_{a_0}^{a_1}[e^x]\,dx+\int_{a_1}^{a_2}[e^x]\,dx+\cdots+\int_{a_{16}}^{a_{17}}[e^x]\,dx+\int_{a_{17}}^{3}[e^x]\,dx=\int_1^{a_0}2dx+\int_{a_0}^{a_1}3dx+\int_{a_1}^{a_2}4dx+\int_{a_2}^{a_3}5dx+\cdots+\int_{a_{16}}^{a_{17}}19dx+\int_{a_{17}}^{3}20dx=2x|_1^{a_0}+3x|_{a_0}^{a_1}+4x|_{a_1}^{a_2}+5x|_{a_2}^{a_3}+\cdots+19x|_{a_{16}}^{a_{17}}+20x|_{a_{17}}^{3}=2(a_0-1)+3(a_1-a_0)+4(a_2-a_1)+\cdots+19(a_{17}-a_{16})+20(3-a_{17})=58-a_0-a_1-a_2-a_3-a_4-\cdots-a_{16}-a_{17}=58-ln3-ln4-ln5-ln6-\cdots-ln19-ln20=58-(ln3+ln4+ln5+\cdots+ln19+ln20)=58-ln3*4*5*\ldots*19*20=58-ln\frac{20!}{2}$

Issue 6. Calculate the integral. $\int_1^6 [x]dx$

Solution: $\int_1^6 [x]dx = \int_1^2 dx + \int_2^3 2dx + \int_3^4 3dx + \int_4^5 4dx + \int_5^6 5dx =$

$= x|_1^2 + 2x|_2^3 + 3x|_3^4 + 4x|_4^5 + 5x|_5^6 = 1 + 2 + 3 + 4 + 5 = 15$

Issue 7. $\int_1^3 \{x^{[x]}\} dx$ calculate the integral.

Solution: $\int_1^3 \{x^{[x]}\} dx = \int_1^3 (x - [x])^{[x]} dx = \int_1^2 (x-1)dx + + \int_2^3 (x-2)^2 dx = \left(\frac{x^2}{2} - x\right)\Big|_1^2 + \left(\frac{x^2}{2} - 2x^2 + 4x\right)\Big|_2^3 = 2 - 2 - \frac{1}{2} + 1 + \frac{9}{2} - -18 + 12 - 2 + 8 - 8 = 5$

Issue 8. $\int_1^4 [x]\{x\}dx$ calculate the integral.

Solution: $\int_1^4 [x]\{x\}dx = \int_1^4 [x](x - [x])dx = \int_1^2 (x-1)dx + + \int_2^3 2(x-2)dx + \int_3^4 3(x-3)dx = \left(\frac{x^2}{2} - x\right)\Big|_1^2 + 2\left(\frac{x^2}{2} - 2x\right)\Big|_2^3 + +3\left(\frac{x^2}{2} - 3x\right)\Big|_3^4 = \frac{1}{2} - 3 + 4 + 15 - \frac{27}{2} = 3$

Issue 9. $\int_1^4 \frac{\{x\}}{[x]} dx$ calculate the integral.

Solution: $\int_1^4 \frac{\{x\}}{[x]} dx = \int_1^4 \frac{x - \{x\}}{[x]} dx = \int_1^2 \frac{x-1}{1} dx + \int_2^3 \frac{x-2}{2} dx + \int_3^4 \frac{x-3}{3} dx = \left(\frac{x^2}{2} - x\right)\Big|_1^2 + \left(\frac{x^2}{4} - x\right)\Big|_2^3 + \left(\frac{x^2}{6} - x\right)\Big|_3^4 = \frac{1}{2} + \frac{1}{4} + \frac{3}{2} - \frac{4}{3} = \frac{11}{12}$ [1]

In conclusion, it will be effective if the methods covered in the subject are used in the preparation of schoolchildren for various Olympiads.

References

1. N. Rahimov. Non-standard problems in mathematics, part 1. Samarkand-2020
2. A.S. Yunusov Interesting mathematics and Olympiad problems. Tashkent. 2017 year
3. N. Rakhimov, B. Mamasidikov. Forming the ability of schoolchildren to solve problems related to whole and fractional parts of numbers. "Science and Education" Scientific Journal / ISSN 2181-0842. March 2022 / Volume 3 Issue 3 [1]