Use of Information Technologies and Computer Mathematics Systems in The Process of Teaching The Topic of Differential Equations

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ABSTRACT

The article provides information about information technology and computer mathematics systems. The advantages of Mathcad and Maple and how to use these programs to solve differential equations are explained in detail. In addition, the article contains several suggestions and recommendations for teachers of mathematics on the use of Mathcad and Maple in the educational process.

Keywords: information technology, computer mathematics systems, Mathcad, Maple, differential equation, educational process, graph, diagram, Runge-Kutta method, Euler method.

It is known that the use of information technologies is one of the new and effective forms of organizing the educational process.

The transition to the information society opens up new opportunities for modernization of educational content and teaching methods. The computer is becoming a powerful tool for systematizing mathematical knowledge and skills, forming a worldview and developing the student's mind. In the systematic use of computers in teaching mathematics, the following should be taken into account: In order to obtain the expected result, it is necessary to use the computer continuously during the educational process; It is necessary for the teacher to be familiar with the computer, to use a flexible methodology to use the educational material in various educational activities aimed at activating students [1].

A new fundamental-scientific direction **"computer mathematics"** has appeared in the field of mathematics and computer science, and it is now widely used in scientific calculations and educational processes. Currently, the rapid development of computer mathematics, computer industry and programming technologies is recognized as the basis of automation of educational, scientific-methodical and scientific research work. At the same time, as a result of the application of achievements in the field of modern information technologies, there are many software tools aimed at automating the solution of scientific-research, scientific-methodical, scientific-technical, engineering, financial and economic, chemical, biological problems. For example: universal software environments such as Mathematica, Maple, Matlab, Mathcad, Derive, Scientific, Workplace, Femlab, FeexPDE are among them. Two of these are widely used by professional mathematicians and researchers.

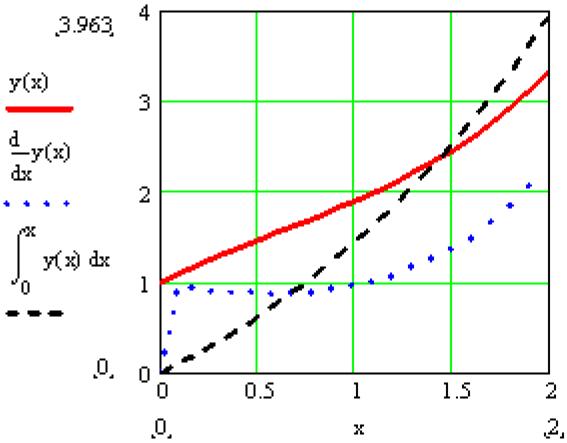
Mathcad was developed as a tool for engineering calculations, and is currently used for complex calculations, research, and various numerical algorithms and analytical substitutions. The use of Mathcad and Maple software environments, considered among the most advanced achievements in the field of information technology in the teaching of mathematics, is one of the main criteria for making the lesson interesting and effective. Mathcad is a computing software for professors, interns, researchers, graduate students, students, technical engineers, physicists, and more. With this program, various professions can solve problems related to their fields and get the necessary graphs and diagrams. Mathcad can also be called a programming language, in other words.

For example, when solving differential equations in Mathcad, the **Given-Odesolve** calculation blog can be used [2]:

Given $\quad y'''(x) - 2^{-x}\cdot y(x) = x\cdot\sin(x)$

$$y(0) = 1 \qquad y'(0) = 1 \qquad y''(0) = -0.5$$

$$y := \mathrm{Odesolve}(x, 2) \qquad x := 0, 0.1 .. 2$$



Here is an algorithm for solving a third-order variable coefficient differential equation.

It is known that in the educational process we are taught to solve a narrow class of differential equations, but in the educational process, we, as a result of the application of computer mathematics systems, have the opportunity to solve a wide class of differential equations, which ensures that the lesson is effective and interesting. Using the **Given-Odesolve** calculation blog in Mathcad, the solution can be derived from the function and integrated. This can be seen in the graph above. It should also be noted that Mathcad is not capable of finding general and analytical solutions of differential equations. This problem can be solved in Maple. The **dsolve(equation, variable, option)** command is used to solve differential equations in Maple, where the **equation** is a differential equation, the **variable** is the solution of a differential equation, and the **option** is an optional parameter, given in the form **keyword=value** [3]. The **dsolve** command in Maple can be used to find general and analytical solutions to a large number of differential equations. If the option **type=exact** is given, an analytical solution will be attempted. If the option **type=series** is given, then the solution will be searched in series form. If the option **type=numeric** is given, a numeric solution is sought. Now we will try to analytically solve the following differential equation:

$$\frac{\partial}{\partial x}\mathrm{y}(x)=\sqrt{x^2-\mathrm{y}(x)}+2x$$

> **eq:=diff(y(x),x)=sqrt(x^2-y(x))+2*x;**

$$eq:=\frac{\partial}{\partial x}\mathrm{y}(x)=\sqrt{x^2-\mathrm{y}(x)}+2x$$

> **dsolve(eq,y(x));**

$$8\frac{\mathrm{y}(x)\sqrt{x^2-\mathrm{y}(x)}}{2\sqrt{x^2-\mathrm{y}(x)}-x}+\frac{4\,\mathrm{y}(x)\,x}{2\sqrt{x^2-\mathrm{y}(x)}-x}-\frac{6x^2\sqrt{x^2-\mathrm{y}(x)}}{2\sqrt{x^2-\mathrm{y}(x)}-x}-\frac{3x^3}{2\sqrt{x^2-\mathrm{y}(x)}-x}-\_C1=0$$

The solution was found implicit.

> **isolate(%,y(x));** with the help of the command, we make the solution analytical:

$$\mathrm{y}(x)=\frac{5}{4}x^2+\frac{1}{2}(-x+\sqrt{-\_C1})\,x+\frac{1}{4}\_C1$$

If we want to find a particular solution satisfying the initial condition $\mathrm{y}(1)=0$, we can find the value of the constant **C1** using Maple's **solve** command:

> **x:=1;y:=0;solve(y = 5/4*x^2+1/2*(-x+sqrt(-C1))*x+1/4*C1,C1);**

$$x:=1$$
$$y:=0$$
$$-9$$

In that case, a private solution would look like this:

$$\mathrm{y}(x)=\frac{5}{4}x^2+\frac{1}{2}(-x+3)\,x-\frac{9}{4}.$$

Here are a few more examples of how to solve differential equations using the Maple program:

> **diff(y(x),x$2)-y(x)=sin(x)*x;**

$$\left(\frac{\partial^2}{\partial x^2}\mathrm{y}(x)\right)-\mathrm{y}(x)=\sin(x)\,x$$

> **dsolve(diff(y(x),x$2)-y(x)=sin(x)*x,y(x));**

$$\mathrm{y}(x)=-\frac{1}{2}\cos(x)-\frac{1}{2}\sin(x)\,x+\_C1\,\mathrm{e}^x+\_C2\,\mathrm{e}^{(-x)}$$

Here, a general solution was found. *C1* and *C2* in the solution are optional constants.

Initial conditions in differential equations are given by commas and combined with the equation:

> **restart;dsolve({diff(v(t),t)+2*t=0,v(1)=5},v(t));**

$$\mathrm{v}(t)=-t^2+6$$

Derivatives are written in the form of the operator in the initial conditions: D(D(y))(0) or D(@@2)(y)(0):

> **de1:=diff(y(t),t$2)+5*diff(y(t),t)+6*y(t)=0;**

$$de1:=\left(\frac{\partial^2}{\partial t^2}\mathrm{y}(t)\right)+5\left(\frac{\partial}{\partial t}\mathrm{y}(t)\right)+6\,\mathrm{y}(t)=0$$

> **dsolve({de1,y(0)=0,D(y)(0)=1},y(t),method=laplace);**

$$\mathrm{y}(t)=-\mathrm{e}^{(-3t)}+\mathrm{e}^{(-2t)}.$$

Now we solve the fourth-order equation:

> **de2:=diff(y(x),x$4)+2*diff(y(x),x$2)-cos(x)=3;**

$$de2:=\left(\frac{\partial^4}{\partial x^4}\mathrm{y}(x)\right)+2\left(\frac{\partial^2}{\partial x^2}\mathrm{y}(x)\right)-\cos(x)=3$$

> **dsolve(de2,y(x)):combine(%);**

$$y(x) = -\cos(x) - \frac{1}{2}\_C1\cos(\sqrt{2}\,x) - \frac{1}{2}\_C2\sin(\sqrt{2}\,x) + \frac{3}{4}x^2 + \_C3\,x + \_C4\,.$$

The solution to the following equation is found using the method of substitution of variables:

```
> restart;q:=(2*sqrt(x*y(x))-x)*diff(y(x),x)+y(x);
```

$$q := (2\sqrt{x\,y(x)} - x)\left(\frac{\partial}{\partial x}y(x)\right) + y(x)$$

To change a variable, the **Dchangevar** command of the **DEtools** package is used:

```
> restart;q:=(2*sqrt(x*y(x))-x)*diff(y(x),x)+y(x);
```

$$q := (2\sqrt{x\,y(x)} - x)\left(\frac{\partial}{\partial x}y(x)\right) + y(x)$$

```
> with(DEtools):f:=Dchangevar({y(x)=v(x)*x},[q],x);
```

$$f := (2\sqrt{x^2\,v(x)} - x)\left(\frac{\partial}{\partial x}v(x)\,x\right) + v(x)\,x\,.$$

Since the 50s of the 20th century, the kinetics of processes that occur under chemical reactions that take place at a very slow and sufficiently high speed at the same time began to be studied. Many such practical problems are brought to the solution of the Cauchy problem for ordinary differential equations and particular types of systems of ordinary differential equations. Such equations can be called *special differential equations* or *systems of special differential equations*. When numerically solving this type of differential equations and their system using the Runge-Kutta method, which is considered to be the most reliable, it was observed that the obtained solution changes slowly in the near-zero part of the integration interval, and suddenly changes when moving to the next part, that is, in the transition phase. The observed phenomenon means that other methods known from the course of computational mathematics, such as Runge-Kutta, Euler, etc., are not suitable for solving this type of equations. In practice, there are such differential equations of this class that it is necessary to integrate millions, billions, or even more points to numerically solve them by the methods mentioned above. The solution of special differential equations or their system consists of two parts. One of them is a function that changes slowly enough, and the other tends to zero with a large speed. There are certain practical difficulties in calculating the values of this second function. Consider the following Cauchy problem for example:

$$y'' + 101\cdot y' + 100\,y = 0 \qquad (1),$$

$$y(0) = 1.01, \qquad y'(0) = -2 \qquad (2).$$

Since the characteristic equation $k^2 + 101\cdot k + 100 = 0$ of this given second-order invariant coefficient homogeneous differential equation has $k_1 = -1$, $k_2 = -100$ solutions, the general solution of equation (1) is written in the form

$$y(x) = C_1\cdot e^{-x} + C_2\cdot e^{-100x} \qquad (3).$$

A particular solution satisfying the given initial conditions will look like this:

$$y(x) = e^{-x} + 0.01\cdot e^{-100x} \qquad (4).$$

The obtained analytical solution consists of the sum of two functions, the values of the first of which change relatively flat and slowly, and the values of the second function change rapidly and tend to zero with great speed.

The following table shows the approximate values of these two functions [0; 0.1] the law of change in the section is given:

Table.

The regularity of change of the approximate values of the function $y(x)=e^{-x}+0.01\cdot e^{-100x}$ *in the cross-section* $[0;\,0.1]$

x	$Y_1=e^{-x}$	$Y_2=0{,}01\cdot e^{-100x}$
0	1	0,01
0,00001	0,99999	0,009999
0,0001	0,9999	0,0099
0,001	0,999	0,009
0,01	0,99	0,004
0,1	0,9	0,0000004

As it can be seen from the values in the table, [0;0.1] is outside the cross section, that is, in the transition phase, the second additive of the solution will have such small values as to be negligible. From this, it can be concluded that it is necessary to numerically find the solution of problem (1)-(2) with a sufficiently small step in the section $[0;\,0.1]$ and to increase the integration step in order to save computer time and reduce rounding errors during the transition phase. Practical calculations have shown that this conclusion is incorrect. Because in order to obtain a stable solution using the familiar methods presented above, a sufficiently small integration step is required, which is the same in the entire part of the integration interval, due to the first function.

Such problems are easily solved in the Maple system. The general solution of the given equation is easily found in Maple as:

```
> restart;
> eq:=diff(y(x),x$2)+101*diff(y(x),x)+100*y(x)=0;
```

$$eq := \left(\frac{d^2}{dx^2}\,y(x)\right)+101\left(\frac{d}{dx}\,y(x)\right)+100\,y(x)=0$$

```
> dsolve(eq,y(x));
```

$$y(x)=\_C1\,e^{(-x)}+\_C2\,e^{(-100\,x)}$$

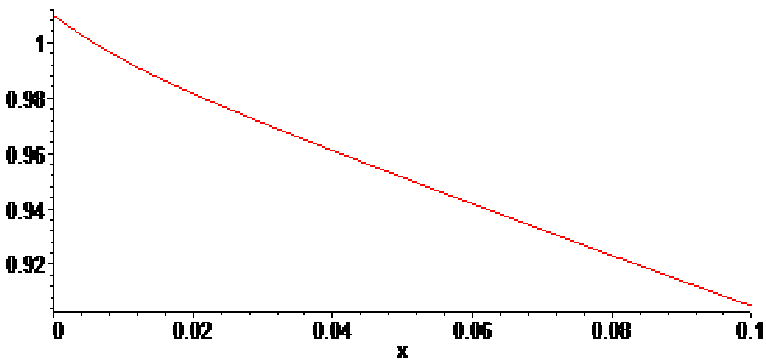
The particular solution of the equation satisfying the initial conditions is found in Maple as follows:

```
> cond:=y(0)=1.01,D(y)(0)=-2;de:=dsolve({eq,cond},y(x));
```

$$cond := y(0)=1.01,\ D(y)(0)=-2$$

$$de := y(x)=e^{(-x)}+\frac{1}{100}\,e^{(-100\,x)}.$$

> **plot(exp(-x)+1/100*exp(-100*x),x=0..0.1);** command automatically draws the graph of the function that is the solution of the differential equation on the section $x=[0;\,0.1]$ and helps the student to have a complete idea of the graph of the function that is the solution of the differential equation.



$y(x)=e^{-x}+0.01\cdot e^{-100x}$ *is the graph of the function on the section* $x=[0;\,0.1]$.

The use of the Maple program in the educational process allows you to see the graph of the solution function in practice, and the students will fully understand how well the solution was found based on the graph of the function. The use of software environments such as Mathematika, Mathlab, Mathcad and Maple, which are among the most advanced achievements in the field of information technology in the teaching of mathematics, allow students to be fully engaged in the lesson and increase students' enthusiasm for learning mathematics and its latest achievements. Students will become aware of the need for in-depth study of mathematics and engage independently in learning the latest advances in mathematics [4].

In conclusion, it should be noted that the use of Mathcad and Maple in the educational process and scientific research works greatly helps in solving many problems.

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