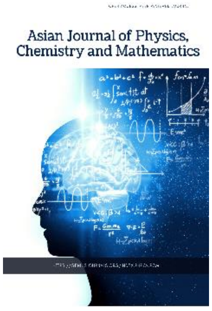

Boundary Problem for a Third Order Equation of a Mixed Composite Type

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ABSTRACT

Investigations of boundary value problems for an equation of composite type are a relatively new direction in the theory of boundary value problems. These problems are of particular interest in connection with their application in various problems of mechanics and physics, such they arise when modeling heat and mass transfer in capillary-porous media of a number of different biological objects and other problems.

Keywords: Nonlocal, nonlocal problem, equations of mixed-composite type, locale, singular integral equation.

Simply connected area D, limited smooth lines σ, based on points $A(0;1)$ and $B(1;0)$ located in the quarter $(x>a,\ y<0)$ and segments AA_1, BE, A_1E directed $x=0$. $x=1$, $y=1$ respectively, where $O,E-$ dots with coefficients $(0;0)$, $(1;1)$ considering equations

$$\frac{\partial}{\partial y}(Lu)=0 \tag{1}$$

where

$$Lu=u_{xx}+\frac{1-\operatorname{sgn} y}{2}u_{yy}-\frac{1+\operatorname{sgn} y}{2}u_{y} \tag{2}$$

Task 1. Find a function $u(x,y)$ with the following properties:

1. Function $u(x,y)$ is a regular solution of the equation (1) in the region of D $(y\neq 0)$

2. Function $u(x,y)$ and its first-order partial derivatives are continuous in a closed domain $\overline{D}$ (it is assumed that at the dots $O(0;0)$, $B(1;0)$ partial derivatives u_x, u_y may go to infinity of order less than one)

3. Function $u(x,y)$ satisfies the boundary conditions

$$u\big|_{\sigma} = f(\xi) \ (\xi\text{-contour dot } \sigma), \ u\big|_{A_1E} = \psi_1(x)$$
$$u\big|_{x=l} = u\big|_{BE} \ (0<l<1), \ u(0,y)+u(0,-y)\big|_{AA_1} = \psi(y) \qquad (3)$$
$$u_x\big|_{AO} = \nu(y)$$

Where $f,\psi,\psi_1,\nu -$ given functions satisfying certain smoothness conditions and matching conditions, $\psi(y) -$ the private function in the study of these problems will use the factor that any regular solution of equation (1) can be represented in the form

$$u(x,y) = z(x,y) + \omega(x) \qquad (4)$$

Respectively (1), $z(x,y) -$ regular solution of the equation

$$z_{xx} + \frac{1-\operatorname{sgn} y}{2} z_{yy} - \frac{1+\operatorname{sgn} y}{2} z_y = 0 \qquad (5)$$

$\omega -$ arbitrary twice continuously differentiable function can be assumed without loss of generality $\omega(0) = \omega'(0) = 0$, it is assumed that σ lies entirely in the strip of bounded straight lines $x=0$, $x=1$ on the grounds of (3)-(5) Problem 1 is reduced to finding a regular solution to Eq. (5) in the domain $D(y \neq 0)$ satisfying the boundary conditions

$$\left.\begin{array}{c} z\big|_{\sigma} = f(\xi) - \omega(x), \ z\big|_{BE} = z\big|_{x=l} = \varphi_1(y) - \omega(1) \\ z\big|_{OA_1} = \varphi(y) \ (0 \le y \le 1), \ z\big|_{AO} = \psi(y) - \varphi(-y), \ (-1 \le y \le 0) \\ z\big|_{A_1E} = \psi_1(x) - \omega(x), \ z_x\big|_{AO} = \upsilon(y) \end{array}\right\} \qquad (6)$$

Let us prove the uniqueness of the solution to Problem 1. If $f = \psi = \psi_1 = \upsilon = 0$, then the function $z(x,y)$ cannot reach a positive maximum (negative minimum) on the segment OB и AA_1. Indeed, suppose that a positive maximum (negative minimum) is reached at some dot $N(x_0,0)$. Then the equations $z_{xx} - z_y = 0$ follows that $z_y(x_0,0) \le 0$ $(z_y(x_0,0) \ge 0)$ on the other hand, from the elliptical part of the region $D = \{(x,y) \in D, \ x>0, \ y<0\}$ have $z_y(x_0,0) > 0$, $(z_y(x_0,0) < 0)$. From the problem statement it follows that $\lim\limits_{y\to 0-} z(x,y) = \lim\limits_{y\to 0+} z(x,y)$, $\lim\limits_{y\to 0-} z_y(x,y) = \lim\limits_{y\to 0+} z_y(x,y)$ hence we conclude that the function $z(x,y)$ cannot reach a positive maximum (negative minimum) on OB.

Let the function $z(x,y)$ reaches a positive maximum (negative minimum) at the dot $N(0,y)$ segment A_1O. Then $z(x,y)$ reaches a positive minimum (negative maximum) at the dot $N_1(0,y)$ segment AO. From the situation $z_x\big|_{AO} = 0$ following that $z(x,y)$ does not reach a positive maximum (negative minimum) on an open segment OA.

Consequently, $z(x,y)$ does not reach a positive maximum (negative minimum) on A_1O. Function $z(x,y)$ cannot reach a positive maximum (negative minimum) on BE. Otherwise, this maximum must be realized inside the region $D_1 = \{(x,y) \in D, \ x>0, \ y<0\}$, which is impossible, it follows that $z(x,y) = 0$ in the region of D_1. Then $\varphi(y)=0$, $\omega(x)=0$ Consequently, $z(x,y)\equiv 0$ and in the region D_2. Thus, it has been proven that $u(x,y)\equiv 0$ in the region of D.

Let us proceed to the proof of the existence of a solution to the problem. Let's assume that for the prostate $\sigma-$ circular arc $x^2+y^2=1$. Regular in the area D_2 solution of the equation (5) satisfying the boundary conditions $z_x\big|_{AO}=\upsilon(y)$, $z_y\big|_{OB}=\upsilon_1(x)$, $z\big|_{\sigma}=f(\xi)-\omega(x)$ is given by the formula

$$z(x,y)=\int_0^1 \upsilon(t)G(x,y,t,0)dt+\int_{\frac{3\pi}{2}}^{2\pi}\bar f(\theta)\frac{\partial G}{\partial n}\bigg|_{|\xi|=1}d\theta-\int_{-1}^{0}\upsilon(t)G(x,y;0,t)dt \qquad (6)$$

$\bar f(\theta)=f(\theta)-\omega(\cos\theta)$; $G(x,y;\xi,\eta)-$ Green's function. From equality (6) we go $\tau(x)$

$$\tau(x)+\frac{1}{\pi}\int_0^1 \upsilon(t)\ln\left|\frac{1-t^2x^2}{t^2-x^2}\right|dt=g_1(x) \qquad (7)$$

Where

$$g_1(x)=\int_{-1}^{0}\upsilon(t)G(x,0;0,t)dt+\int_{\frac{3\pi}{2}}^{2\pi}\bar f(\theta)\frac{\partial G}{\partial n}\bigg|_{|\xi|=1}d\theta$$

Due to the continuity of the first derivatives of the function $z(x,y)$ from the parabolic part D_1 областиin the area of D we get the ratio between $\tau(x)$ and $\upsilon(x)$

$$\tau^4(x)-\upsilon_1(x)=0 \qquad (8)$$

Using Conditions $\tau(0)=\tau(1)=0$, and (8) get

$$\tau'(x)-\int_0^x \upsilon_1(t)dt+\int_0^1(1-t)\upsilon_1(t)dt=0 \qquad (9)$$

Excluding $\tau'(x)$ and (8) and (9), we have

$$\frac{1}{\pi}\int_0^1 \upsilon_1(t)\left(\frac{2t^2x}{1-x^2t^2}-\frac{2x}{t^2-x^2}\right)dt=F(x) \qquad (10)$$

Where

$$F(x)=\int_0^x \upsilon(t)dt-\int_0^1(1-t)\upsilon_1(t)dt-g_1'(x)$$

Using the change of variables

$$\frac{t^2}{1+t^4}=\nu,\ \frac{x^2}{1+x^4}=s$$

Equations (10) are reduced to the form

$$\frac{1}{\pi}\int_0^1\frac{m(\nu)}{\nu-s}d\nu=P(s) \qquad (11)$$

Where

$$m(v)=\upsilon_1(t)\frac{1+t^4}{2t},\quad P(s)=F(x)\frac{1+x^4}{2x}$$

Inverting the integral equation (11), we have

$$m(v)=-\frac{1}{\pi}\int_0^1\left[\frac{s(1-v)}{v(1-s)}\right]^{\frac{1}{2}}\frac{p(v)}{v-s}=dv$$

Returning to the old variables x and t, we obtain the Fredholm integral equation of the second kind

$$\upsilon_1(x)-\frac{1}{\pi}\int_0^1 k_0(x,t)\upsilon_1(t)dt=f_0(x) \qquad (12)$$

where $K_0(x,t)-$ kernel resolvent

$$f_0(x)=\tilde{f}(x)+\int_0^1 g(x,t)\omega(t)dt$$

where $\tilde{f}_0(x)$, $g(x,t)-$ known features.

Substituting values $\upsilon_1(x)$ into the formula (9) define $\tau(x)$

$$\tau(x)=\int_0^1 p_1(x,t)\omega(t)dt+F_1(x) \qquad (13)$$

Solution of equation (5) satisfying the boundary conditions $z|_{OA_1}=\varphi(y)$, $z|_{BE}=\varphi_1(y)-\omega(1)=\bar{\varphi}_1(y)$, $z|_{OB}=\tau(x)$ is given by the formula

$$z(x,y)=\frac{1}{2\sqrt{\pi}}\left[\int_0^1\varphi(\eta)G^*_\xi(x,y;0,t)dt-\int_0^y\bar{\varphi}_1(t)G^*_\xi(x,y;1,t)dt+\int_0^1\tau(t)G^*(x,y;t,0)dt\right] (14)$$

Where $G^*(x,y;\xi,\eta)-$ Green's function.

Realizing the condition $z|_{x=l}=\bar{\varphi}_1(y)$ relatively $\bar{\varphi}_1(y)$, we obtain a Volterra integral equation of the second kind with sufficiently smooth kernel

$$\bar{\varphi}_1(y)+\frac{1}{2\sqrt{\pi}}\int_0^y\bar{\varphi}_1(t)G^*_\xi(l,y;1,t)dt=F_2(y) \qquad (15)$$

where

$$F_2(y)=\frac{1}{2\sqrt{\pi}}\int_0^y\varphi(t)G^*_\xi(l,y;0,t)dt+\frac{1}{2\sqrt{\pi}}\int\tau(t)G^*(l,y;t,0)dt$$

Equation (15) has a unique solution

Imlement the conditions $z|_{OA}=\psi(y)-\varphi(-y)$ has

$$\varphi(y)=\psi(y)-\int_0^1 \upsilon_1(t)G(0,-y;t,0)dt-\int_{\frac{3\pi}{2}}^{2\pi}\bar{f}(\theta)\frac{\partial G(0,-y;\cos\theta,\sin\theta)}{\partial n}d\theta+$$

$$+\int_0^1 \upsilon_1(-t)G(0,-y;0,-t)dt$$

Substituting values $\varphi(y)$, $\bar{\varphi}_1(y)$ into the formula (14) and using the conditions $z\big|_{A_1E}=\psi_1(x)-\omega(x)$, for determining $\omega(x)$ $(0\le x\le 1)$ we obtain an integral Fredholm equation of the second kind, the solvability of which follows from the uniqueness of the solution of the problem.

References:

1. Vragov V.N. On one equation of mixed composite type. Differential equations. 1973.
2. F.M. Muminov, Z.M. Miratoev, U.A. Utabov. On One Boundary Value Problem for a Third-Order Composite Type Equation. CENTRAL ASIAN JOURNAL OF THEORETICAL & APPLIED SCIENCES. 2021.04.08. Vol.02, Issue:04, pp.17-22.
3. Muminov F.M., Miratoev Z.M. On a nonlocal boundary value problem for a nonclassical equation. Scientific progress, 2021. Vol.01, Issue:06, pp.922-927.
4. Muminov F.M., Dushatov N.T. Nonlocal boundary value problem for linear equations of mixed type. Central Asian Journal of Theoretical and Applied Sciences, 2021. Vol.02, Issue:05, pp.191-196.
5. Srazhdinov I.F. Initial-boundary value problem for one composite type system. Bulletin of the Mathematical Institute. 2021. Volume 4, No. 2, ISSN-2181-9483, pp. 90-95.
6. Srazhdinov I.F. To investigation of the mixed problem for system of equations of composite type. CENTRAL ASIAN JOURNAL OF THEORETICAL & APPLIED SCIENCES. April 2021. Vol.02, Issue 04. стр.23-32.