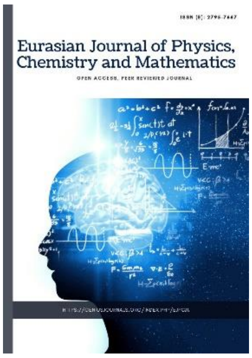

Using the Elzaki Transform to solve systems of partial differential equations

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ABSTRACT

In this research, we applied Elzaki transform to obtain the general formulas of solution for a system of partial differential equations with constant coefficients in dimension two. After that, these formulas used for solving some system, whether they are homogeneous or non-homogeneous.

Keywords: Elzaki Transform, partial differential equations

1-Introduction:

Differential equations and their applications continue to pique people's curiosity, and this enthusiasm and effort have led to the resolution of a variety of problems involving differential equations and mathematical analysis. Researchers are still discovering novel applications of differential equations in all branches of science, particularly in the study of endless processes[1]. Integral transforms, such as Sumudu and Laplace[2,3], are among the most essential methods for solving differential equations.Other transforms include Elzaki, Al Temime, Shehu, and Noval[4,5,6,7].

Elzaki Tarig proposed a new integral transformation that is an integral transformation of the Laplace kind ,the Elzaki transformation, in 2011[8].The Elzaki transform has been successfully extended to solve ordinary, partial, and fractional differential equations[9,10]. as follows:

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$$E[Đ(t), ɲ)] = \int_0^{\infty} \exp\left(\frac{-t}{ɲ}\right) Đ(t)dt = T(ɲ) \quad , ɲ \in (-ƛ_1, ƛ_2), \quad ƛ_1, ƛ_2 > 0$$

$$= \lim_{\alpha\to\infty} \int_0^{\alpha} \exp\left(\frac{-t}{ɲ}\right) Đ(t)dt = \Omega(ɲ) \ ; \quad ɲ > 0$$

Where $Đ(t)$is a real function, $\exp\left(\frac{-t}{ɲ}\right)$ is the kernel function, and E is the operator of Elzaki transform

The Elzaki transform was used to solve the system of partial differential equations in this paper. The fundamental aspects and assumptions of this change were discussed in the second section. General formulas for solving systems of partial differential equations with constant coefficients are derived in the third section. Finally, these formulas used to solve several examples of systems of partial differential equations in the fourth section.

2 Preliminaries

We introduced certain theorems and properties that would be used in our work in this part.

Definition (2.1):[11]

The purpose of this research is study of bilinear multiplication of solutions (like for QCR equations) for systems of non-homogeneous linear partial differential equations of first order of the form

$$\sum_{i=0}^{m}\sum_{j=0}^{n} Đ_{ij}\, k(x_1, x_2, \dots, x_n)\frac{\partial Đ_i}{\partial x_j} = f(x), \qquad k = 1,2, \dots, r,$$

where $x_1, x_2, \dots, x_n$are independent variables (real or complex) and $Đ_1, Đ_2, \dots, Đ_m$are dependent variables, and $Đ_{ij}k$ are given functions of class C^1

Property:[12]

1) If $Đ_1(x)$, $Đ_2(x), \dots, Đ_n(x)$ have Elzaki transform, then

$E(ß_1Đ_1(x) + ß_2Đ_2(x) + \cdots + ß_n Đ_n(x)) = ß_1E(Đ_1(x)) + ß_2 E(Đ_2(x)) + \dots + ß_n E(Đ_n(x))$

where $ß_1, ß_2, \dots, ß_n$ are constants $Đ_1(x)$, $Đ_2(x), \dots, Đ_n(x)$ are defined function.

2) If $E^{-1} Đ_1(ɲ) = \gamma_1(t)$, $E^{-1} Đ_2(ɲ) = \gamma_2(t), \dots, E^{-1} Đ_n(ɲ) = \gamma_n(t)$, then:
$E^{-1}[ß_1Đ_1(ɲ) + ß_2Đ_2(ɲ) + \cdots + ß_nĐ_n(ɲ)] = ß_1\gamma_1(t) + ß_2\gamma_2(t) + \cdots + ß_n \gamma_n(t)$

where $ß_1, ß_2, \dots ß_n$ are constants.

Theorem(2.3):[13]

ELzaki transform of the function $Đ(t)$ is defined as:

$E[Đ(t)]=T(ɲ) = ɲ\int_0^{\infty} Đ(t)e^{-t/ɲ}\, dt$, $t > 0$, $ɲ \in (-ƛ_1, ƛ_2)$, $ƛ_1, ƛ_2 > 0$

To get ELzaki transform of partial derivatives, can be used integration by parts. [9]:

1) $E\left[\frac{\partial Đ}{\partial t}(x,t)\right] = \frac{T(x,ɲ)}{ɲ} - ɲĐ(x,0).$

2) $E\left[\frac{\partial^2 Đ}{\partial t^2}(x,t)\right] = \frac{1}{ɲ^2}\, T(x,ɲ) - Đ(x,0) - ɲ\frac{\partial Đ}{\partial t}(x,0).$

3)This result can easily be extended to the n partial derivative by using mathematical induction. Such that,

$$E\left[\frac{\partial^n Đ}{\partial t^n}(x,t)\right] = \frac{T(x,ɲ)}{ɲ^n} - \sum_{\kappa=0}^{n-1} ɲ^{2-n+\kappa} Đ^{(\kappa)}(x,0)$$

If n=1 then ,

$$E\left[\frac{\partial Đ}{\partial t}(x,t)\right] = \frac{T(x,ɲ)}{ɲ} - ɲĐ(x,0).$$

Is true from (1).
We suppose the relation is true for $n = m$

$$E\left[\frac{\partial^m Đ}{\partial t^m}(x,t)\right] = \frac{T(x,ɲ)}{ɲ^m} - \sum_{\kappa=0}^{m-1} ɲ^{2-m+\kappa} Đ^{(\kappa)}(x,0)$$

If $n = m + 1$, can be showed that the relation

$$E[Đ^{(m+1)}(x,t)] = \frac{T(ɲ)}{ɲ^{m+1}} - \sum_{\kappa=0}^{m} ɲ^{1-m+\kappa} Đ^{(\kappa)}(x,0),$$

is true. it is enough put $Đ^{(m)}(x,t) = g(t)$, so we have

$$E[Đ^{(m+1)}(x,t)] = E[g'(t)]$$

and the result obtained.

Remark(2.4): [14]

Using Leibniz's rule, to obtain Elzaki transform for partial derivatives as follows:

$$E\left[\frac{\partial Đ(x,ɲ)}{\partial x}\right] = \int_0^\infty \exp\left(\frac{-t}{ɲ}\right)\frac{\partial Đ(x,ɲ)}{\partial x}dt = \frac{\partial}{\partial x}\int_0^\infty \exp\left(\frac{-t}{ɲ}\right) Đ(x,ɲ)\,dt = \frac{\partial}{\partial x}\left[E[Đ(x,ɲ)]\right] = \frac{d}{dx}E[Đ(x,ɲ)]$$

$$E\left[\frac{\partial^2 Đ(x,ɲ)}{\partial x^2}\right] = \int_0^\infty \exp\left(\frac{-t}{ɲ}\right)\frac{\partial^2 Đ(x,ɲ)}{\partial x^2}dt = \frac{\partial^2}{\partial x^2}\int_0^\infty \exp\left(\frac{-t}{ɲ}\right) Đ(x,ɲ)dt = \frac{\partial^2}{\partial x^2}\left[E[Đ(x,ɲ)]\right] = \frac{d^2}{dx^2}E[Đ(x,ɲ)]$$

$$E\left[\frac{\partial^n Đ(x,ɲ)}{\partial x^n}\right] = \int_0^\infty \exp\left(\frac{-t}{ɲ}\right)\frac{\partial^n Đ(x,ɲ)}{\partial x^n}dt = \frac{\partial^n}{\partial x^n}\int_0^\infty \exp\left(\frac{-t}{ɲ}\right) Đ(x,ɲ)dt = \frac{\partial^n}{\partial x^n}\left[E[Đ(x,ɲ)]\right] = \frac{d^n}{dx^n}\left[E[Đ(x,ɲ)]\right]$$

Table(1): Elzaki transformation for some functions:[15]

ID	Function $\gamma(t)$	$E(\gamma(t)) = \mu\int_0^\infty \gamma(t)\, e^{-t/\mu}\, dt = T(\mu)$
1	1	μ^2
2	t^n	$n!\,\mu^{n+2}$
3	e^{at}	$\frac{\mu^2}{1-a\mu}$
4	$\sin(at)$	$\frac{a\mu^3}{1+a^2\mu^2}$
5	$\cos(at)$	$\frac{\mu^2}{1+a^2\mu^2}$

3. The general Formulas of the sets solution for systems are derived using Elzaki Transformation

Formula(1):

Consider the non-homogeneous system of first order:

$$\left.\begin{array}{l} Ŧ_t(x,t) + Đ_x(x,t) = f_1(x,t) \\ Đ_t(x,t) + Ŧ_x(x,t) = f_2(x,t) \end{array}\right\} \quad (3.1)$$

with the conditions $Ŧ(x,0) = ƛ_1(x)$, $Ŧ(x,0) = ƛ_2(x)$ and $Ŧ(0,t) = Ŧ(1,t) = 0$, $Đ(0,t) = Đ(1,t) = 0$.
By applying Elzaki transform to both sides:

$$\frac{1}{ɲ} Ŧ(x,ɲ) - ɲ\, Ŧ(x,0) + \frac{d}{dx} Đ(x,ɲ) = E(f_1(x,t))$$

$$\frac{1}{ɲ} Đ(x,ɲ) - ɲ\, Đ(x,0) + \frac{d}{dx} Ŧ(x,ɲ) = E(f_2(x,t))$$

After substitute the initial conditions, we get:

$$\frac{d}{dx} Đ(x,ɲ) + \frac{1}{ɲ} Ŧ(x,ɲ) = E(f_1(x,t)) + ɲ\, ƛ_1(x) \quad (3.2)$$

$$\frac{d}{dx} Ŧ(x,ɲ) + \frac{1}{ɲ} Đ(x,ɲ) = E(f_2(x,t)) + ɲ\, ƛ_2(x) \quad (3.3)$$

Simple steps for equation (3.2),get

$$\frac{d}{dx} Ŧ(x,ɲ) = ɲ \frac{d}{dx} E\big(f_1(x,t)\big) + ɲ^2 ƛ_1'(x) - ɲ \frac{d^2}{dx^2} Đ(x,ɲ) \quad (3.4)$$

Now, substitute equation (3.4) in (3.3).

$$\frac{d^2}{dx^2} Đ(x,ɲ) - \frac{1}{ɲ^2} Đ(x,ɲ) = \frac{d}{dx} E\big(f_1(x,t)\big) + ɲ\, ƛ_1'(x) - \frac{1}{ɲ} E\big(f_2(x,t)\big) - ƛ_2(x) \quad (3.5)$$

The equation (3.5) represent non-homogeneous ordinary differential equation of order two has the solution.

$Đ(x,ɲ) = Đ_c(x,ɲ) + Đ_p(x,ɲ)$,

Where

$$Đ_c(x,ɲ) = ß_1 e^{\frac{1}{ɲ}x} + ß_2\, e^{\frac{-1}{ɲ}x}$$

And using variation of parameters:

$$Đ_p(x,ɲ) = ß_1(x) e^{\frac{1}{ɲ}x} + ß_2(x) e^{\frac{-1}{ɲ}x}$$

Then

$$ß_1'(x)\, e^{\frac{1}{ɲ}x} + ß_2'(x)\, e^{\frac{-1}{ɲ}x} = 0$$

$$\frac{1}{ɲ} ß_1'(x) e^{\frac{1}{ɲ}x} - \frac{1}{ɲ} ß_2'(x)\, e^{\frac{-1}{ɲ}x} = \frac{d}{dx} E\big(f_1(x,t)\big) + ɲ\, ƛ_1'(x) - \frac{1}{ɲ} E\big(f_2(x,t)\big) - ƛ_2(x)$$

Where

$$\Delta = \begin{vmatrix} e^{\frac{1}{ɲ}x} & e^{\frac{-1}{ɲ}x} \\ \frac{1}{ɲ} e^{\frac{1}{ɲ}x} & -\frac{1}{ɲ} e^{\frac{-1}{ɲ}x} \end{vmatrix} = -2\frac{1}{ɲ}$$

$$ß́_1(x) = \frac{1}{\Delta}\begin{vmatrix} 0 & e^{\frac{-1}{ȵ}x} \\ \frac{d}{dx}E\big(f_1(x,t)\big) + ȵ\,ƛ́_1(x) - \frac{1}{ȵ}E\big(f_2(x,t)\big) - ƛ_2(x) & -\frac{1}{ȵ}e^{\frac{-1}{ȵ}x} \end{vmatrix}$$

$$ß́_1(x) = \frac{\frac{d}{dx}E\big(f_1(x,t)\big) + ȵ\,ƛ́_1(x) - \frac{1}{ȵ}E\big(f_2(x,t)\big) - ƛ_2(x)e^{\frac{-1}{ȵ}x}}{2\frac{1}{ȵ}}$$

So

$$ß_1(x) = \frac{ȵ}{2}\int\left(\frac{d}{dx}E\big(f_1(x,t)\big) + ȵ\,ƛ́_1(x) - \frac{1}{ȵ}E\big(f_2(x,t)\big) - ƛ_2(x)\right)e^{\frac{-1}{ȵ}x}\,dx$$

In similar way:

$$ß́_2(x) = \frac{1}{\Delta}\begin{vmatrix} e^{\frac{1}{ȵ}x} & 0 \\ \frac{1}{ȵ}e^{\frac{1}{ȵ}x} & \frac{d}{dx}E\big(f_1(x,t)\big) + ȵ\,ƛ́_1(x) - \frac{1}{ȵ}E\big(f_2(x,t)\big) - ƛ_2(x) \end{vmatrix}$$

$$ß́_2(x) = \frac{\left(\frac{d}{dx}E\big(f_1(x,t)\big) + ȵ\,ƛ́_1(x) - \frac{1}{ȵ}E\big(f_2(x,t)\big) - ƛ_2(x)\right)e^{\frac{1}{ȵ}x}}{-2\frac{1}{ȵ}}$$

$$\therefore ß_2(x) = -\frac{ȵ}{2}\int\left(\frac{d}{dx}E\big(f_1(x,t)\big) + ȵ\,ƛ́_1(x) - \frac{1}{ȵ}E\big(f_2(x,t)\big) - ƛ_2(x)\right)e^{\frac{1}{ȵ}x}\,dx$$

$$Đ(x,ȵ) = ß_1e^{\frac{1}{ȵ}x} + ß_2\,e^{\frac{-1}{ȵ}x} + \left[\left(\frac{ȵ}{2}\int\left(\frac{d}{dx}E\big(f_1(x,t)\big) + ȵ\,ƛ́_1(x) - \frac{1}{ȵ}E\big(f_2(x,t)\big) - ƛ_2(x)\right)e^{\frac{-1}{ȵ}x}\,dx\right)e^{\frac{1}{ȵ}x} + \left(-\frac{ȵ}{2}\int\left(\frac{d}{dx}E\big(f_1(x,t)\big) + ȵ\,ƛ́_1(x) - \frac{1}{ȵ}E\big(f_2(x,t)\big) - ƛ_2(x)\right)e^{\frac{1}{ȵ}x}\,dx\right)e^{\frac{-1}{ȵ}x}\right]$$

by utilizing to the boundary conditions, obtained $ß_1 = ß_2 = 0$, then the solution of $Đ(x,ȵ)$ is:

$$Đ(x,ȵ) = \left[\left(\frac{ȵ}{2}\int\left(\frac{d}{dx}E\big(f_1(x,t)\big) + ȵ\,ƛ́_1(x) - \frac{1}{ȵ}E\big(f_2(x,t)\big) - ƛ_2(x)\right)e^{\frac{-1}{ȵ}x}\,dx\right)e^{\frac{1}{ȵ}x} + \left(-\frac{ȵ}{2}\int\left(\frac{d}{dx}E\big(f_1(x,t)\big) + ȵ\,ƛ́_1(x) - \frac{1}{ȵ}E\big(f_2(x,t)\big) - ƛ_2(x)\right)e^{\frac{1}{ȵ}x}\,dx\right)e^{\frac{-1}{ȵ}x}\right]$$

Taking the inverse of Elzaki transform to both sides:

$$Đ(x,t) = E^{-1}\left[\left(\frac{ȵ}{2}\int\left(\frac{d}{dx}E\big(f_1(x,t)\big) + ȵ\,ƛ́_1(x) - \frac{1}{ȵ}E\big(f_2(x,t)\big) - ƛ_2(x)\right)e^{\frac{-1}{ȵ}x}\,dx\right)e^{\frac{1}{ȵ}x} + \left(-\frac{ȵ}{2}\int\left(\frac{d}{dx}E\big(f_1(x,t)\big) + ȵ\,ƛ́_1(x) - \frac{1}{ȵ}E\big(f_2(x,t)\big) - ƛ_2(x)\right)e^{\frac{1}{ȵ}x}\,dx\right)e^{\frac{-1}{ȵ}x}\right] \quad (3.6)$$

Moreover, in similar way it can by obtaind $Ŧ(x,ȵ)$:

$$Ŧ(x,ȵ) = ȵE\big(f_1(x,t)\big) + ȵ^2\,ƛ_1(x) - ȵ\frac{d}{dx}\left[\left(\frac{ȵ}{2}\int\left(\frac{d}{dx}E\big(f_1(x,t)\big) + ȵ\,ƛ́_1(x) - \frac{1}{ȵ}E\big(f_2(x,t)\big) - ƛ_2(x)\right)e^{\frac{-1}{ȵ}x}\,dx\right)e^{\frac{1}{ȵ}x} + \left(-\frac{ȵ}{2}\int\left(\frac{d}{dx}E\big(f_1(x,t)\big) + ȵ\,ƛ́_1(x) - \frac{1}{ȵ}E\big(f_2(x,t)\big) - ƛ_2(x)\right)e^{\frac{1}{ȵ}x}\,dx\right)e^{\frac{-1}{ȵ}x}\right]$$

Taking the inverse of Elzaki transform to both sides for the above equation :

$$Ŧ(x,t)=E^{-1}\Bigg[ɲE\big(f_1(x,t)\big) + ɲ^2 ƛ_1(x) - ɲ\frac{d}{dx}\Bigg[\bigg(\frac{ɲ}{2}\int\Big(\frac{d}{dx}E\big(f_1(x,t)\big) + ɲ\acute{ƛ}_1(x) - \frac{1}{ɲ}E\big(f_2(x,t)\big) - ƛ_2(x)\Big)e^{\frac{-1}{ɲ}x}dx\bigg)e^{\frac{1}{ɲ}x} + \bigg(-\frac{ɲ}{2}\int\Big(\frac{d}{dx}E\big(f_1(x,t)\big) + ɲ\acute{ƛ}_1(x) - \frac{1}{ɲ}E\big(f_2(x,t)\big) - ƛ_2(x)\Big)e^{\frac{1}{ɲ}x}\,dx\bigg)e^{\frac{-1}{ɲ}x}\Bigg]\Bigg] \quad (3.7)$$

So, $Đ(x,t)$ and $Ŧ(x,t)$ represent the general formula of the system (3.1)

Formula(2):

Consider the nonhomogeneous system of second order :

$$\left.\begin{aligned} Ŧ_{tt}(x,t) + Đ_x(x,t) &= f_1(x,t) \\ Đ_{tt}(x,t) + Ŧ_x(x,t) &= f_2(x,t) \end{aligned}\right\} \quad (3.8)$$

with the conditions $Ŧ(x,0) = ƛ_1(x)$, $Đ(x,0) = ƛ_2(x)$, $Ŧ_t(x,0) = ð_1(x)$, $Đ_t(x,0) = ð_2(x)$ *and* $Ŧ(0,t) = Ŧ(1,t) = 0$, $Đ(0,t) = Đ(1,t) = 0$

By Applying Elzaki transform to both sides:

$$\frac{1}{ɲ^2}Ŧ(x,ɲ) - Ŧ(x,0) - ɲŦ_t(x,0) + \frac{d}{dx}Đ(x,ɲ) = E(f_1(x,t))$$

$$\frac{1}{ɲ^2}Đ(x,ɲ) - Đ(x,0) - ɲĐ_t(x,0) + \frac{d}{dx}Ŧ(x,ɲ) = E(f_2(x,t))$$

After substitute the initial conditions, we get:

$$\frac{d}{dx}Đ(x,ɲ) + \frac{1}{ɲ^2}Ŧ(x,ɲ) = E(f_1(x,t)) + ƛ_1(x) + ɲð_1(x) \quad (3.9)$$

$$\frac{d}{dx}Ŧ(x,ɲ) + \frac{1}{ɲ^2}Đ(x,ɲ) = E(f_2(x,t)) + ƛ_2(x) + ɲð_2(x) \quad (3.10)$$

Next, simple calculation of equation (3.9),get

$$\frac{d}{dx}Ŧ(x,ɲ) = \frac{d}{dx}ɲ^2E(f_1(x,t)) + ɲ^2\acute{ƛ}_1(x) + ɲ^3\acute{ð}_1(x) - ɲ^2\frac{d^2}{dx^2}Đ(x,ɲ) \quad (3.11)$$

Now, substitute equation (3.11) in (3.10).

$$\frac{d^2}{dx^2}Đ(x,ɲ) - \frac{1}{ɲ^4}Đ(x,ɲ) = \frac{d}{dx}E(f_1(x,t)) + \acute{ƛ}_1(x) + ɲ\acute{ð}_1(x) - \frac{1}{ɲ^2}E(f_2(x,t)) - \frac{1}{ɲ^2}ƛ_2(x) - \frac{1}{ɲ}ð_2(x) \quad (3.12)$$

The equation (3.12) represent non-homogeneous equation of order two has the solution

$Đ(x,ɲ) = Đ_c(x,ɲ) + Đ_p(x,ɲ)$,

Where

$$Đ_c(x,ɲ) = ß_1 e^{\frac{1}{ɲ^2}x} + ß_2 e^{\frac{-1}{ɲ^2}x}$$

And $Đ_p$ can be obtained by:

$$Đ_p(x,ɲ) = ß_1(x)e^{\frac{1}{ɲ^2}x} + ß_2(x)e^{\frac{-1}{ɲ^2}x}$$

Since

$$\acute{ß}_1(x)\,e^{\frac{1}{ɲ^2}x} + \acute{ß}_2(x)\,e^{\frac{-1}{ɲ^2}x} = 0$$

$\frac{1}{ɲ^2}ẞ_1'(x)e^{\frac{1}{ɲ^2}x}$ - $\frac{1}{ɲ^2}ẞ_2'(x)\ e^{\frac{-1}{ɲ^2}x}$= $\frac{d}{dx}E(f_1(x,t)) + ƛ_1'(x) + ɲð_1'(x) - \frac{1}{ɲ^2}E(f_2(x,t)) - \frac{1}{ɲ^2}ƛ_2(x) - \frac{1}{ɲ}ð_2(x)$

Where

$$\Delta = \begin{vmatrix} e^{\frac{1}{ɲ^2}x} & e^{\frac{-1}{ɲ^2}x} \\ \frac{1}{ɲ^2}e^{\frac{1}{ɲ^2}x} & -\frac{1}{ɲ^2}e^{\frac{-1}{ɲ^2}x} \end{vmatrix} = -2\frac{1}{ɲ^2}$$

$$\therefore\ ẞ_1(x) = \frac{ɲ^2}{2}\int\left(\frac{d}{dx}E(f_1(x,t)) + ƛ_1'(x) + ɲð_1'(x) - \frac{1}{ɲ^2}E(f_2(x,t)) - \frac{1}{ɲ^2}ƛ_2(x) - \frac{1}{ɲ}ð_2(x)\ \right)e^{\frac{-1}{ɲ^2}x}\,dx$$

In similar way:

$$\therefore ẞ_2(x) = -\frac{ɲ^2}{2}\int\left(\frac{d}{dx}E(f_1(x,t)) + ƛ_1'(x) + ɲð_1'(x) - \frac{1}{ɲ^2}E(f_2(x,t)) - \frac{1}{ɲ^2}ƛ_2(x) - \frac{1}{ɲ}ð_2(x)\ \right)e^{\frac{1}{ɲ^2}x}\ dx$$

$$Đ(x,ɲ) = ẞ_1e^{\frac{1}{ɲ^2}x} + ẞ_2\ e^{\frac{-1}{ɲ^2}x} + \left[\left(\frac{ɲ^2}{2}\int\left(\frac{d}{dx}E(f_1(x,t)) + ƛ_1'(x) + ɲð_1'(x) - \frac{1}{ɲ^2}E(f_2(x,t)) - \frac{1}{ɲ^2}ƛ_2(x) - \frac{1}{ɲ}ð_2(x)\right)e^{\frac{-1}{ɲ^2}x}\ dx\right)e^{\frac{1}{ɲ^2}x} + \left(-\frac{ɲ^2}{2}\int\left(\frac{d}{dx}E(f_1(x,t)) + ƛ_1'(x) + ɲð_1'(x) - \frac{1}{ɲ^2}E(f_2(x,t)) - \frac{1}{ɲ^2}ƛ_2(x) - \frac{1}{ɲ}ð_2(x)\right)e^{\frac{1}{ɲ^2}x}\ dx\right)e^{\frac{-1}{ɲ^2}x}\right]$$

$ẞ_1 = ẞ_2 = 0$, then the solution of $Đ(x,ɲ)$ is:

$$Đ(x,ɲ) = \left[\left(\frac{ɲ^2}{2}\int\left(\frac{d}{dx}E(f_1(x,t)) + ƛ_1'(x) + ɲð_1'(x) - \frac{1}{ɲ^2}E(f_2(x,t)) - \frac{1}{ɲ^2}ƛ_2(x) - \frac{1}{ɲ}ð_2(x)\right)e^{\frac{-1}{ɲ^2}x}\ dx\right)e^{\frac{1}{ɲ^2}x} + \left(-\frac{ɲ^2}{2}\int\left(\frac{d}{dx}E(f_1(x,t)) + ƛ_1'(x) + ɲð_1'(x) - \frac{1}{ɲ^2}E(f_2(x,t)) - \frac{1}{ɲ^2}ƛ_2(x) - \frac{1}{ɲ}ð_2(x)\right)e^{\frac{1}{ɲ^2}x}\ dx\right)e^{\frac{-1}{ɲ^2}x}\right]$$

Taking the inverse of Elzaki transform to both sides:

$$Đ(x,t) = E^{-1}\left[\left[\left(\frac{ɲ^2}{2}\int\left(\frac{d}{dx}E(f_1(x,t)) + ƛ_1'(x) + ɲð_1'(x) - \frac{1}{ɲ^2}E(f_2(x,t)) - \frac{1}{ɲ^2}ƛ_2(x) - \frac{1}{ɲ}ð_2(x)\right)e^{\frac{-1}{ɲ^2}x}\ dx\right)e^{\frac{1}{ɲ^2}x} + \left(-\frac{ɲ^2}{2}\int\left(\frac{d}{dx}E(f_1(x,t)) + ƛ_1'(x) + ɲð_1'(x) - \frac{1}{ɲ^2}E(f_2(x,t)) - \frac{1}{ɲ^2}ƛ_2(x) - \frac{1}{ɲ}ð_2(x)\right)e^{\frac{1}{ɲ^2}x}\ dx\right)e^{\frac{-1}{ɲ^2}x}\right]\right] \quad (3.13)$$

Moreover, in similar way it can by obtaind $Ŧ(x,ɲ)$:

$$Ŧ(x,ɲ) = ɲ^2E(f_1(x,t)) + ɲ^2\,ƛ_1(x) + ɲ^3ð_1(x) - ɲ^2\frac{d}{dx}\left[\left(\frac{ɲ^2}{2}\int\left(\frac{d}{dx}E(f_1(x,t)) + ƛ_1'(x) + ɲð_1'(x) - \frac{1}{ɲ^2}E(f_2(x,t)) - \frac{1}{ɲ^2}ƛ_2(x) - \frac{1}{ɲ}ð_2(x)\right)e^{\frac{-1}{ɲ^2}x}\ dx\right)e^{\frac{1}{ɲ^2}x} + \left(-\frac{ɲ^2}{2}\int\left(\frac{d}{dx}E(f_1(x,t)) + ƛ_1'(x) + ɲð_1'(x) - \frac{1}{ɲ^2}E(f_2(x,t)) - \frac{1}{ɲ^2}ƛ_2(x) - \frac{1}{ɲ}ð_2(x)\right)e^{\frac{1}{ɲ^2}x}\ dx\right)e^{\frac{-1}{ɲ^2}x}\right]$$

Taking the inverse of Elzaki transform to both sides for the above equation :

$$Ŧ(x,t)=E^{-1}\left[ɲ^2\,ƛ_1(x)+ɲ^3ð_1(x)-ɲ^2\frac{d}{dx}\left[\left(\frac{ɲ^2}{2}\int\left(\frac{d}{dx}E(f_1(x,t))+ƛ_1'(x)+ɲð_1'(x)-\frac{1}{ɲ^2}E(f_2(x,t))-\frac{1}{ɲ^2}ƛ_2(x)-\frac{1}{ɲ}ð_2(x)\right)e^{\frac{-1}{ɲ^2}x}\,dx\right)e^{\frac{1}{ɲ^2}x}+\left(-\frac{ɲ^2}{2}\int\left(\frac{d}{dx}E(f_1(x,t))+ƛ_1'(x)+ɲð_1'(x)-\frac{1}{ɲ^2}E(f_2(x,t))-\frac{1}{ɲ^2}ƛ_2(x)-\frac{1}{ɲ}ð_2(x)\right)e^{\frac{1}{ɲ^2}x}\,dx\right)e^{\frac{-1}{ɲ^2}x}\right]\right] \qquad (3.14)$$

So, Ð(x, t) and Ŧ(x, t) represent the general formula of the system (3.8)

1. **Applications**

The utility and effectiveness of Elzaki transformation are demonstrated in the following section using the exact solution of the partial differential equations scheme.

Example (1): To solve the system

$$\left.\begin{aligned}Ð_t(x,t)+Ŧ_x(x,t)=0\\ Ŧ_t(x,t)+Ð_x(x,t)=0\end{aligned}\right\} \qquad (4.1)$$

with the conditions $Ð(x,0)=e^{5x}$, $Ŧ(x,0)=e^{-5x}$ and $Ð(0,t)=Ð(1,t)=0$, $Ŧ(0,t)=Ŧ(1,t)=0$
which the system is homogenous $f_1(x,t)=f_2(x,t)=0$

$$Ð(x,t)=E^{-1}\left[\left(\frac{ɲ}{2}\int(ɲ\,ƛ_1'(x)-ƛ_2(x))e^{\frac{-1}{ɲ}x}dx\right)e^{\frac{1}{ɲ}x}+\left(-\frac{ɲ}{2}\int(ɲ\,ƛ_1'(x)-ƛ_2(x))e^{\frac{1}{ɲ}x}\,dx\right)e^{\frac{-1}{ɲ}x}\right]$$

$$=E^{-1}\left[\left(\frac{ɲ}{2}\int(5\,ɲ\,e^{5x}-e^{-5x})e^{\frac{-1}{ɲ}x}dx\right)e^{\frac{1}{ɲ}x}-\left(\frac{ɲ}{2}\int(5\,ɲ\,e^{5x}-e^{-5x})e^{\frac{1}{ɲ}x}\,dx\right)e^{\frac{-1}{ɲ}x}\right]$$

$$=E^{-1}\left[\left(\frac{ɲ}{2}\int 5\,ɲ\,e^{(5-\frac{1}{ɲ})x}e^{\frac{-1}{ɲ}x}dx\right)e^{\frac{1}{ɲ}x}-\left(\frac{ɲ}{2}\int\left(e^{\left(-5-\frac{1}{ɲ}\right)x}e^{\frac{-1}{ɲ}x}dx\right)e^{\frac{-1}{ɲ}x}\right)e^{\frac{1}{ɲ}x}-\left(\frac{ɲ}{2}\int 5\,ɲ\,e^{(5+\frac{1}{ɲ})x}e^{\frac{1}{ɲ}x}dx\right)e^{\frac{-1}{ɲ}x}-\left(\frac{ɲ}{2}\int\left(e^{\left(-5+\frac{1}{ɲ}\right)x}e^{\frac{1}{ɲ}x}dx\right)e^{\frac{-1}{ɲ}x}\right)\right]$$

$$=E^{-1}\left[\frac{5ɲ^2}{2}.\frac{1}{5-\frac{1}{ɲ}}e^{5x}-\frac{ɲ}{2}.\frac{1}{-5-\frac{1}{ɲ}}e^{-5x}-\frac{5ɲ^2}{2}.\frac{1}{5+\frac{1}{ɲ}}e^{5x}+\frac{ɲ}{2}.\frac{1}{-5+\frac{1}{ɲ}}e^{-5x}\right]$$

$$=E^{-1}\left[\left(\frac{-5ɲ^3}{1-25ɲ^2}e^{5x}+\frac{ɲ^2}{1-25ɲ^2}e^{-5x}\right)\right]$$

$$\therefore Ð(x,t)=-e^{5x}sinh5t+e^{-5x}cosh5t.$$

Moreover, for the second variable Ŧ(x, t):

$$Ŧ(x,t)=E^{-1}\left[ɲ^2ƛ_1(x)-ɲ\frac{d}{dx}\left[\left(\frac{ɲ}{2}\int(ɲ\,ƛ_1'(x)-ƛ_2(x))e^{\frac{-1}{ɲ}x}dx\right)e^{\frac{1}{ɲ}x}+\left(-\frac{ɲ}{2}\int(ɲ\,ƛ_1'(x)-ƛ_2(x))e^{\frac{1}{ɲ}x}\,dx\right)e^{\frac{-1}{ɲ}x}\right]\right]$$

$$=E^{-1}\left[ɲ^2e^{5x}-ɲ\frac{d}{dx}\left[\frac{-5ɲ^3}{1-25ɲ^2}e^{5x}+\frac{ɲ^2}{1-25ɲ^2}e^{-5x}\right]\right]$$

$$=E^{-1}\left[ɲ^2e^{5x}+\frac{25ɲ^4}{1-25ɲ^2}e^{5x}+\frac{5ɲ^3}{1-25ɲ^2}e^{-5x}\right]$$

$$=E^{-1}\left[\frac{ɲ^2}{1-25ɲ^2}e^{5x}+\frac{5ɲ^3}{1-25ɲ^2}e^{-5x}\right]$$

$$\therefore Ŧ(x,t)=e^{5x}cosh5t+e^{-5x}sinh5t.$$

Example(2):To solve the system

$$\left.\begin{aligned}Ð_x(x,t)+Ŧ_t(x,t)=3x^2\\ Ŧ_x(x,t)+Ð_t(x,t)=-12t\end{aligned}\right\} \qquad (4.2)$$

with the conditions $Đ(x,0)=0, Ŧ(x,0)=-12$ and $Đ(0,t)=Đ(1,t)=0, Ŧ(0,t)=Ŧ(1,t)=0$,
Using the formula (1) with the equations (3.6) and (3.7)

$$Đ(x,t)=E^{-1}\left[\left(\frac{ɲ}{2}\int\left(\frac{d}{dx}E(f_1(x,t))+ɲ\acute{ƛ}_1(x)-\frac{1}{ɲ}E(f_2(x,t))-ƛ_2(x)\right)e^{\frac{-1}{ɲ}x}dx\right)e^{\frac{1}{ɲ}x}+\left(-\frac{ɲ}{2}\int\left(\frac{d}{dx}E(f_1(x,t))+ɲ\acute{ƛ}_1(x)-\frac{1}{ɲ}E(f_2(x,t))-ƛ_2(x)\right)e^{\frac{1}{ɲ}x}dx\right)e^{\frac{-1}{ɲ}x}\right]$$

$$=E^{-1}\left[\left(\frac{ɲ}{2}\int\left(6xɲ^2+\frac{12ɲ^3}{ɲ}+12\right)e^{\frac{-1}{ɲ}x}dx\right)e^{\frac{1}{ɲ}x}-\left(\frac{ɲ}{2}\int\left(6xɲ^2+\frac{12ɲ^3}{ɲ}+12\right)e^{\frac{1}{ɲ}x}dx\right)e^{\frac{-1}{ɲ}x}\right]$$

$$=E^{-1}\left[\left(3ɲ^2\int xe^{\frac{-1}{ɲ}x}dx+6ɲ^3\int e^{\frac{-1}{ɲ}x}dx+6ɲ\int e^{\frac{-1}{ɲ}x}dx\right)e^{\frac{1}{ɲ}x}-\left(3ɲ^2\int xe^{\frac{-1}{ɲ}x}dx+6ɲ^3\int e^{\frac{-1}{ɲ}x}dx+6ɲ\int e^{\frac{-1}{ɲ}x}dx\right)e^{\frac{-1}{ɲ}x}\right]$$

$$=E^{-1}[-6xɲ^4-12ɲ^4-12ɲ^2]$$

$$\therefore Đ(x,t)=-3xt^2-6t^2-12.$$

Moreover, for the second variable $Ŧ(x,t)$:

$$Ŧ(x,t)=E^{-1}\left[ɲE(f_1(x,t))+ɲ^2ƛ_1(x)-ɲ\frac{d}{dx}\left[\left(\frac{ɲ}{2}\int\left(\frac{d}{dx}E(f_1(x,t))+ɲ\acute{ƛ}_1(x)-\frac{1}{ɲ}E(f_2(x,t))-ƛ_2(x)\right)e^{\frac{-1}{ɲ}x}dx\right)e^{\frac{1}{ɲ}x}+\left(-\frac{ɲ}{2}\int\left(\frac{d}{dx}E(f_1(x,t))+ɲ\acute{ƛ}_1(x)-\frac{1}{ɲ}E(f_2(x,t))-ƛ_2(x)\right)e^{\frac{1}{ɲ}x}dx\right)e^{\frac{-1}{ɲ}x}\right]\right]$$

$$=E^{-1}\left[3x^2ɲ^3-ɲ\frac{d}{dx}[-6xɲ^4-12ɲ^4-12ɲ^2]\right]$$

$$=E^{-1}[3x^2ɲ^3-ɲ[-6ɲ^4]]$$

$$=E^{-1}[3x^2ɲ^3+6ɲ^5]$$

$$\therefore Ŧ(x,t)=3x^2t+t^3.$$

Example(3):To solve the system:

$$\left.\begin{aligned}Đ_{tt}(x,t)+Ŧ_x(x,t)=0\\ Ŧ_{tt}(x,t)+Đ_x(x,t)=0\end{aligned}\right\} \quad (4.3)$$

with the conditions $Đ(x,0)=4x, Ŧ(x,0)=-4x$,and $Đ_t(x,0)=4$, $Ŧ_t(x,0)=-4$
and $Đ(0,t)=Đ(1,t)=0, Ŧ(0,t)=Ŧ(1,t)=0$
which the system is homogenous $f_1(x,t)=f_2(x,t)=0$

$$Đ(x,t)=E^{-1}\left[\left(\frac{ɲ^2}{2}\int\left(\acute{ƛ}_1(x)+ɲ\acute{ð}_1(x)-\frac{1}{ɲ^2}ƛ_2(x)-\frac{1}{ɲ}ð_2(x)\right)e^{\frac{-1}{ɲ^2}x}dx\right)e^{\frac{1}{ɲ^2}x}+\left(-\frac{ɲ^2}{2}\int\left(\acute{ƛ}_1(x)+ɲ\acute{ð}_1(x)-\frac{1}{ɲ^2}ƛ_2(x)-\frac{1}{ɲ}ð_2(x)\right)e^{\frac{1}{ɲ^2}x}dx\right)e^{\frac{-1}{ɲ^2}x}\right]$$

$$=E^{-1}\left[\left(\frac{ɲ^2}{2}\int\left(4+\frac{4}{ɲ^2}x+\frac{4}{ɲ}\right)e^{\frac{-1}{ɲ^2}x}dx\right)e^{\frac{1}{ɲ^2}x}-\left(\frac{ɲ^2}{2}\int\left(4+\frac{4}{ɲ^2}x+\frac{4}{ɲ}\right)e^{\frac{1}{ɲ^2}x}dx\right)e^{\frac{-1}{ɲ^2}x}\right]$$

$$=E^{-1}[-4ɲ^4-4xɲ^2-4ɲ^3]$$

$$\therefore Đ(x,t)=-2t^2-4x-4t.$$

Also, for the second variable $Ŧ(x,t)$:

$$Ŧ(x,t)=E^{-1}\left[ɲ^2ƛ_1(x)+ɲ^3ð_1(x)-ɲ^2\frac{d}{dx}\left[\left(\frac{ɲ^2}{2}\int\left(\acute{ƛ}_1(x)+ɲ\acute{ð}_1(x)-\frac{1}{ɲ^2}ƛ_2(x)-\frac{1}{ɲ}ð_2(x)\right)e^{\frac{-1}{ɲ^2}x}dx\right)e^{\frac{1}{ɲ^2}x}+\left(-\frac{ɲ^2}{2}\int\left(\acute{ƛ}_1(x)+ɲ\acute{ð}_1(x)-\frac{1}{ɲ^2}ƛ_2(x)-\frac{1}{ɲ}ð_2(x)\right)e^{\frac{1}{ɲ^2}x}dx\right)e^{\frac{-1}{ɲ^2}x}\right]\right]$$

$$=E^{-1}[4ɲ^2x+4ɲ^3-ɲ^2[-4ɲ^2]]$$

$$\therefore Ŧ(x,t)=4x+4t+2t^2.$$

Example(4): To solve the system

$$\left.\begin{aligned}Ŧ_{tt}(x,t)+Đ_x(x,t)=0\\ Đ_{tt}(x,t)+Ŧ_x(x,t)=2t-2\end{aligned}\right\} \quad (4.2)$$

with the conditions $Đ(x,0)=2x$, $Ŧ(x,0)=-2x$,and $Đ_t(x,0)=2$, $Ŧ_t(x,0)=-2$
and $Đ(0,t)=Đ(1,t)=0$, $Ŧ(0,t)=Ŧ(1,t)=0$,
Using the formula (2) with the equations (3.13) and (3.14)

$$Đ(\mathrm{x},\mathrm{t}) = \mathrm{E}^{-1}\left[\left[\left(\frac{ɲ^2}{2}\int\left(\frac{\mathrm{d}}{\mathrm{dx}}E(f_1(x,t)) + \acute{ƛ}_1(\mathrm{x}) + ɲ\acute{ð}_1(x) - \frac{1}{ɲ^2}E(f_2(x,t)) - \frac{1}{ɲ^2}ƛ_2(\mathrm{x}) - \frac{1}{ɲ}ð_2(x)\right)e^{\frac{-1}{ɲ^2}x}\,dx\right)e^{\frac{1}{ɲ^2}x} + \left(-\frac{ɲ^2}{2}\int\left(\frac{\mathrm{d}}{\mathrm{dx}}E(f_1(x,t)) + \acute{ƛ}_1(\mathrm{x}) + ɲ\acute{ð}_1(x) - \frac{1}{ɲ^2}E(f_2(x,t)) - \frac{1}{ɲ^2}ƛ_2(\mathrm{x}) - \frac{1}{ɲ}ð_2(x)\right)e^{\frac{1}{ɲ^2}x}\,dx\right)e^{\frac{-1}{ɲ^2}x}\right]\right]$$

$$= \mathrm{E}^{-1}\left[\left[\left(\frac{ɲ^2}{2}\int\left(2 - \frac{1}{ɲ^2}(2ɲ^3 - 2ɲ^2)\right) + \frac{2}{ɲ^2}x + \frac{2}{ɲ}\right)e^{\frac{-1}{ɲ^2}x}\,dx\right)e^{\frac{1}{ɲ^2}x} - \left(\frac{ɲ^2}{2}\int\left(2 - \frac{1}{ɲ^2}(2ɲ^3 - 2ɲ^2)\right) + \frac{2}{ɲ^2}x + \frac{2}{ɲ}\right)e^{\frac{1}{ɲ^2}x}\,dx\right)e^{\frac{-1}{ɲ^2}x}\right]\right]$$

$$= \mathrm{E}^{-1}\left[\left(ɲ^2\int e^{\frac{-1}{ɲ}x}dx - ɲ^3\int e^{\frac{-1}{ɲ}x}dx + ɲ^2\int e^{\frac{-1}{ɲ}x}dx + \int \mathrm{x}e^{\frac{-1}{ɲ}x} + ɲ\int e^{\frac{-1}{ɲ}x}\right)e^{\frac{1}{ɲ}x} - \left(ɲ^2\int e^{\frac{1}{ɲ}x}dx - ɲ^3\int e^{\frac{1}{ɲ}x}dx + ɲ^2\int e^{\frac{1}{ɲ}x}dx + \int \mathrm{x}e^{\frac{1}{ɲ}x} + ɲ\int e^{\frac{1}{ɲ}x}\right)e^{\frac{-1}{ɲ}x}\right]$$

$$= \mathrm{E}^{-1}[-4ɲ^4 + 2ɲ^5 - 2xɲ^2 - 2ɲ^3]$$

$$\therefore Đ(\mathrm{x},\mathrm{t}) = -2t^2 + \frac{1}{3}t^3 - 2x - 2t.$$

Moreover, for the second variable $Ŧ(\mathrm{x},\mathrm{t})$:

$$Ŧ(\mathrm{x},\mathrm{t}) = \mathrm{E}^{-1}\left[ɲ^2ƛ_1(\mathrm{x}) + ɲ^3ð_1(x) - ɲ^2\frac{\mathrm{d}}{\mathrm{dx}}\left[\left(\frac{ɲ^2}{2}\int\left(\frac{\mathrm{d}}{\mathrm{dx}}E(f_1(x,t)) + \acute{ƛ}_1(\mathrm{x}) + ɲ\acute{ð}_1(x) - \frac{1}{ɲ^2}E(f_2(x,t)) - \frac{1}{ɲ^2}ƛ_2(\mathrm{x}) - \frac{1}{ɲ}ð_2(x)\right)e^{\frac{-1}{ɲ^2}x}\,dx\right)e^{\frac{1}{ɲ^2}x} + \left(-\frac{ɲ^2}{2}\int\left(\frac{\mathrm{d}}{\mathrm{dx}}E(f_1(x,t)) + \acute{ƛ}_1(\mathrm{x}) + ɲ\acute{ð}_1(x) - \frac{1}{ɲ^2}E(f_2(x,t)) - \frac{1}{ɲ^2}ƛ_2(\mathrm{x}) - \frac{1}{ɲ}ð_2(x)\right)e^{\frac{1}{ɲ^2}x}\,dx\right)e^{\frac{-1}{ɲ^2}x}\right]\right]$$

$$= \mathrm{E}^{-1}\left[2\mathrm{x}ɲ^2 + 2ɲ^3 - ɲ^2\frac{\mathrm{d}}{\mathrm{dx}}[-4ɲ^4 + 2ɲ^5 - 2xɲ^2 - 2ɲ^3]\right]$$

$$= \mathrm{E}^{-1}[2\mathrm{x}ɲ^2 + 2ɲ^3 - ɲ^2[-2ɲ^2]]$$

$$= \mathrm{E}^{-1}[2\mathrm{x}ɲ^2 + 2ɲ^3 + 2ɲ^4]$$

$$\therefore Ŧ(\mathrm{x},\mathrm{t}) = 2x + 2t + t^2.$$

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