

# Strong Equality Co-Neighborhood Domination in Graphs

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ABSTRACT

Let  $G = (V, E)$  be a simple and undirected graph without isolated vertices. A subset  $D \subset V$  such that  $D \neq V$  is called strong equality co-neighborhood dominating set of  $G$  (SENDS), if satisfy the following condition that  $\forall v \in D$  has property that it is adjacent to the same number of vertices in the set  $V - D$  where  $\deg(v) \geq \deg(u) \forall v \in D$  and  $\forall u \in V - D$ .

In this paper, SENDS in graphs is defined. Some properties of SENDS are determined for some special graphs.

**Keywords:**

Strong co-neighborhood dominating set, Strong co-neighborhood domination number.

## 1. Introduction

Let  $G = (V, E)$  be an undirected, simple, and finite graph of order  $|V|$ . The open neighborhood of a vertex  $v$  in  $G$  is  $N_G(v) = \{u \in V(G) : uv \in E(G)\}$  and  $N_G[v] = N_G(v) \cup \{v\}$  is the closed set [12].

A subset  $D \subseteq V$  is a dominating set of  $G$ , if  $N_G[D] = V$ . The domination number  $\gamma(G)$  is the minimum cardinality of a dominating set in  $G$ . The  $N_G(v) = \{u \in V(G) : uv \in E(G)\}$  is the open neighborhood of a vertex  $v \in G$  and  $N_G[v] = N_G(v) \cup \{v\}$ . Domination in graphs has wide range to solve variant problems life. So there are many appeared many parameters of domination as in [1-8] for domination by vertices and [22] for domination by edges. Also, there is a study of domination polynomial of certain graph as in [9,14,15,16,18,19]. And Chromatic polynomials and chromaticity of zero-divisor graphs as in [17]. The domination deal with many fields as a strong graph and fuzzy graph [11, 20], topological graph [13], and others. C. Berge in [10] is the first person Proposition 1.2. [20].

who introduced the domination parameter. In [21] the co-neighborhood domination is defined as a subset  $D \subset V$  is equality co-neighborhood dominating set of  $G$  (ENDS), if  $|N[v] \cap (V - D)|$  equal  $\forall v \in D$ . In [18] the inverse co-neighborhood domination is introduced. In this paper, the new concept of parameter domination is introduced which is called the strong EN domination.

To prove our main results we need the following results

Definition.1.1[20]. Let  $G$  be a simple graph, a proper subset  $D \subset V$  is called equally co-neighborhood dominating set of  $G$  (ENDS), if every vertex in set  $D$  is adjacent to equally number of vertices in  $V - D$ . The set  $D$  is called minimal ENDS (MENDES) if it has no proper ENDS. The equally domination number denoted by  $\gamma_{en}(G)$  for simplicity  $\gamma_{en}(G)$  is the minimum cardinality of a MENDES. The MENDES of cardinality  $\gamma_{en}$  is called  $\gamma_{en}$ - set.

For a complete bipartite graph  $K_{n,m}, \gamma_{en}(K_{n,m}) = \min\{m, n, |m - n| + 2\}$ .

Theorem 1.3. [20].

If  $G_1$  and  $G_2$  are two graphs, then in general 1)  $\gamma_{en}(G_1 \odot G_2) = |G_1|$ . 2)  $\gamma_{en}(G_2 \odot G_1) = |G_2|$ .

2 SENDS for some certain graphs

Definition 2.1.

Consider  $G$  is a simple graph,  $D \subset V$  is called the strong equality co-neighborhood dominating set of the graph  $G$  (SENDS), if  $|N[v] \cap (V - D)|$  equal  $\forall v \in D$ , and  $\deg(v) \geq \deg(u) \forall v \in D$  and  $\forall u \in V - D$ . The strong co-equally domination number denoted by  $\gamma_{en}^s(G)$  is the minimum cardinality of SENDS. The MSENDS of cardinality is called  $\gamma_{en}^s$ -set. (For example see Fig .1)

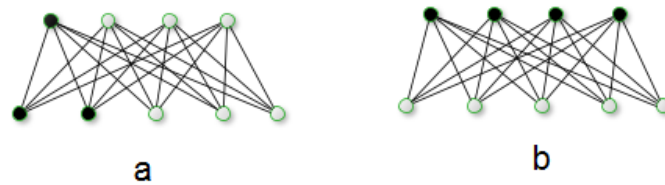


Figure 1: (a)  $\gamma_{en}(G)$  (b)  $\gamma_{en}^s(G)$

Proposition 2.2. Assume that the graph  $G$  has order  $n$ , then

1.  $v \notin D$ , if  $v$  is a pendant vertex such that  $n \geq 3$  and  $G$  be connected graph.
2.  $\gamma_{en}^s(G) = \gamma_{en}(G)$  If  $\gamma_{en}(G) = 1$
3.  $1 \leq \gamma_{en}^s(G) \leq n - 1$ .
4.  $\gamma_{en}^s(G) \geq \gamma_{en}(G)$ .
5. If  $G$  be a graph has ENDS, then  $G$  not necessary has SENDS

Proof.

Let  $v$  be a pendant vertex in  $G$  and let  $D$  be a  $\gamma_{en}^s(G)$ -set. since  $n \geq 3$  and  $G$  is connected graph, then there is  $u \in G$  such that  $\deg(u) \geq 2$ , since  $\deg(u) > \deg(v)$  hence  $v \notin D$ , according to Definition 1.1 and Definition 2.1.

If  $\gamma_{en}(G) = 1$ , then there is vertex ( $v$ ) such that  $\deg(v) = n - 1$ , therefore  $\gamma_{en}^s(G) = \gamma_{en}(G)$ .

The lower bound occurs by (2) and upper bound occurs when  $G = P_2$ .

If  $G = K_{4,5}$ , then  $\gamma_{en}(G) = 3$  but  $\gamma_{en}^s(G) = 4$ , then  $\gamma_{en}^s(G) > \gamma_{en}(G)$  and by (2) therefore  $\gamma_{en}^s(G) \geq \gamma_{en}(G)$ . (For example see Fig .1)

It is straightforward by the following example.

Example .2.3

Let  $G$  be a bipartite graph see figure 2. We have  $D_1 = \{v_1, u_1, u_2\}, D_2 = \{v_2, u_1, u_2\}$  and  $D_3 = \{v_3, v_4, u_3, u_4, u_5\}$  are ENDS, but not SENDS because  $\deg(v_2) \geq \deg(u_1), \deg(v_1) \geq \deg(u_1), \deg(v_2) \geq \deg(u_3)$ , respectively for  $v_1$  or  $v_2 \in V - D$ , then  $G$  has no SENDS.

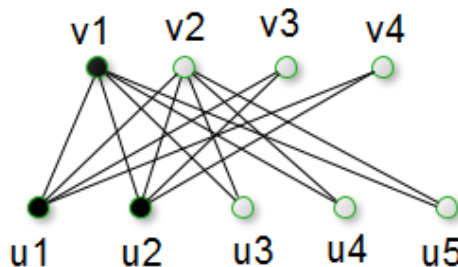


Figure 1:  $G$  has no SENDS

Proposition 2.4. For complete  $K_n$ , wheel  $W_n$ , and star  $S_n$  graphs, then

$$\gamma_{en}^s(S_n) = \gamma_{en}^s(K_n) = \gamma_{en}^s(W_n) = 1, \forall n \geq 3.$$

Proof.

It is straightforward from Proposition 2.2.(2).

Proposition 2.5. For a complete bipartite graph, then

$$\gamma_{en}^s(K_{n,m}) = \begin{cases} \min\{n, m\}, & \text{if } n \neq m \\ 2, & \text{if } n = m \end{cases}$$

Proof.

Let  $K_{n,m}$  be complete bipartite graph and let  $X, Y$  be partite subsets of  $K_{n,m}$  such that  $|X|=n$  and  $|Y|=m$ . Since  $X, \text{ and } Y$  are ENDS according to Proposition 1.2, if  $m > n$  then  $\deg(v) > \deg(u) \forall v \in X \text{ and } u \in Y$  according to definition of complete bipartite graph, then only  $X$  is SENDS, therefore  $\gamma_{en}^s(K_{n,m}) = \min\{n, m\}$ .

If  $n=m$ , then all vertices has equal degree, hence the set  $\{v,u\}$  is SENDS  $\forall v \in X \text{ and } u \in Y$  according to Definition 2.1., therefore  $\gamma_{en}^s(K_{n,m}) = 2$

Theorem 2.6. Let  $G$  be a  $r$ -regular graph of order  $n$ , then

$$\gamma_{en}^s(G_n) = \gamma_{en}(G_n) = \left\lfloor \frac{n}{r+1} \right\rfloor, \forall n \geq r + 1.$$

Proof.

Since  $\deg(v) = r \forall v \in G$ , then for every  $r$  of vertices in  $G$  there is one vertex that dominates these vertices, so we could have  $\frac{n}{r+1}$  set if  $n \equiv 0 \pmod{r+1}$  and  $\gamma_{en}(G_n) = \frac{n}{r+1}$ , and if  $n \equiv j \pmod{r+1}$ , then there are set has  $j \leq r$  of vertices one vertex dominates the rest, therefore  $\gamma_{en}(G_n) = \left\lfloor \frac{n}{r+1} \right\rfloor$ .

Now Since  $\deg(v) = r \forall v \in G_n$ , then  $\gamma_{en}^s(G_n) = \gamma_{en}(G_n) = \left\lfloor \frac{n}{r+1} \right\rfloor$  according to Definition 2.1.

Proposition 2.7. Let  $C_n$  be a cycle graph with order  $n$ , then

$$\gamma_{en}^s(C_n) = \gamma_{en}(C_n) = \left\lfloor \frac{n}{3} \right\rfloor, \forall n \geq 3.$$

Proof.

Since  $\deg(v) = 2 \forall v \in C_n$ , then  $\gamma_{en}^s(C_n) = \gamma_{en}(C_n) = \left\lfloor \frac{n}{3} \right\rfloor$  according to Definition 2.1. and Theorem 2.6.

Proposition 2.8. Let  $P_n$  be a path graph with order  $n$ , then

$$\gamma_{en}^s(P_n) = \gamma_{en}(P_n) = \left\lfloor \frac{n}{3} \right\rfloor, \forall n \geq 2.$$

Proof.

Since  $\deg(v) = 2 \forall v \in P_n$  except for the pendant vertices, then according to Definition 2.1 and Proposition 2.2 the pendant vertices are not in  $D$ , and they must be in  $V - D$ , then  $\gamma_{en}^s(P_n) = \gamma_{en}(P_n) = \left\lfloor \frac{n}{3} \right\rfloor$ .

### 3 SENDS of the join two graphs

Theorem 3.1. Let  $G_1$  and  $G_2$  are two graphs with order  $n, m$  respectively, and let  $D_i = \gamma_{en}^s(G_i)$  - set if  $G_i$  has SENDS or  $D_i = G_i$  if  $G_i$  has no SENDS such that  $i=1,2$ , then

$$\gamma_{en}^s(G_1 + G_2) =$$

$$\min \begin{cases} 2 & \text{if } G_1 \equiv G_2 \\ |D_1| & \text{if } m + s_1 \geq n + p_2 \text{ such that } s_1 \text{ is the smallest degree in the vertices} \\ & \text{of } D_1 \text{ and } p_1 \text{ is the largest degree in the vertices of } D_2 \\ |D_2| & \text{if } n + s_2 \geq m + p_1 \text{ such that } s_2 \text{ is the smallest degree in the vertices} \\ & \text{of } D_2 \text{ and } p_1 \text{ is the largest degree in the vertices of } D_1 \\ \min \{|D_1|, |D_2|\}, & \text{if } m + s_1 = n + p_2 \text{ and } n + s_2 = m + p_1 \\ |S| & \text{where } S \text{ is the minimum set such that } S \cap V(G_1) \neq \emptyset \text{ and } S \cap V(G_2) \neq \emptyset \text{ and} \\ & N(v_i) \cap V - S \text{ is equal } \forall v_i \in S \text{ and } \deg(v) \geq \deg(u) \forall v \in S \text{ and } u \in V - S \end{cases}$$

Proof.

There are three cases as follows.

Case 1. For  $G_1 \equiv G_2$ , and according to definition of  $(G_1 + G_2)$  every vertex in  $G_1$  is adjacent to all vertices in  $G_2$  and vice versa, then  $\gamma_{en}^s(G_1 + G_2)$ -set =  $\{v, u\}$  such that  $v \in V(G_1)$  and it has largest degree in  $G_1$ , as well as  $u \in V(G_2)$  and it has largest degree in  $G_2$  and  $v$  is a correspondent to vertex  $u$ , therefore  $\gamma_{en}^s(G_1 + G_2) = 2$ .

Case 2. If  $G_1$  and  $G_2$  have SENDS, then  $D_1$  be  $\gamma_{en}^s(G_1)$ -set and  $D_2$  be  $\gamma_{en}^s(G_2)$ -set. And if  $G_1$  and  $G_2$  have no SENDS, then  $D_1 = G_1$  and  $D_2 = G_2$ , then  $D_1$  and  $D_2$  are SENDS of  $(G_1 + G_2)$ , There are three subcases of MSENDs of  $(G_1 + G_2)$  depending on whether  $G_1$  and  $G_2$  have SENDS or not, as follows.

If  $m + s_1 \geq n + p_2$ , then  $\deg(v_1) \geq \deg(v_2) \forall v_1 \in D_1$  and  $\forall v_2 \in D_2$ , since  $D_1$  and  $D_2$  are SENDS of  $(G_1 + G_2)$ , therefore  $\gamma_{en}^s(G_1 + G_2) = |D_1|$ .

If  $n + s_2 \geq m + p_1$ , then  $\deg(v_2) \geq \deg(v_1) \forall v_1 \in D_1$  and  $\forall v_2 \in D_2$ , since  $D_1$  and  $D_2$  are SENDS of  $(G_1 + G_2)$ , therefore  $\gamma_{en}^s(G_1 + G_2) = |D_2|$ .

If  $m + s_1 = n + p_2$  and  $n + s_2 = m + p_1$ , then is clear  $\gamma_{en}^s(G_1 + G_2) = \min \{|D_1|, |D_2|\}$

Case 3. Since  $S$  is minimum subset of  $(G_1 + G_2)$  such that has some vertices from  $G_1$  and some vertices from  $G_2$  and since  $(v_i) \cap V - D$  is equal  $\forall v_i \in S$  and  $\deg(v) \geq \deg(u) \forall v \in S$  and  $u \in V - S$ , then  $S$  is SENDS and strong according definition of the operation of join of two graphs and Definition 2.1. Therefore,  $\gamma_{en}^s(G_1 + G_2) = |S|$  in this case.

Assuming that all or some cases are fulfilled, then  $\gamma_{en}^s(G_1 + G_2) = \min \{ \text{case. 1, case. 2, case. 3} \}$ . Thus the proof is done.

Corollary 3.2.

$F_n = P_n + K_1$  be a fan graph with order  $n + 1$ , then  $\gamma_{en}^s(F_n) = 1 \forall n \geq 2$  (For example see Fig. 3(a))

$C_{n,m} = C_n + \overline{K_m}$  be a cone graph with order  $n + m$ ,  $\overline{K_m} \equiv N_m$  then

$$\gamma_{en}^s(C_{n,m}) = \begin{cases} \gamma_{en}^s(C_n) = \left\lceil \frac{n}{3} \right\rceil, & \text{if } m + 2 > n \\ m, & \text{if } m + 2 \leq n \end{cases} \quad (\text{For example see Fig. 3 (b,c)})$$

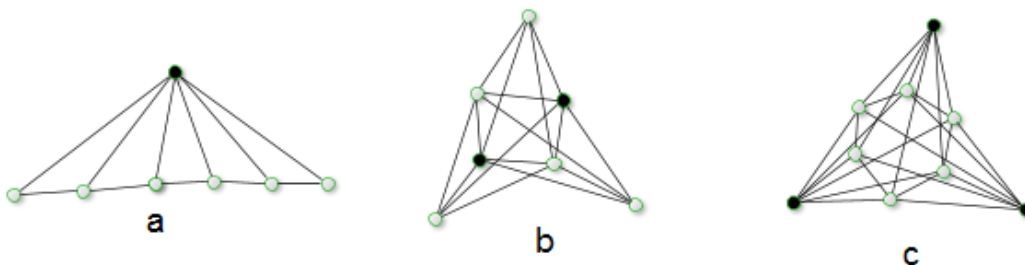


Figure 3: (a) SENDS in fan  $F_6$  (b) SENDS in cone  $C_{4,3}$  (c) SENDS in cone  $C_{6,3}$

Proposition 3.3.

1. For ladder graph  $L_n$ ,  $\gamma_{en}^s(L_n) = \gamma_{en}(L_n) = 2 \left\lceil \frac{n}{3} \right\rceil$  for  $n \geq 2$ .
2. For helm graph  $H_n$ ,  $\gamma_{en}^s(H_n) = n$  if  $n = 3, 4$  and  $H_n$  has no SENDS if  $n > 4$ .
3. For barbell graph  $B_{n,n}$ ,  $\gamma_{en}^s(B_{n,n}) = 2$ .
4. For a windmill graph  $W_n^m$ ,  $\gamma_{en}^s(W_n^m) = 1$  for  $m \geq 3$  and  $n \geq 2$

Proof.

1. According to definition of ladder graph  $L_n = P_n \times P_2$ , for  $n \geq 2$  and the product each vertex in the first path is associated with the corresponding vertex in second path, then  $\gamma_{en}(L_n) = 2 \gamma_{en}(P_n) = 2 \lfloor \frac{n}{3} \rfloor$ . Since  $\deg(v) = 3 \forall v \in \gamma_{en}(L_n) - set$  and  $\deg(u) \leq 3 \forall u \notin \gamma_{en}(L_n) - set$ , then  $\gamma_{en}^s(L_n) = \gamma_{en}(L_n) = 2 \lfloor \frac{n}{3} \rfloor$ . (For example see Fig .4(a) )

2. According to definition of helm graph  $H_n = W_n \cup N_n \cup \{u_i v_i \in E(H_n): \text{for } v_i \in C_n \text{ and } u_i \in N_n, \forall i = 1, 2, 3, \dots, n\}$ . We have  $\deg(v_i) = 4$  and  $\deg(p_i) = 1 \forall p_i \in N_n$  we get the  $\gamma_{en}(H_n) - set = C_n$  then if  $n \geq 4 \deg(v) > \deg(v_i) \forall v_i \in C_n$  and  $v$  is centre of  $W_n$  and  $v \notin \gamma_{en}(H_n) - set$ , but if  $n=3,4$ , then  $\deg(v) \leq \deg(v_i)$ . Thus the prove is done

3. Since  $B_{n,n} = K_n \cup K_n \cup \{uv \in E(B_{n,n}): \text{for } u \in K_n^1 \text{ and } v \in K_n^2\}$ , then It is clear  $\gamma_{en}^s(B_{n,n}) - set = \{u, v\}$ , because  $\deg(v) = \deg(u) = n$ , then  $\gamma_{en}^s(B_{n,n}) = 2$ .

4. According to definition of windmill graph we get  $n$  copies of complete graph  $K_m$  are joining by one common vertex ( $v$ ), then  $\deg(v) = m(n - 1)$ , then it is clear  $\gamma_{en}^s(W_n^m) - set = \{v\}$  and  $\gamma_{en}^s(W_n^m) = 1$

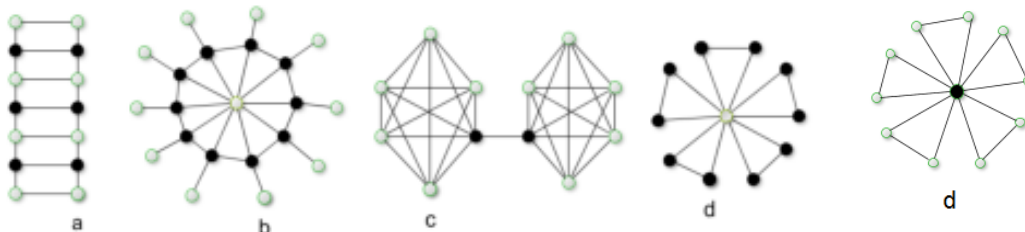


Figure 4: SENDS in (a) ladder graph  $L_7$  (b) helm graph  $H_{10}$  (c) barbell graph  $B_{6,6}$  (d) in windmill graph  $W_3^5$

4 SENDS of complement of certain graphs

Proposition 3.1.

1.  $\overline{C_n}$  and  $\overline{P_n} \forall n \leq 3$  has no SENDS
2.  $\gamma_{en}^s(\overline{P_n}) = 2$ , for  $n \geq 4$ .
3.  $\gamma_{en}^s(\overline{C_n}) = 2$ , for  $n \geq 4$ .

Proof.

1. Since  $\overline{C_n}$  and  $\overline{P_n} \forall n \leq 3$  has isolated vertex then has no ENDS and no SENDS.

2. There are two pendants vertices in  $P_n$   $v_1$ , and  $v_n$ , such that the vertex  $v_1$  is adjacent to all other vertices in the graph  $\overline{P_n}$ , except the vertex  $v_2$  and  $v_n$  is adjacent to all vertices in  $\overline{P_n}$ , except  $v_{n-1}$ . Since  $\deg(v_1) = \deg(v_n) = 1$  in  $P_n$ , then  $\deg(v_1) = \deg(v_n) = n - 2$  and  $\{v_1, v_n\}$  is ENDS in  $\overline{P_n}$ , but  $\deg(v_i) = 2 \forall 1 < i < n$  in  $P_n$ , then  $\deg(v_i) = n - 3$  in  $\overline{P_n}$ , then  $\{v_1, v_n\}$  is SENDS in  $\overline{P_n}$  and  $\gamma_{en}^s(\overline{P_n}) = 2$

3. Since  $C_n$  is 2-regular graph then  $\overline{C_n}$  is  $n - 3$ -regular graph, then  $\gamma_{en}(\overline{C_n}) = \gamma_{en}^s(\overline{C_n}) = 2$  according to Theorem 1.3. The proof is done.

Proposition 4.2.  $\gamma_{en}^s(\overline{K_{n,m}}) = \begin{cases} \text{has no SENDS,} & \text{if } n \neq m \\ 2, & \text{if } n = m \end{cases}$

Proof. Let  $m \geq n$ , since  $\overline{K_{n,m}}$  and  $K_n \cup K_m$  are isomorphic graphs, then must be  $\gamma_{en}(\overline{K_{n,m}}) = m - n + 2$ , and  $\gamma_{en}(\overline{K_{n,m}}) - set$  has one vertex from  $K_n$  and  $m - n + 1$  vertices from  $K_m$ , according to Definition 1.1. and according to Proposition 1.2. then

If  $m = n$ , then  $\gamma_{en}(\overline{K_{n,m}}) = 2$ , since  $\deg(v) = n - 1 \forall v \in \overline{K_{n,m}}$ , then  $\gamma_{en}^s(\overline{K_{n,m}}) = 2$  according to Definition 2.1.

If  $m > n$  then  $\deg(v) > \deg(u) \forall v \in K_n$  and  $u \in K_m$ , then  $\gamma_{en}(\overline{K_{n,m}})$  is not SENDS. Therefore  $\overline{K_{n,m}}$  has no SENDS.

5 SENDS of the corona graphs

Theorem 4.1. Let  $G_1$  and  $G_2$  are two graphs, then

$$\gamma_{en}^s(G_1 \odot G_2) = \gamma_{en}(G_1 \odot G_2) = |G_1|$$

Proof.

If  $G_1$  and  $G_2$  are two graphs of order  $n \geq 2$  and  $m \geq 1$  respectively and since  $\gamma_{en}(G_1 \odot G_2) = |G_1|$  according to Theorem 1.3, then  $\deg(v) \geq m$  and  $\deg(u) \leq m - 1 \forall v \in G_1$  and  $\forall u \in G_2$  in  $(G_1 \odot G_2)$ , then  $\gamma_{en}^s(G_1 \odot G_2) = \gamma_{en}(G_1 \odot G_2) = |G_1|$ .

Proposition 4.2. If  $G_1$  and  $G_2$  are two graphs of order  $n \geq 2$  and  $m \geq 1$  respectively, then

$$\gamma_{en}^s(\overline{G_1 \odot G_2}) = \gamma_{en}(\overline{G_1 \odot G_2}) = 2$$

Proof.

Since  $G_1$  and  $G_2$  are two graphs of order  $n \geq 2$  and  $m \geq 1$  respectively, then we have every vertex in  $i_{th}$  copy of  $G_2$  is adjacent to all the vertices of  $(\overline{G_1 \odot G_2})$  except  $i_{th}$  vertex

in  $G_1$  and all other vertices in same copies of  $G_2$ . Let  $D = \{v, u: v \text{ in } i_{th} \text{ copy of } G_2 \text{ and } u \text{ in}$

$(i + 1)_{th}$  copy of  $G_2$ , such that  $\deg(v) \geq \deg(v_i) \forall v_i \in i_{th} \text{ copy of } G_2$ , as well as  $u\}$ , then set  $D$  is  $\gamma_{en}^s(\overline{G_1 \odot G_2})$  -set. Therefore,  $\gamma_{en}^s(\overline{G_1 \odot G_2}) = 2$

Conclusion.

Throughout this paper, the modern and strong of this modern concept of domination have been defined. Many propositions, theorems, and corollary are proved. Also, for most the certain graphs this number is determined. Moreover, some operation on graphs are calculated as a complement, join, and corona.

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