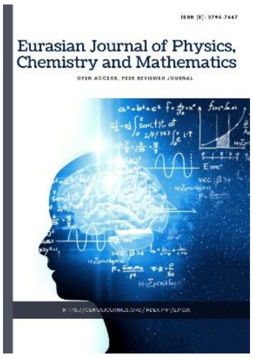

Foundations Of The Theory Of Empirical Characteristic Functions

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ABSTRACT

Probabilities theory random of events event to give probability based on other of events probability determination with practitioner mathematics from networks is one. Each of the event face to give probability is close to 1 if so , it in practice almost to consider inevitable possible. Therefore for probabilities in theory probability is close to 1 was situations big importance profession will reach.

Keywords: Characteristic Of Functions, Extreme Value, Asymptotic Properties, Asymptotic Distribution, Mathematical Expectation, Variance

1. Characteristic of functions definition and simple properties

Probabilities theory random of events event to give probability based on other of events probability determination with practitioner mathematics from networks is one. Each of the event face to give probability is close to 1 if so , it in practice almost to consider inevitable possible. Therefore for probabilities in theory probability is close to 1 was situations big importance profession will reach.

Most real processes random, mutual weak connected factors under the influence happened to be because of , probabilities theory this factors between laws in determining important role plays. For example, a large numbers law such inevitability provider to events is an example .

In Uzbekistan probabilities theory of the 20th century at the beginning take shape started in this field academician V.I.Romanovsky , as well as T.A.Sarimsakov, S.X.Sirojiddinov, T.A.Azlarov, and others important contributions added. Today 's on the day this theory mathematics high at the level developed field is considered .

Probabilities theory central from the concepts one this is random quantity is the concept of randomness . quantity this is a coincidence under the influence different numerical values acceptance doer is a quantity . Example for , products between defects number , random shooting successes number or device work duration etc. random quantities as is considered .

Such amounts usually of experience results to the ball relatively determined to be , them elementary events in space described to consider as functions possible . Any optional function not , maybe probability expressive and known set to the class suitable descending functions random quantity as is taken .

If ξ by chance your quantity $F_\xi(x)$ distribution function all $x \in R$ at points continuous if , then such random to the amount continuous random quantity It is called .

Definition : If ξ continuous random of the amount distribution function

$$F(x) = F_\xi(x) = \int_{-\infty}^{x} p(u)\,du$$

in appearance expression possible if so , then random to the amount absolute continuous random quantity is called . Here, the function p(x) is ξ random of the amount density function It is called .

Along with real-valued random variables, the theory of characteristic functions requires the involvement of complex-valued random variables. A complex-valued random variable is called

$$\zeta(\omega) = \xi_1(\omega) + i\xi_2(\omega)$$

is called a function of the form , where $\omega \in \Omega$ and (ξ_1, ξ_2) are random vectors. Many definitions and properties related to random variables are also valid for complex random variables. For example, if $M\xi_1$ and $M\xi_2$ mathematician expectations there is if so, then without $\zeta = \xi_1 + i\,\xi_2$ complex random of the amount mathematician wait

$$M\zeta = M\xi_1 + i\,M\xi_2$$

can be expressed by the formula.

If (ξ_1, η_1) and (ξ_2, η_2) are independent random vectors, then $\varsigma_1 = \xi_1 + i\eta_1$ and $\varsigma_2 = \xi_2 + i\eta_2$ are complex random variables.

Mathematician your expectation main features complex random quantities for also preserved remains.

Description . [5] $\xi -$ random of the amount **characteristic function** that the real t argument this

$$f_\xi(t) = Me^{itx} = \int_{\Omega} e^{itx} dP = \int_{-\infty}^{\infty} e^{itx} dF_\xi(x) \tag{1}$$

is called a function.

If ξ has a discrete distribution, then the characteristic function

$$f_\xi(t) = \sum_k e^{itx_k} P(\xi = x_k) = \sum_k e^{itx_k} p(x_k) \tag{2}$$

expressed through equality.

If ξ a random variable has an absolutely continuous distribution and $p(x)$ its density function is, then

$$f_\xi(t) = \int_{-\infty}^{\infty} e^{itx} p(x)\,dx \tag{3}$$

is, that is, the characteristic function of an absolutely continuous random variable $p(x)$ is the Fourier transform of the density function.

Euler to the formula according to $e^{i\alpha} = \cos\alpha + i\sin\alpha$ equality appropriate happened for (1. 1. 1) from the equality

$$f_\xi(t) = M\cos t\xi + iM\sin t\xi$$

come It 's out .

This $\left|f_\xi(t)\right| = \left|Me^{it\xi}\right| \le 1$ inequality k to i x theoretical random How much is the medicine? characteristic function existence come It 's out .

Characteristic function one how many the most basic x- axis there is and below them brought Let's go .

Theorem 1. $f_\xi(t) - \xi$ random of the amount x characteristic function if , then

1 ^{0}. $f_\xi(\circ) = 1$, $\left|f_\xi(t)\right| \le 1;$

2 ^{0}. I x theoretical a and b unchanging real numbers for $f_{a+b\xi}(t) = e^{iat} f_\xi(bt)$ equality suitable ;

3 ^{0}. $\overline{f}_\xi(t) = f_\xi(-t) = f_{-\xi}(t);$

4 ^{0}. If $\xi_1, \xi_2, \dots, \xi_n$ independent random quantities if , then

$f_{\xi_1+\dots+\xi_n}(t) = f_{\xi_1}(t)\cdot f_{\xi_2}(t)\cdots f_{\xi_n}(t)$

equality suitable ;

5 ^{0}. $f_\xi(t)$ x characteristic function $R = (-\infty, \infty)$ flat on continuous ;

6 ^{0}. $f_\xi(t)$ x characteristic function real to be for ξ random of the amount $F_\xi(x)$ distribution function symmetrical to be necessary and enough .

Proof . 1 ^{0} x ossify proof $e^{i0} = 1$ and $\left|Me^{it\xi}\right| \le M\left|e^{it\xi}\right| = M1 = 1$ from relationships come It turns out that 2 ^{0} and 3 ^{0} are x oscillating . x characteristic function by definition brought release possible . Indeed ,

$f_{a+b\xi} = Me^{it(a+b\xi)} = e^{ita}Me^{itb\xi} = e^{ita}f_\xi(bt)$

and

$\bar{f}_\xi(t) = \overline{Me^{it\xi}} = Me^{\overline{it\xi}} = Me^{-it\xi} = f_\xi(-t).$

4 ^{0} x oss proof . $e^{it\xi_1}, e^{it\xi_2}, \dots, e^{it\xi_n}$ complex random quantities independent . So ,

$f_{\xi_1+\xi_2+\dots+\xi_n}(t) = Me^{it(\xi_1+\xi_2+\dots+\xi_n)} = Me^{it\xi_1}e^{it\xi_2}\dots e^{it\xi_n} = Me^{it\xi_1}Me^{it\xi_2}\dots Me^{it\xi_n} = f_{\xi_1}(t)f_{\xi_2}(t)\dots f_{\xi_n}(t).$

This on the ground independent complex values acceptance doer limited random quantities of the lot mathematician unexpected suitable mathematician expectations to a large extent equal because it is used .

Now $f_\xi(t)$ function flat continuity We prove . Optional real $x, y \in R$ numbers for

$$\left|e^{ix} - e^{iy}\right| \le 2 \tag{4}$$

and

$$\left|e^{ix}-e^{iy}\right|=\left|\int_x^y e^{iu}du\right|\le\left|\int_x^y du\right|=\left|x-y\right| \tag{5}$$

inequalities It is appropriate .

N optional being a positive number , $A=\{\omega\in\Omega;|\xi(\omega)|\le N\}$ Let it be . Then

$$\left|f_\xi(t_1)-f_\xi(t_2)\right|=\left|M\left(e^{it_1\xi}-e^{it_2\xi}\right)\right|\le M\left|e^{it_1\xi}-e^{it_2\xi}\right|I_A+M\left|e^{it_1\xi}-e^{it_2\xi}\right|I_{\bar A}$$

from the relationship $|\xi|>N$ if , (1.1.4) and $|\xi|\le N$ If , then (1.1.5) inequality using this

$$\left|f_\xi(t_1)-f_\xi(t_2)\right|\le\left|t_1-t_2\right|N+2P(|\xi|>N) \tag{6}$$

inequality appropriate that come it comes out .

Optional $\varepsilon>0$ number given Let it be . Initially N so we choose , as a result $P(|\xi|>N)<\varepsilon/4$ let it be , then $\delta=\dfrac{\varepsilon}{2N}$ we can say and from this $|t_1-t_2|<\delta$ inequality satisfactory all t_1,t_2 numbers from the relation (1.1.6) for

$$\left|f_\xi(t_1)-f_\xi(t_2)\right|<\varepsilon$$

inequality suitable that come it comes out .

6 ^{0} x ossani proof for us different distribution to functions various x characteristic functions suitable from the arrival We use . Indeed ,

$$\overline{f}_\xi(t)=f_\xi(t)$$

if and only that's all $f_\xi(t)$ real , that is , 3 ^{0} x ossa according to , ξ and $-\xi$ a year is characteristic to the function has if and only that's all she is real is considered . But this own in turn ξ and $-\xi$ one year to distribution has that is , $F_\xi(x)$ symmetrical to the fact that is equivalent .

Characteristic functions application mainly 4 ^{0} x oss is based on . Independent random quantities add very complicated was action that is participants distribution functions to the composition is brought . Characteristic functions for this complicated action x characteristic functions multiplication practice with interchangeable That's it .

Characteristic of functions special properties

Theorem 2. If $M|\xi|^k<\infty$ if , then $f_\xi(t)$ x characteristic function k - orderly continuous to the product has divided into the following relationship appropriate will be :

$$f_\xi^{(\nu)}(t)=i^\nu\int_{-\infty}^{\infty}x^\nu e^{itx}dF_\xi(x),\quad \nu=1,2,\dots,k, \tag{7}$$

$$f_\xi^{(\nu)}(0)=i^\nu M\xi^\nu, \tag{8}$$

$$f_\xi(t)=\sum_{\nu=0}^{k}\frac{(it)^\nu}{\nu!}M\xi^\nu+o(t^k),\quad t\to0. \tag{9}$$

Proof . (7) Eq. $\nu=1$ for We prove the following .

$$\frac{f_\xi(t+h)-f_\xi(t)}{h}=M\,\frac{e^{it\xi}(e^{ih\xi}-1)}{h}$$

in relation (5) to inequality see $\left|\frac{e^{ih\xi}-1}{h}\right|\le 1$ and of the theorem on condition see and $M\left|\xi\right|<\infty$ happened for Lebesgue's majorant approximation about to the theorem according to $h\to 0$ to the limit to pass possible and then ,

$$f_\xi^{'}(t)=\lim_{h\to 0}\frac{f_\xi(t+h)-f_\xi(t)}{h}=\lim_{h\to 0}M\,\frac{e^{it\xi}(e^{ih\xi}-1)}{h}=iM\xi e^{it\xi}$$

equality that come it comes out .

(7) relationship general without proof for we this

$$\left|e^{ix}-\left(1+\frac{ix}{1!}+\cdots+\frac{(ix)^{n-1}}{(n-1)!}\right)\right|\le\min\left\{\frac{|x|^n}{n!},\frac{2|x|^{n-1}}{(n-1)!}\right\} \tag{10}$$

from inequality we use .

(10) of the inequality $n=1$ was from the inequalities (4) and (5) come (10) relationship any n for appropriate Let it be . Then

$$\int_0^\infty\left(e^{in}-\sum_{k=0}^{n-1}\frac{(in)^k}{k!}\right)du=\frac{1}{i}(e^{ix}-\sum_{k=0}^{n}\frac{(ix)^k}{k!})$$

equality suitable since it happened

$$\left|e^{ix}-\left(1+\frac{ix}{1!}+\cdots+\frac{(ix)^n}{n!}\right)\right|\le\int_0^{|x|}\left|e^{iu}-\sum_{k=0}^{n-1}\frac{(iu)^k}{k!}\right|du\le\int_0^{|x|}\min\left\{\frac{|u|^n}{n!},\frac{2|u|^{n-1}}{(n-1)!}\right\}du\le$$

$$\le\min\left\{\int_0^{|x|}\frac{u^n}{n!}du,2\int_0^{|x|}\frac{u^{n-1}}{(n-1)!}du\right\}=\min\left\{\frac{|x|^{n+1}}{(n+1)!},2\frac{|x|^n}{n!}\right\}.$$

So , completely mathematician induction to the method According to inequality (10), optional $n\ge 1$ for suitable. (7) formula $\nu<k$ for suitable Let it be . Then

$$\frac{f_\xi^{(\nu)}(t+h)-f_\xi^{(\nu)}(t)}{h}=i^\nu M\xi^\nu\,\frac{e^{it\xi}\left(e^{ih\xi}-1\right)}{h}$$

from equality (10) to inequality mainly

$$\left|\xi^\nu e^{it\xi}\,\frac{\left(e^{ih\xi}-1\right)}{h}\right|\le|\xi|^{\nu+1}$$

happened and of the theorem on condition see $M|\xi|^{\nu+1}<\infty$ happened for Lebesgue's majorant approximation to the theorem according to

$$\lim_{h\to 0}i^\nu M\xi^\nu\,\frac{e^{it\xi}\left(e^{it\xi}-1\right)}{h}=i^{\nu+1}M\xi^{\nu+1}.$$

S h his for

$$f_\xi^{(\nu+1)}(t)=\lim_{h\to 0}\frac{f_\xi^{(\nu)}(t+h)-f_\xi^{(\nu)}(t)}{h}=i^{\nu+1}M\xi^{\nu+1}.$$

In the relation (7) $t = 0$ that, we use formula (8) harvest We do . In relation (9) residue limit assessment for (10) inequality

$$R_k(t) = \left| f_\xi(t) - \sum_{\nu=0}^{k} \frac{(it)^\nu}{\nu!} M\xi^\nu \right| = \left| M\left(e^{it\xi} - \sum_{\nu=0}^{k} \frac{(it\xi)^\nu}{\nu!} \right) \right|$$

to separate we use :

$$R_k(t) \le \frac{|t|^{k+1}}{(k+1)!} M|\xi|^{k+1} I_A + 2M\frac{|t|^k|\xi|^k}{k!} I_{\bar{A}}, \tag{11}$$

this on the ground $A = \{\omega \in \Omega : |\xi(\omega)| \le N\}$.

If $|\xi| > N$ if $I_A = 0$ happened because of (11) this

$$R_k(t) \le \frac{NM|\xi|^k|t|^{k+1}}{(k+1)!} + \frac{2|t|^k}{k!} M|\xi|^k I_{\bar{A}}$$

inequality come it comes out . $\varepsilon > 0$ optional Let be a positive number . First N number so we choose , as a result

$$M|\xi|^k I_{\bar{A}} < \frac{\varepsilon}{4}$$

poverty Let it be , then $\delta = \frac{\varepsilon(k+1)}{2NM|\xi|^k}$ that we get , as a result $|t| < \delta$ inequality satisfactory all $t \in R$ numbers for $R_k(t) \le \frac{|t|^k}{k!}\varepsilon$ that is $R_k(t) = o\left(t^k\right), t \to 0$ attitude to has Let 's be . proof demand made was .

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