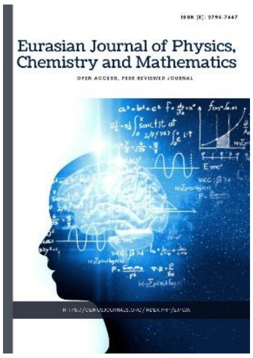

Mathematical Model Of Regulation Of Dynamic Processes In Systems With Distributed Parameters (On The Example Of Flooding In Oil Fields)

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ABSTRACT

The paper considers an applied problem of regulating dynamic flooding processes in the development of oil fields. Mathematical models, computational algorithms, and the results of a computational experiment are presented.

Keywords: optimal control, regulation, dynamic process, flooding, system, filtration theories, oil wells, numerical methods, finite differences, differential equation, maximum principle

Introduction: Mathematical modeling of dynamic regulation and control processes in systems with distributed parameters is a rapidly developing area of applied mathematics, since it allows describing and analyzing complex systems where parameters change not only in time, but also in space. For such systems, methods of regulation and control are developed that use mathematical models to predict the behavior of the system and achieve the desired state by influencing it [1,3,10,12,].

There are numerous works devoted to the control of dynamic processes in systems with distributed parameters, applied to oil and gasfields,in particular, the determination of optimal modes of operation of various oil and gas production facilities, using the methods of optimal control theory, is considered [1-3]. The aim is to develop some techniques that make it possible to reasonably determine the optimal modes of periodic operation of injection wells. Existing mathematical models of hydrodynamic filtration processes and existing methods of optimal control theory, as well as modern methods of computational experiment based on numerical analysis methods, make such applied problems of software and optimal control solvable without any difficulties [4-7, 11].

Problem statement: Suppose that the mathematical model of the liquid filtration process in a medium with non-uniform permeability has the form (in dimensionless form) [2,4,6,7]:

$$\frac{\partial P}{\partial \tau}+\gamma_1\frac{\partial^2 P}{\partial \tau^2}-\gamma_2\frac{\partial^3 P}{\partial \xi^2 \partial \tau}-\gamma_3\frac{\partial^2 P}{\partial \xi^2}=0 \qquad (1)$$

Let be at the point $\xi=0$ an injection well is located. The injection pressure in it determines the pressure distribution over the entire length of the reservoir

$$0\le\xi\le1$$

The control action is taken as a lumped function of a time variable $U(\tau)$, which is the pressure at the point $\xi=0$, i.e.

$$P(\xi,\tau)/_{\xi=0}=U(\tau). \qquad (2)$$

Assume that the initial moment of the dynamic process under consideration coincides with the well start-up, i.e. the system is in an equilibrium state. Then you can set the following conditions:

$$P(\xi,\tau)/_{\xi=1}=0, \qquad (3)$$

$$P(\xi,\tau)/_{\tau=0}=0, \qquad (4)$$

$$\frac{\partial P(\xi,\tau)}{\partial \tau}/_{\tau=0}=0 \qquad (5)$$

The discharge pressure cannot exceed the maximum level for which the pump is designed, so there is a natural technological limitation, written in dimensionless form:

$$0\le U(\tau)\le 1. \qquad (6)$$

Main part: Reservoir flooding is one of the main ways to maintain reservoir pressure. Therefore, the following can be taken as the management goal:

$$Z=\int_0^T (P(\xi,T)-P^*(\xi))^2 d\xi \to \min, \qquad (7)$$

that is, it is required to select the control - discharge pressure in such a way that by the end of the period $T=1$ square of the root-mean-square deviation of the true pressure distribution $P(\xi,T)$ by layer from the specified one $P^*(\xi)$ was minimal. Moreover, the moment of time *T*, i.e. the duration of water injection, is also subject to determination. Therefore, the problem under consideration must be solved in the following sequence.

1. For a given *T*, find such a control $U(\tau)$, which will satisfy constraint (6) and provide a minimum of functionality (7).

2. Define the control $U(\tau)$, according to condition (6), so that in the minimum time the functional satisfies the inequality $Z\le\gamma$, where $\gamma>0$ the specified number.

So, first we consider the following problem: for equation (1) satisfying conditions (2) - (5), choose a control function that is subject to constraint (6) and delivers a minimum to functional (7).

The solution is carried out by the finite difference method of numerical analysis [8, 9].

Разобь Divide the time coordinate into steps τ, and the coordinate with respect to the spatial variable in steps h, i.e.

$$x_i=ik; t_j=j\tau; i=\overline{1,N}; j=\overline{1,M}.$$

On the grid $\overline{W}_{\tau h}=\left\{x_i=ih, t_j=j\tau, (i=\overline{1,N}; j=\overline{1,M})\right\}$ let us construct an unconditionally stable implicit scheme, a finite-difference approximation of equation (1):

$$\frac{P_i^{j+1}-P_i^j}{\tau}+\gamma_1\frac{P_i^{j-1}-2P_i^j+P_i^{j+1}}{\tau^2}-\gamma_2\left(\frac{P_{i-1}^{j-1}+2P_i^{j+1}-P_{i-1}^{j+1}-P_{i-1}^j+2P_i^j-P_{i+1}^j}{\tau h^2}\right)-$$

$$-\gamma_3\frac{P_{i-1}^{j+1}-2P_i^{j+1}+P_{i+1}^{j+1}}{h^2}=0. \qquad (8)$$

Assuming $P_i^{j+1}=P_i$, equation (8) is written as:

$$a_i P_{i-1} - b_i P_i + c_i P_{i+1} = -d_i, \quad (9)$$

where

$$a_i = c_i = \frac{\gamma_2}{\tau h^2} + \frac{\gamma_3}{h^2}; b_i = \frac{1}{\tau} + \frac{\gamma_1}{\tau^2} + \frac{2\gamma_2}{\tau h^2} + \frac{2\gamma_3}{h^2};$$

$$d_i = -\frac{\gamma_2}{\tau h^2}(P_{i+1}^j + P_{i-1}^j) - \frac{\gamma_1}{\tau^2} P_i^{j-1} + \left(\frac{1}{\tau} + \frac{2\gamma_1}{\tau^2} + \frac{2\gamma_2}{\tau h^2}\right) P_i^j.$$

Equation (9) is solved by the run-through method [8], where

$$P_{i-1} = A_i P_i + B_i \quad (10)$$

$$A_i = a_i / (b_i - c_i A_{i-1}) \quad (11)$$

$$B_i = (F_{i-1} + bB_{i-1}) / (b_i - c_i A_{i-1}) \quad (12)$$

$$A_2 = 0; \quad B_2 = U^j \quad (13)$$

$$P_N = 0. \quad (14)$$

The computational experiment is performed in the following sequence:

1. According to formulas (11), (12), taking into account (13), the coefficients are determined A_i, B_i;

2. Using (14), formula (10) gives the values

$$P_{N-1}, P_{N-2}, \ldots, P_1.$$

Function values d_i known when $j = 1,2$. Indeed, the initial conditions are $P(\xi,0) = 0, \quad \frac{\partial P}{\partial \tau}/_{\tau=0} = 0$ equivalent to: $P_i^1 = 0, P_i^2 = 0.$ Therefore, at the first iteration, the values of the function on the 3rd time layer are determined by the run method. Then, if necessary P_i^3, P_i^2 defined by P_i^4 and so on until $j = M$.

The solution by the method of successive approximations is carried out according to the usual scheme.

1. Under the given conditions (2)-(5), equation (1) was integrated and determined by $P^K(\xi,1)$.

2. The functional value is calculated using formula (7). If

$$\left|Z^K - Z^{K-1}\right| \le \varepsilon,$$

then the invoice process ends, if not, then step 3 is executed.

3. Based on the search results $P^K(\xi,1)$ taking into account (2)-(6), the conjugate equation (1) is integrated.

4. Using the formula $\frac{\partial Z}{\partial U} = -\left(\gamma_2 \frac{\partial^2 f(0,\tau)}{\partial\xi\partial\tau} + \gamma_3 \frac{\partial f(0,\tau)}{\partial\xi}\right)$ the functional gradient is calculated.

5. Improved control is being built [2,11]:

$$U_j^{K+1} = U_j^K + \delta \frac{\partial Z}{\partial U_j}.$$

The calculations are repeated starting from point 1. The above method determines the duration of exposure and optimal control of the injection pressure for focal injection wells.

The heterogeneity of the reservoir caused the fact that water was pumped into the focal injection wells periodically. Changes in reservoir pressure in producing wells lead to the fact that the operation of injection wells must be regulated according to the current situation.

Calculation of technological parameters of periodic operation of injection wells was carried out at the following values:

$P = 900 кг / м^3; \mu = 0{,}1 П_a \cdot c; K_1 = 9 \cdot 10^{-14} м^2;$

$K_2 = 9\cdot 16^{-14}\,м^2; M_1 = 0,3; M_2 = 0,05; \tau_M = 86,4\cdot 10^3 c;$

$T = 8,64\cdot 10^5 c; \alpha_P = 0,5\cdot 10^{-10}\,Па^{-1}; \beta_c = 10^{-10}\,Па^{-1};$

$e = 0,4\cdot 10^{-6}\,м; \alpha = 10^{-11}; L = 200м; P_{\max} = 20МПа;$

$\varepsilon = 0,03; \delta = 0,01; \gamma = 0,4.$

At the first stage of the solution, various time points were set T and calculated the corresponding controls $U(\tau)$, delivering a minimum to the functional (7). At the second stage, according to Table 1, the minimum time for which the functional reached a value less than 0.3 was determined and the optimal value was chosen $T = 1$ (in dimensions T *=10* days). Analysis of the results of a computational experiment allows us to draw the following conclusions:

1) the type of optimal discharge pressure is the same for any T;
2) time management T it has characteristic areas of growth and decline, and the maximum injection pressure of subsequent growth is higher than the previous one.

Based on the physical concepts of periodic water injection, it can be assumed that these pressure fluctuations will intensify the flow processes.

Convergence of functional values Table 1.

Iteration number	of the functional value at T=0.5	Iteration number	of the functional value at T=0.8	Iteration number	of the functional value at T=0,9
2	6.0432	2	3.0115	2	3.6614
5	3.7866	5	1.3688	5	1.6476
9	3.5286	10	0.5500	10	0.6237
		15	0.5206	17	0.5451
				18	0.5152
at T=1		at T=1,2			
2	4.2582	2	4.0452		
5	2.1873	5	2.5451		
10	1.3450	10	1.5658		
15	0.9328	15	1.0731		
20	0.6845	20	0.7593		
25	0.4988	24	0.5664		
29	0.3152	25	0.5157		
30	0.2862	27	0.3800		
		28	0.3527		

In practice, it is important to specify the cycle duration value for each injection well. To solve this problem using the above method, it would be necessary to additionally study the reservoir structure for individual injection wells, which is very difficult. Therefore, it is advisable to develop recommendations for adjusting the cycle duration based on current fishing information.

It is known that the flow rate and injection pressures are systematically measured for individual injection wells, and the flow rate is the most informative factor, since any changes in the reservoir will necessarily affect it.

It is assumed, similarly, to build a mathematical model of the injection well operation using linear identification methods. The input is taken as the discharge pressure $P(t)$, and per output expense $q(t)$. The definition of the mathematical model of such an object is based on the relationship between the mutual correlation function of the input and output signals R_{qp} with the input correlation function R_{pp} [11]. As a result of statistical processing of the average daily flow rate and injection pressure values for wells, the following values were obtained: R_{pp} , R_{qp} . For the specified type of correlation function, the object's differential equation is

$$d\frac{d^2q}{dt^2}+b\frac{dq}{dt}+q=cp \ . \qquad (15)$$

The coefficients of equation (15) are determined from the existing standard albums of linear identification[11]. Numerical values of the coefficients are shown in Table 2.

Table 2.
Linear identification results

Well number	74.65*10⁸*d/c/*	8.64*10⁴*b/c/*	10*см³/мП*a*
1	28.57	5.50	0.63
2	22.12	5.39	0.51
3	34.97	3.12	0.32
4	31.65	1.19	0.75
5	34.36	4.10	0.58

In case of periodic impact on the deposit, the flooding process can be regulated by selecting the cycle duration and injection pressure. Using the example of well No. 1, we will consider the following problem. Sets the time when the process will continue T^1, it is required to select the discharge pressure, the duration of exposure in this way T_b and the duration of the stop T_0 so that during this time T^1 pump the maximum amount of water:

$$\int_0^{T^1} q(t)dt \to \max .$$

It is assumed that at the start of the review process:

$$q(0)=q_0, \quad q(0)=0,$$

and at the end point in time:

$$q(T^1)=0, \quad q(T^1)=0.$$

The discharge pressure must satisfy the natural constraint:

$$0 \le P(t) \le P_{\max}.$$

Existing methods of the mathematical theory of optimal processes allow us to solve this problem[12]. We write down the problem statement in terms of the theory of optimal processes. Let's transform (15):

$$d\frac{d^2q}{dt^2}+T\frac{dq}{dt}+T_1q=ap,$$

where

$$T=\frac{b}{d}, T_1=\frac{1}{d}, a=\frac{c}{d}.$$

For control, we will take the discharge pressure:

$$U(t)=P(t).$$

Let's denote: $x_1(t)=q(t);\ \ x_2(t)=\frac{dx_1}{dt};\ \ x_3(t)=-\int_0^{T^1} x_1(t)dt;$

$$x_1^0=q_0;\ \ x_2^0=0;\ \ V^1=P_{\max}.$$

The following problem is considered. In phase space X two points are given x_1^0, x_2^0. Among all valid piecewise continuous controls $U(t), \quad 0 \le t \le T$ find one that satisfies the constraint:

$$0 \le V(t) \le V^1,$$

and brings up the system:

$$\begin{cases} \frac{dx_1}{dt} = x_2 = f_1, \\ \frac{dx_2}{dt} = -T_1x_1 - Tx_2 + \alpha U = f_2, \end{cases} \quad (16)$$

from the position:

$$x_1(0) = x_1^0, \quad x_2(0) = x_2^0; \quad (17)$$

to the position:

$$x_1(T^1) = 0; \quad x_2(T^1) = 0 \quad (18)$$

so, what

$$-\int_0^T x_1(t)dt \to \min .$$

Let's add anew equation to system(16):

$$\frac{dx_3}{dt} = -x_1.$$

Using the equations of system (16), we make equations for coprime variables:

$$\begin{cases} \frac{d\psi_1}{dt} = -\left[\frac{\partial f_1}{\partial x_1}\psi_1 + \frac{\partial f_2}{\partial x_1}\psi_2 + \frac{\partial f_3}{\partial x_1}\psi_3\right] = T_1\psi_2 + \psi_3, \\ \frac{d\psi_2}{dt} = -\left[\frac{\partial f_1}{\partial x_2}\psi_1 + \frac{\partial f_2}{\partial x_2}\psi_2 + \frac{\partial f_3}{\partial x_2}\psi_3\right] = \psi_1 + T\psi_2, \\ \frac{d\psi_3}{dt} = -\left[\frac{\partial f_1}{\partial x_3}\psi_1 + \frac{\partial f_2}{\partial x_3}\psi_2 + \frac{\partial f_3}{\partial x_3}\psi_3\right] = 0. \end{cases} \quad (19)$$

Finding the values ψ_2 and ψ_3, $\psi_3 = 0$,

$$\frac{d^2\psi_2}{dt^2} = -T\frac{d\psi_2}{dt} + T_1\psi_2 = 0. \quad (20)$$

The characteristic equation for (20) for the corresponding values of the coefficients has two roots:

$$K_{1,2} = \beta \pm \gamma_i = 0,0965 \pm 0,1607i,$$

therefore:

$$\psi_2(t) = C_1\exp(\beta t)\cos\gamma t + C_2\exp(\beta t)\sin\gamma t.$$

Let's make up the Hamiltonian:

$$H = \psi_1x_2 + \psi_2(-T_1x_2 - Tx_2 + \alpha U) - \psi_3x_1,$$

in order for the Hamiltonian to have a maximum value, as it is called by the Pontryagin maximum principle, it is necessary:

$$U = \begin{cases} u^1, & if \quad \psi_2(t) > 0, \\ 0, & if \quad \psi_2(t) < 0. \end{cases}$$

Let's analyze at what values t function $\psi_2(t)$ changes the sign. Converting it $\psi_2(t)$:

$$\psi_2(t) = \sqrt{C_1^2 + C_2^2}\exp(\beta t)\sin(\gamma t + \alpha), \quad (21)$$

$$\sin\alpha = \frac{C_1}{\sqrt{C_1^2 + C_2^2}}; \cos\alpha = \frac{C_2}{\sqrt{C_1^2 + C_2^2}}.$$

Let's put $C_1 = 0, C_2 = 1$ to $\alpha = 0$. Since (21) includes a sine, the control switching points will satisfy the equation:

$$\sin \gamma t_k = 0, \text{ i.e. } t_k = \frac{\pi}{\gamma} k, \; k = 1,2,...,.$$

При $\gamma = 0{,}1607, t_k \approx 20.8{,}64 \cdot 10^4 \cdot k(c)$. It follows that the optimal value of the exposure duration is $T_b = 20 \cdot 8{,}64 \cdot 10^4 (c)$, duration of the stop $T_0 = 20 \cdot 8{,}64 \cdot 10^4 (c)$ (Fig. 1). Let's calculate the total volume of water for well No.1 at such a CEC, if

$$T^1 = 8{,}64 \cdot 10^4 \cdot 60(c); x_1^0 = 100(m^3); U^1 = 16(M\Pi_a). \qquad (22)$$

At the moment $t = 0$ the conditions (19) must bemet, and the control of $U(t) = U^1$, therefore, the change in the flow rate at the first exposure satisfies the relation:

$$x_1(t) = C_1' \exp(-\beta t) \cos \gamma t + C_2' \exp(-\beta t) \sin \gamma t + aU',$$

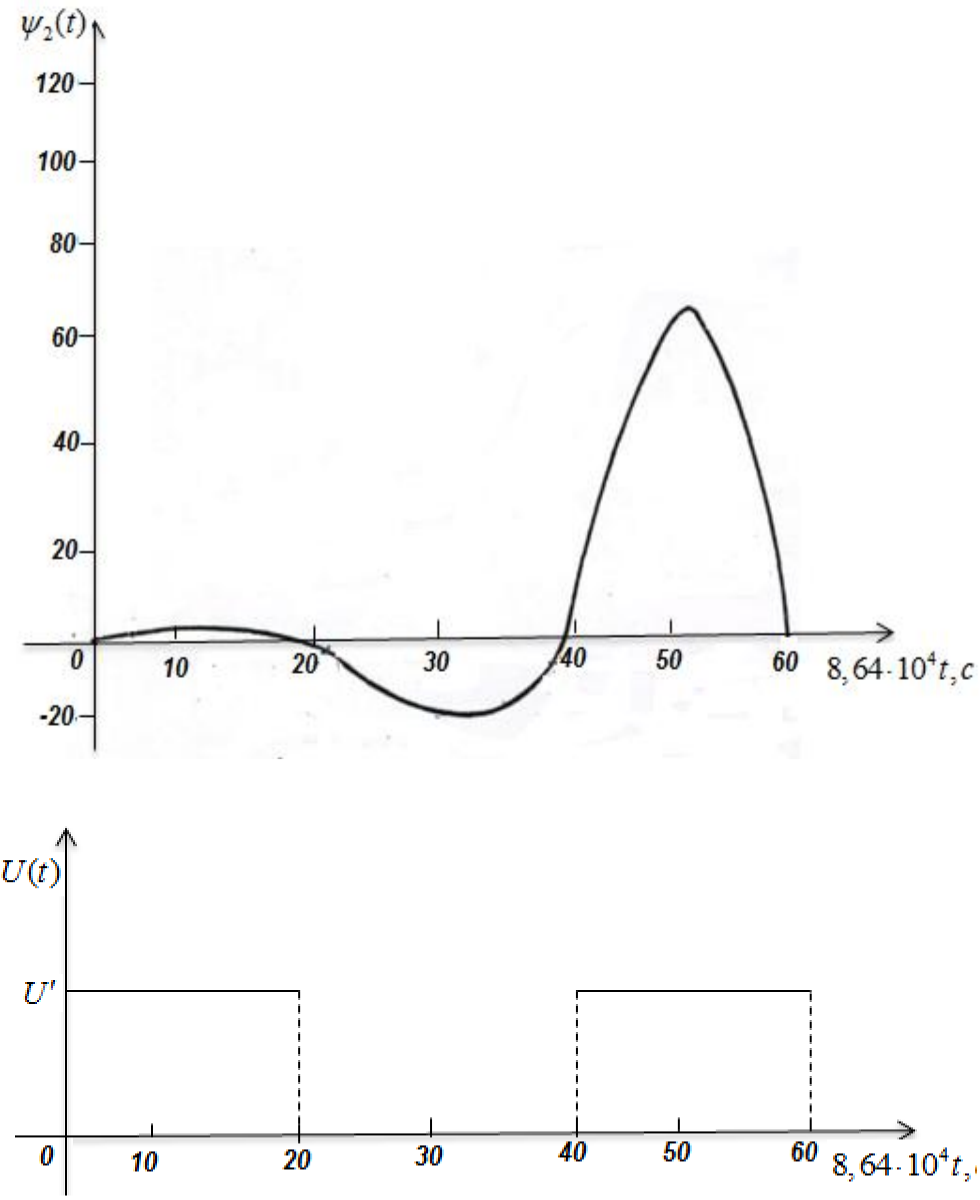

Figure 1. Dependence of the optimal control on the conjugate function.

where $$C_1' = x_1^0 - aU', \quad C_2' = \left(\frac{(x_1^0 - aU')\beta}{4} \right) / \gamma \; . \qquad (23)$$ Converting it $x_1(t)$

$$x_1(t) = \sqrt{(C_1')^2 + (C_2')^2} \exp(-\beta t)\sin(\gamma t + \alpha) + aU', \qquad (24)$$

$$\sin\alpha = \frac{C_1'}{(C_1')^2 + (C_2')^2}; \quad \cos\alpha = \frac{C_2'}{(C_1')^2 + (C_2')^2}. \qquad (25)$$

Integrating $\exp(-\beta t)\sin(\gamma t + \alpha)$, twice in parts, we get:

$$\int_0^{\frac{\pi}{\gamma}} \exp(-\beta t)\sin(\gamma t + \alpha)dt = \frac{\frac{1}{\beta}(\exp(-\beta\frac{\pi}{\gamma}) + 1)(\sin\alpha + \frac{\gamma}{\beta}\cos\alpha)}{1 + \left(\frac{\gamma}{\beta}\right)^2};$$

Taking into account (22) - (25), we determine the volume of water injected during the first cycle:

$$Q_1 = \frac{\sqrt{(C_1')^2 + (C_2')^2} \cdot \frac{1}{\beta}\left(\exp\left(-\beta\frac{\pi}{\gamma} + 1\right)\right)\frac{C_1'}{(C_1')^2 + (C_2')^2} + \frac{\gamma}{\beta}\frac{C_2'}{(C_1')^2 + (C_2')^2}}{1 + \left(\frac{\gamma}{\beta}\right)^2} +$$

$$+aU'\frac{\pi}{\gamma} = 703(m^3).$$

Start time of the second cycle $t_2 = \frac{2\pi}{\gamma}$; the end of the second cycle coincides with the time of reviewing the process $T^1 = t_3 = \frac{3\pi}{\gamma}$. Since T^1 relations (20) must be fulfilled at the final time T 120, the law of flow rate change in the second cycle will be:

$$x_1(t) = \sqrt{(C_1'')^2 + (C_2'')^2} \cdot \exp(-\beta t)\sin(\gamma t + \alpha) + aU', \qquad (26)$$

$$C_1'' = \frac{aU'}{\exp\left(-\beta\frac{3\pi}{\gamma}\right)}, \quad C_2'' = \frac{C_1''\beta}{\gamma}$$

To get the download volume in the second cycle, integrate (26) from $t_2 = \frac{2\pi}{\gamma}$, before $t_3 = \frac{3\pi}{\gamma}$.

$$Q_2 = \frac{\sqrt{(C_1'')^2 + (C_2'')^2} \cdot \frac{1}{\beta}\left(\exp\left(-\beta\frac{3\pi}{\gamma}\right) + \exp\left(-\beta\frac{2\pi}{\gamma}\right)\right)\left(\frac{C_1''}{(C_1'')^2 + (C_2'')^2} + \frac{\gamma}{\beta}\frac{C_2''}{(C_1')^2 + (C_2')^2}\right)}{1 + \left(\frac{\gamma}{\beta}\right)^2} +$$

$$+aU'\frac{\pi}{\gamma} = 244(m^3).$$

The total consumption for two cycles will be: $Q = 947(m^3)$.

When solving the same problem for injection wells No. 1,2,3,4, the following values for the duration of exposure and shutdown were obtained:

for well # 1, $T_b = T_0 = 31 * 8{,}64 * 10^4$ (c);

for well No. 2, $T_b = T_0 = 20 * 8{,}64 * 10^4$ (c);

for well No. 3, $T_b = T_0 = 20 * 8{,}64 * 10^4$ (c);

for well No. 4, $T_b = T_0 = 20 * 8{,}64 * 10^4$ (c).

The pressure must be maintained at the highest possible level during the entire discharge time.

Conclusion. The proposed recommendations on choosing the optimal values for the duration of exposure and stopping during periodic operation of injection wells were included in the "Technology for regulating periodic exposure to the productive reservoir of an oil field", which is being implemented in the development of oil fields.

It is known that in some cases direct methods of monitoring injection wells are difficult. In addition, during periodic operation of injection wells, it is advisable to have information about the future injectivity values of those injection wells that have been stopped by the time the process is considered. In this regard, the task of predicting the injectivity of injection wells becomes urgent.

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