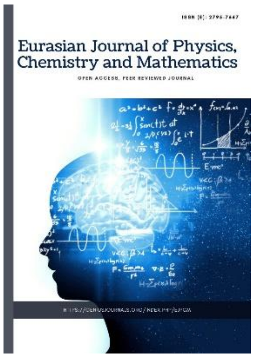

Applying some lemmas and theorems in calculating limits

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ABSTRACT

The article considers the issues of facilitating the process of calculating the limits of some non-standard sequences and functions using linear permutations, lemmas and theorems. Also, applications of calculating the limits of some sequences and classes of functions using simple permutations to theoretical and practical problems are presented.

Keywords: Sequence, function, limit theory, linear permutations, L'Hôpital's rule, non-standard sequence, contextual problems, problem problems.

Introduction

Research has been conducted in our country and abroad on the methods of teaching fundamental mathematical disciplines in higher education institutions using software-didactic complexes, person-centered education, contextual issues, problem issues, large, medium and small modular technologies, and using computer mathematical programs by D. Yunusova [1], M. Tojiyev [2], J.B. Ergashev [3], D.N. Ashurova [4], A.J. Khurramov [5], G.N. Goyibnazarova [6], D.Q. Durdiyev [7], I.V. Kuznetsova [8], J.I. Zaytseva [9], I.S. Novikova [10], Elizabeth Eckerman-Hicks [11] and other scientists. In the works of A. Hakimov, D. N. Ashurova and others, some innovative methods for calculating the limits of sequences and classes of functions have been developed [13-16].

Discussion

It is known that a number of studies have been conducted on the limits of sequences and functions, but the calculation of limits is not fully justified by lemmas and theorems. The main purpose of this article is to apply lemmas and theorems that facilitate the calculation of limits to the first and second great limits. The article focuses on the problems of improving the effectiveness of teaching the subject of calculating sequences and function limits and improving teaching methods in the process of training future mathematics teachers.

Problem Statement And Conformal Mapping

Lemma . $x = x_0$around the point

$$\lim_{x\to x_0} f(x) = \lim_{x\to x_0} \varphi(x) = 0$$

if done , then following attitude appropriate

$$\lim_{x\to x_0} \frac{a^{f(x)} - a^{\varphi(x)}}{x - x_0} = qlna \quad (a > 0, a \neq 1).$$

Here

$$\lim_{x\to x_0} \frac{a^{\varphi(x)}(a^{f(x)-\varphi(x)} - 1)}{f(x) - \varphi(x)} = lna\,, \quad q = \lim_{x\to x_0} \frac{f(x) - \varphi(x)}{x - x_0}$$

Proof .

$$\lim_{x\to x_0} \frac{a^{f(x)} - a^{\varphi(x)}}{x - x_0} = \lim_{x\to x_0} \frac{a^{\varphi(x)}(a^{f(x)-\varphi(x)} - 1)}{x - x_0} = \lim_{x\to x_0} \frac{a^{\varphi(x)}(a^{f(x)-\varphi(x)} - 1)}{f(x) - \varphi(x)} \lim_{x\to x_0} \frac{f(x) - \varphi(x)}{x - x_0}$$
$$= qlna.$$

Based on the above lemma following theorems easily proof possible .

Theorem 1. $x = x_0$around the point$f(x)$, $\varphi(x)$, $g(x)$ functions for

$$\lim_{x\to x_0} f(x) = \lim_{x\to x_0} \varphi(x) = \lim_{x\to x_0} g(x) = 0 \ (a > 0, a \neq 1)$$

conditions if done , then following attitude appropriate

$$\lim_{x\to x_0} \frac{a^{f(x)} - a^{\varphi(x)}}{g(x)} = qlna. \qquad (1)$$

Here $q = \lim_{x\to x_0} \frac{f(x)-\varphi(x)}{g(x)}$.

Theorem 2 : $x = x_0$in the vicinity

$$\lim_{x\to x_0} f(x) = \lim_{x\to x_0} \varphi(x) = \lim_{x\to x_0} g(x) = 0 \ \ va \ \ g'(x_0) \neq 0$$

attitude appropriate if , then following equality appropriate ($b > 0, b \neq 1, a > 0, a \neq 1$),

$$\lim_{x\to x_0} \frac{a^{f(x)} - b^{\varphi(x)}}{g(x)} = mlna - nlnb.$$

this on the ground

$$m = \lim_{x\to x_0} \frac{f(x)}{g(x)}; \ \ n = \lim_{x\to x_0} \frac{\varphi(x)}{g(x)}.$$

This theorems based on the above lemma is proven .

Now above to theorems related practical issues seeing let's go out :

1^0. $$\lim_{x\to 0} \frac{a^x - a^{sinx}}{x} = \lim_{x\to 0} \frac{a^{sinx}(a^{x-sinx} - 1)}{x} = \lim_{x\to 0} \frac{a^{x-sinx} - 1}{x - sinx}.$$
$$\lim_{x\to 0} \frac{x - sinx}{x} = lna \cdot \lim_{x\to 0} \left(1 - \frac{sinx}{x}\right) = lna \cdot (1 - 1) = 0$$

2^0. $$\lim_{x\to 0} \frac{a^{sinx} - a^{tgx}}{x^3} = \lim_{x\to 0} \frac{a^{sinx-tgx} - 1}{sinx - tgx} \cdot \lim_{x\to 0} \frac{sinx(cosx - 1)}{x^3 cosx} =$$
$$= lna \ \cdot (-2 \lim_{x\to 0} \frac{sinx}{x} \cdot \lim_{x\to 0} \frac{sin^2 \frac{x}{2}}{\left(\frac{x}{2}\right)^2 4} \cdot \lim_{x\to 0} \frac{1}{cosx}) = -\frac{1}{2} lna.$$

3^0. $$\lim_{x\to 1} \frac{a^{lnx} - b^{\sin(x-1)}}{x - 1} = \lim_{x\to 1} \frac{a^{lnx} - 1}{x - 1} - \lim_{x\to 1} \frac{1 - b^{\sin(x-1)}}{1 - x} = lna \cdot 1 - lnb \cdot 1 = ln\frac{a}{b}$$

Practical issue and examples solution process facilitator following theorem Let's prove it .

Theorem 3 : $x = a$For functions around a point$f(x)$, $\varphi(x)$

$$\varphi'(x) \neq 0 \text{and} \lim_{x\to a} f(x) = a, \ \lim_{x\to a} \varphi(x) = 0\,, \ a > 0, \ a \neq 1$$

if , then following attitude appropriate

$$\lim_{x\to a} \frac{a^{f(x)} - f(x)^a}{\varphi(x)} = a^a \, ln\frac{a}{e} \lim_{x\to a} \frac{f(x) - a}{\varphi(x)} \qquad (2)$$

Proof . Theorem. proof in the process complicated not happened following replacement Let's do it :

$$\frac{a^{f(x)}-f(x)^a}{\varphi(x)}=-\frac{\left(1-a^{f(x)-a}\right)a^a}{\varphi(x)}=\frac{a^a\left(1-\left(1+\frac{f(x)-a}{a}\right)^a\right)}{\varphi(x)}.$$

Here limit to calculate grass

$$\lim_{x\to a}\frac{a^{f(x)}-f(x)^a}{\varphi(x)}=a^a\lim_{x\to a}\frac{1-a^{f(x)-a}}{a-f(x)}\cdot\lim_{x\to a}\frac{f(x)-a}{\varphi(x)}=a^{a-1}\lim_{x\to a}\frac{1-\left(1+\frac{f(x)-a}{a}\right)^a}{(f(x)-a)a}\cdot\lim_{x\to a}\frac{f(x)-a}{\varphi(x)}$$

$$=a^a\lim_{x\to a}\frac{f(x)-a}{\varphi(x)}(lna-lne)=a^a ln\frac{a}{e}\lim_{x\to a}\frac{f(x)-a}{\varphi(x)}. \qquad (3)$$

Theorem in proof **[12]** cited following from relationships used

$$\lim_{x\to a}\frac{(1-(a-f(x)))^\mu}{f(x)-a}=\mu,\lim_{x\to a}\frac{1-a^{f(x)-a}}{a-f(x)}=lna$$

(2) formula practical for example Let's use it .

If$\varphi'(x)\neq 0$ condition if done , then

$$\lim_{x\to e}\frac{f(x)^e-e^{f(x)}}{\varphi(x)}=0$$

of relationships appropriate that come [13-14].

1^0. $\lim_{x\to a}\frac{x^a-a^x}{\sin(a-x)}=\lim_{x\to a}\frac{a^a(1-a^{x-a})}{\sin(a-x)}+\lim_{x\to a}\frac{a^a\left(\left(1+\frac{x-a}{a}\right)^a-1\right)}{\frac{a-x}{a}}=$

$$=a^a\lim_{x\to a}\frac{1-a^{x-a}}{a-x}\cdot\lim_{x\to a}\frac{a-x}{\sin(a-x)}-a^a\lim_{x\to a}\frac{\left(1+\frac{x-a}{a}\right)^a-1}{\frac{x-a}{a}}\cdot\lim_{x\to a}\frac{a-x}{a\sin(a-x)}=$$

$$=a^a lna-\frac{a^a\cdot a}{a}=a^a(lna-1)=a^a ln\frac{a}{e}$$

If $a=e$so,

$$\lim_{x\to e}\frac{x^e-e^x}{\sin(e-x)}=\lim_{x\to e}\frac{x^e-e^x}{\mathrm{tg}(e-x)}=\lim_{x\to e}\frac{x^e-e^x}{\arcsin(e-x)}=\lim_{x\to e}\frac{(x^e-e^x)}{\mathrm{arctg}(e-x)}=0$$

that come comes out .

2^0. $\lim_{x\to 2}\frac{2^{2sin\frac{\pi x}{4}}-\left(2sin\frac{\pi x}{4}\right)^2}{x^2-4}=0$ relationship prove .

This We use (3) for :

$$\lim_{x\to a}\frac{f(x)-f(a)}{\varphi(x)}=\lim_{x\to 2}\frac{2-2sin\frac{\pi x}{4}}{4-x^2}=2\lim_{x\to 2}\frac{1-sin\frac{\pi x}{4}}{4-x^2}=-4\lim_{x\to 2}\frac{sin^2\left(\frac{\pi}{8}(x-2)\right)}{x^2-4}=$$

$$=-4\lim_{x\to 2}\frac{sin^2\left(\frac{\pi}{8}(x-2)\right)}{\left(\frac{\pi}{8}(x-2)\right)^2}\cdot\frac{\left(\frac{\pi}{8}(x-2)\right)^2}{x-2}=-4\left(\frac{\pi}{4}\right)^2\lim_{x\to 2}(x-2)=0.$$

So ,

$$\lim_{x\to 2}\frac{2^{2sin\frac{\pi x}{4}}-\left(2sin\frac{\pi x}{4}\right)^2}{x^2-4}=0$$

will be That's it .

3^0. $\lim_{x\to x_0}f(x)=\lim_{x\to x_0}\varphi(x)=a,\ \lim_{x\to x_0}g(x)=0$ if , then following limit count .

$$\lim_{x\to x_0}\frac{f(x)^{\varphi(x)}-\varphi(x)^{f(x)}}{g(x)}=\lim_{x\to x_0}\frac{a^{\varphi(x)\log_a f(x)}-a^{f(x)\log_a \varphi(x)}}{g(x)}=$$

$$=a^a\left(\lim_{x\to x_0}\frac{a^{\varphi(x)\log_a f(x)-a}-1}{g(x)}-\lim_{x\to x_0}\frac{a^{f(x)\log_a \varphi(x)-a}-1}{g(x)}\right)=$$

$$=a^a\left(\lim_{x\to x_0}\frac{a^{\varphi(x)\log_a f(x)-a}-1}{\varphi(x)\log_a f(x)-a}\cdot\frac{\varphi(x)\log_a f(x)-a}{g(x)}-\lim_{x\to x_0}\frac{a^{f(x)\log_a \varphi(x)-a}-1}{f(x)\log_a \varphi(x)-a}\cdot\frac{f(x)\log_a \varphi(x)-a}{g(x)}\right)=$$

$$=a^a\ln a\left(\lim_{x\to x_0}\frac{\varphi(x)\log_a f(x)-a}{g(x)}-\lim_{x\to x_0}\frac{f(x)\log_a \varphi(x)-a}{g(x)}\right)$$

4^0. Calculate $\lim_{x\to a}\left(\frac{(2x-a)^{\frac{x^2}{a}}-\left(\frac{x^2}{a}\right)^{2x-a}}{x^2-a^2}\right)$.

limit two kind in a way calculation possible .

Method 1: Hospital rule using we work

$$\lim_{x\to a}\frac{\left(\frac{x^2}{a}\right)^{2x-a}-(2x-a)^{\frac{x^2}{a}}}{a^2-x^2}=\lim_{x\to a}\frac{\left(\left(\frac{x^2}{a}\right)^{2x-a}\right)'-\left((2x-a)^{\frac{x^2}{a}}\right)'}{(a^2-x^2)'}$$

$$=\lim_{x\to a}\frac{\frac{2x}{a}(2x-a)^{\frac{x^2}{a}}\left(\ln(2x-a)+\frac{x}{2x-a}\right)-2\left(\frac{x^2}{a}\right)^{2x-a}\left(ln\left(\frac{x^2}{a}\right)+\frac{2x-a}{x}\right)}{2x}$$

$$=\frac{2a^a(lna+1)-2a^a(lna+1)}{2a}=0$$

Method 2: Limit We use Theorem 3 in the calculation .

$$\lim_{x\to x_0}\frac{f(x)^{\varphi(x)}-\varphi(x)^{f(x)}}{g(x)}=a^a\ln a(\lim_{x\to x_0}\frac{\varphi(x)\log_a f(x)-a}{g(x)}-\lim_{x\to x_0}\frac{f(x)\log_a \varphi(x)-a}{g(x)})$$

$$\lim_{x\to a}\frac{(2x-a)^{\frac{x^2}{a}}-\left(\frac{x^2}{a}\right)^{2x-a}}{x^2-a^2}=a^a lna\cdot\lim_{x\to a}\frac{\frac{x^2}{a}\log_a(2x-a)-(2x-a)\log_a\frac{x^2}{a}}{x^2-a^2}$$

$$=a^a lna$$

$$\cdot\left[\lim_{x\to a}\frac{\frac{x^2}{a}\log_a(2x-a)-a\log_a(2x-a)}{x^2-a^2}-\lim_{x\to a}\frac{(2x-a)\log_a\frac{x^2}{a}-a\log_a\frac{x^2}{a}}{x^2-a^2}\right.$$

$$\left.+\lim_{x\to a}\frac{a\log_a(2x-a)-a\log_a\frac{x^2}{a}}{x^2-a^2}\right].$$

1) $\lim_{x\to a}\left(\frac{\frac{x^2}{a}\log_a(2x-a)-a\log_a(2x-a)}{x^2-a^2}\right)=\lim_{x\to a}\left(\frac{\left(\frac{x^2}{a}-a\right)\log_a(2x-a)}{x^2-a^2}\right)=\lim_{x\to a}\left(\frac{\log_a(2x-a)}{a}\right)=\frac{1}{a}$

2) $\lim_{x\to a}\left(\frac{(2x-a)\log_a\frac{x^2}{a}-a\log_a\frac{x^2}{a}}{x^2-a^2}\right)=\lim_{x\to a}\left(\frac{2(x-a)\log_a\frac{x^2}{a}}{x^2-a^2}\right)=\lim_{x\to a}\left(\frac{2\log_a\frac{x^2}{a}}{x+a}\right)=\frac{1}{a}$

3) $\lim\limits_{x\to a}\left(\frac{a\left(\log_a(2x-a)-\log_a\frac{x^2}{a}\right)}{x^2-a^2}\right)=\lim\limits_{x\to a}\left(\frac{a\log_a\frac{(2x-a)a}{x^2}}{x^2-a^2}\right)=a\lim\limits_{x\to a}\left(\frac{\log_a(\frac{2a}{x}-\frac{a^2}{x^2}-1+1)}{x^2-a^2}\right)=a\lim\limits_{x\to a}\left(\frac{\log_a(1-(\frac{a}{x}-1)^2)}{x^2-a^2}\right)=$
$a\lim\limits_{x\to a}\left(\log_a\left(1-(\frac{x-a}{x})^2\right)^{\frac{1}{x^2-a^2}}\right)=a\log_a\lim\limits_{x\to a}\left(\left(1-\frac{x-a}{x}\right)^{\frac{1}{x^2-a^2}}\cdot\left(1+\frac{x-a}{x}\right)^{\frac{1}{x^2-a^2}}\right)=a\log_a\left(e^{-\frac{1}{2a^2}}\cdot e^{\frac{1}{2a^2}}\right)=$
$a\cdot 0=0$

So , using Theorem 3, intended result This was taken . theorem above similar theories and practical issues calculation and proof process somewhat makes it easier and time saves .

Theorem 4. $\lim\limits_{x\to x_0}u(x)=1,\quad \lim\limits_{x\to x_0}v(x)=\infty$ When , $\lim\limits_{x\to x_0}u^{\,v}=e^{\left(\lim\limits_{x\to x_0}(u-1)v\right)}$ the equality is done .

Proof . $\lim\limits_{x\to x_0}u^{\,v}=\lim\limits_{x\to x_0}\left((1+(u-1))^{\frac{1}{u-1}}\right)^{(u-1)v}=e^{\left(\lim\limits_{x\to x_0}(u-1)v\right)}$

Regarding Theorem 4 following practical examples Let's look at [15]:

1. $\lim\limits_{x\to\infty}\left(\frac{a+x}{b+x}\right)^{kx}=e^{k(a-b)}$.
   $\lim\limits_{x\to\infty}\left(\frac{1+x}{2+x}\right)^{x}=exp\left\{\lim\limits_{x\to\infty}x(\frac{1+x}{2+x}-1)\right\}=exp\left\{\lim\limits_{x\to\infty}\frac{(-x)}{2+x}\right\}=e^{-1}$.
2. $\lim\limits_{x\to 0}\left(\frac{1+x}{1+2x}\right)^{\frac{1}{x}}$. $\lim\limits_{x\to x_0}u(x)=1$ and We use it when $\lim\limits_{x\to x_0}v(x)=\infty$it $\lim\limits_{x\to x_0}u^{v}=\lim\limits_{x\to x_0}\left((1+(u-1))^{\frac{1}{u-1}}\right)^{(u-1)v}=exp\left\{\lim\limits_{x\to x_0}(u-1)v\right\}$is.
   $\lim\limits_{x\to 0}\left(\frac{1+x}{1+2x}\right)^{\frac{1}{x}}=exp\left\{\lim\limits_{x\to 0}\frac{\frac{1+x}{1+2x}-1}{x}\right\}=exp\left\{\lim\limits_{x\to 0}\frac{(-1)}{1+2x}\right\}=e^{-1}$.

Conclusion

The formation of the dynamics of students' learning of mathematics, the analysis of the level of their acquired theoretical knowledge, practical skills and competencies, and the study of the possibilities of mathematics in shaping the level of mastery, create a methodological basis for the development of students' independent learning skills, thereby increasing the effectiveness of the educational process.

This knowledge creates a methodological basis for developing the skills of independent study of the "Theory of Limits" section, analyzing the level of theoretical and practical skills and qualifications learned, increasing the level of mastery, as well as exploring the possibilities of the "Theory of Limits" to increase the effectiveness of practical training.

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