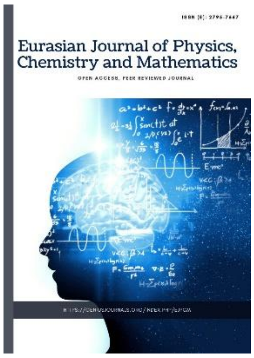

Basic Concepts Of Integral-Differential Equations

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ABSTRACT

This article analyzes the essence of integro-differential equations, their role in mathematical modeling, and their significance for describing physical, technical, and biological processes. Integro-differential equations represent a class of complex equations that combine differential and integral operations. The article discusses their main types, solution methods, and the advantages of analytical and numerical approaches. The influence of the integral kernel and boundary conditions on the equation's solution is also theoretically substantiated.

Keywords: integrodifferential equation, integral kernel, analytical solution, numerical solution, boundary condition, integral operator, mathematical modeling.

Introduction. In the 21st century, mathematical modeling has become a fundamental tool for the in-depth analysis of natural, technical, and economic processes. Many processes—heat transfer, elasticity, wave propagation, the dynamics of biophysical systems, and economic growth models—cannot be fully described by ordinary differential equations. In such cases, integrodifferential equations, which combine differential and integral operators, represent the historical process of action or accumulation through an integral operator. In such equations, the current state depends not only on the current argument but also on changes that occurred in the past. Therefore, equations of this type are of great importance in modeling systems with memory.

The scope of their application is very wide:

physics (thermal conductivity, propagation of electromagnetic waves),

mechanics (weight distribution, deformations),

biology (delayed effects in population growth models),

economics (circulation of capital, inertial processes of supply and demand).

Thus, integrodifferential equations are an important theoretical tool in applied areas of modern mathematics.

An equation in which the unknown function is represented by a differential (derivative) and integral signs is called an integro-differential equation. The simplest example of a linear integro-differential equation of this order is the following:n

$$p_0(x)y^{(n)}(x)+p_1(x)y^{(n-1)}(x)+\dots+p_{n-1}(x)y'(x)+$$
$$+p_n(x)y(x)+P[y]=f(x), x\in(a,b)$$

has the form , where is an unknown function, and are continuous functions defined on an interval, and is some linear integral or differential operator of fractional order (less than).$a,b\in R, a<b, y=y(x) f(x) p_j(x) (j=\overline{0,n}) [a,b] p_0(x)\not\equiv 0, x\in[a,b]; P[y] n$ Arab $P[y]$ If the Fredholm integral operator exists,

$$p_0(x)y^{(n)}(x)+p_1(x)y^{(n-1)}(x)+\dots+p_{n-1}(x)y'(x)+p_n(x)y(x)+$$
$$+\int_a^b K(x,t)\,y(t)dt=f(x) x\in(a,b) \qquad 1.1$$

We obtain an integro-differential equation of the form, where is a function considered at four angles.$K(x,t)-\Delta=\{(x,t): a\le x\le b, a\le t\le b\}$
must be present in equation (1.1), otherwise (1.1) remains a differential equation. If it is present in equation (1.1), then the Volterra integral operator is involved.$K(x,t)\not\equiv 0,(x,t)\in\bar{\Delta} K(x,t)\equiv 0, t\in[x,b]$

$$p_0(x)y^{(n)}(x)+p_1(x)y^{(n-1)}(x)+\dots+p_{n-1}(x)y'(x)+p_n(x)y(x)+$$
$$+\int_a^b K(x,t)\,y(t)dt=f(x)\ ,\quad x\in(a,b) \qquad 1.2$$

we obtain an integro-differential equation of the form
An integro-differential equation of the form (1.2) can also be considered in an interval - if it is an integral or differential operator of fractional order,$[a,+\infty) P[y]$

$$p_0(x)y^{(n)}(x)+p_1(x)y^{(n-1)}(x)+\dots+p_{n-1}(x)y'(x)+$$
$$+p_n(x)y(x)+w(x)D_{ax}^{\alpha}\omega(x)y(x)=f(x), x\in(a,b) \qquad (1.3)$$

we have an integro-differential equation of the form, where and are given functions,$w(x)\omega(x) w(x)\not\equiv 0,\omega(x)\neq 0, x\in[a,b]; \alpha\in(0,n)$- all
The original problem for the integro-differential equations (1.1), (1.2) and (1.3) is formulated as follows: (1.1) [(1.2) or (1.3)] is continuous and$[a,b]$

$$y(a)=k_0,\qquad y'(a)=k_1,\dots,y^{(n-1)}(a)=k_{n-1}$$

Find a solution that satisfies the initial conditions herek_j =constant,$j=\overline{0,n-1}$ —numbers are given. If equations (1.2) and (1.3) are considered in the interval , then it is required to find their solution, satisfying the initial conditions specified above, also in the interval .$[a,+\infty)[a,+\infty)$
NoteIn integro-differential equations (1.2) and (1.3)

$$\int_a^x K(x,t)y(t)dt \text{ va } D_{ax}^{\alpha}\omega(x)y(x)$$

Instead of operators, respectively

$$\int_x^b K(x,t)y(t)dt \text{ va } D_{xb}^{\alpha}\omega(x)y(x)$$

Operator derivation is also possible. There are many methods for finding solutions to initial value problems for integro-differential equations, and below we will introduce one of them with examples. Issue 1.1.

$$y''(x)+\frac{1}{7}\sin\ xy'(x)+\frac{1}{7}y(x)+$$

$$\frac{1}{7}\int_0^1 \sin(x-t)\,y(t)dt = \ \cos x\,, \qquad x\in(0,1) \qquad 1.4$$

The equation is continuous in the interval and$[0,1]$

$$y(0)=1, y'(0)=2 \tag{1.5}$$

Find a solution that satisfies the initial conditions.

YesGo away. In equation (1.4), we replace with and integrate the resulting equality over the interval . Taking into account the initial conditions (3.21), we obtain$xzz[0,x]$

$$y'(x)+\frac{1}{7}\sin\ xy(x)+\frac{1}{7}\int_0^x (1-\cos\ z)y(z)dz+$$

$$+\frac{1}{7}\int_0^1 [\cos\ z-\cos\ (x-z)]y(z)dz=2+\sin\ x$$

An equation has been derived. This equation can also be obtained by taking into account the integral function and the condition on the interval, as indicated above:$[0,x]y(0)=1$

$$y(x)+\frac{1}{7}\int_0^x \sin\ ty(t)dt+\frac{1}{7}\int_0^x dt\int_0^t (1-\cos\ z)y(z)dz+$$

$$+\frac{1}{7}\int_0^1 [x\cos\ t-\sin\ (x-t)-\sin\ t]y(t)dt=2(x+1)-\cos\ x$$

We obtain an equation. Here, by changing the order of integration in the iterated integral, we obtain an equation that is equivalent in strength (in the sense of having a solution) to the problem.$\{(3.20),(3.21)\}$

$$y(x)+\frac{1}{7}\int_0^x [\sin\ t+(x-t)(1-\cos\ t)]y(t)dt+$$

$$+\frac{1}{7}\int_0^1 [x\cos\ t-\sin\ (x-t)-\sin\ t]y(t)dt=2(x+1)-\cos\ x$$

Let's make an equation. Here, if

$$K(x,t)=\begin{cases}x-t+t\cos\ t-\sin\ (x-t), & x\geqslant t\\ x\cos\ t-\sin\ (x-t)-\sin\ t, & x\leqslant t\end{cases} \tag{1.6}$$

If we introduce the notations, the last equation can be written as follows:

$$y(x)+\frac{1}{7}\int_0^1 K(x,t)y(t)dt=2(x+1)-\cos\ x, x\in(0,1) \tag{1.7}$$

(1.6) y (x) is a Fredholm integral equation of the second kind with respect to the unknown function . Considering that and the inequalities , the inequality follows from (3.22). Obviously, the parameter of the integral equation (1.6) is less than the number , i.e. . Considering this, as well as the continuity of the kernel at , the finite jump at , and the continuity of the right-hand side of equation (1.6), a solution to the integral equation (1.6) exists and is unique according to the Fredholm alternative. Suppose that the function is a solution to equation (1.6), i.e.$x,t\in[0,1]|\sin\ x|\le 1, |\cos\ x|\le 1\sup|K(x,t)|<3\lambda=(1/7)[\sup|K(x,t)|]^{-1}(1/7)<(1/3)K(x,t)x\neq tx=ty_0(x)$

$$y_0(x)+\frac{1}{7}\int_0^1 K(x,t)y_0(t)dt=2(x+1)-\cos\ x$$

Let equality be appropriate. This is equality.

$$y_0(x) = -\frac{1}{7}\int_0^x K(x,t)y_0(t)dt -$$
$$-\frac{1}{7}\int_x^1 K(x,t)y_0(t)dt + 2(x+1) - \cos\, x$$

can be written as. Taking into account (1.5), the last equality has the form

$$y_0(x) = -\frac{1}{7}\int_0^x [x - t + t\cos\, t - \sin\,(x-t)]y_0(t)dt -$$
$$-\frac{1}{7}\int_x^1 [x\cos\, t - \sin\, t - \sin\,(x-t)]y_0(t)dt + 2(x+1) - \cos\, x$$

takes the form. Using this equality and taking into account that and , it can be shown by direct calculation that . Taking this into account and the equivalence of the integral equation (1.6) to the problem, it follows that the solution to the integral equation (1.6) is also a solution to the problem.$y_0(x) \in C[0,1] y_0(x) \in C^2[0,1]\{(3.20),(3.21)\}\{(1.4),(1.5)\}$

Issue 1.2.

$$y''(x) + xy'(x) + x^3 y(x) + \int_0^x (1+t^2)y(t)dt = \frac{1}{1+x}, x \in (0,1)$$

Find a solution to the equation that is continuous on the interval and satisfies the initial conditions (3.21).[0,1]

YesGo away. Here also, replacing with in this equation, and then integrating twice over the interval,$xzz[0,x]$

$$y(x) + \int_0^x \left[t + (t^3 - 1)(x-t) + \frac{1}{2}(1+t^2)(x-t)^2\right] y(t)dt =$$
$$= (1+x)\ln(1+x) + x \qquad x \in (0,1) \qquad 1.8$$

We arrive at the second type of Volterra integral equation in the form (1.24), which is the core of the resulting integral equation.

$$K(x,t) = t + (t^3 - 1)(x-t) + \frac{1}{2}(1+t^2)(x-t)^2$$

The function is continuous on a square and has second-order derivatives, and the function on the right-hand side of the equation is continuous on the interval and has continuous second-order derivatives. Consequently, Volterra-type integral equations have a solution, equation (1.24), which is also a solution to problem 1.2.$\{(x,t): 0 \le x \le 1, 0 \le t \le 1\} f(x) = (1+x)\ln\,(1+x) + x[0,1] y_0(x) \in C^2[0,1]$

Conclusion. Integro-differential equations are a natural generalization of differential and integral equations used to express the evolution of complex systems over time. These equations take into account the system's previo
us state and historical influences, allowing them to be used in many scientific fields. Analytical and numerical methods are seamlessly combined to find their solutions. Research shows that approaches using integral operators provide high accuracy and stability when modeling complex systems.

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