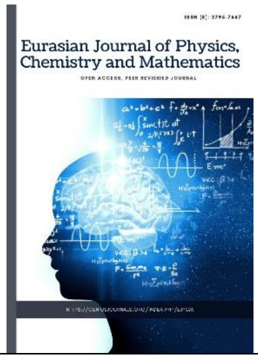

A Matlab-Based Least-Squares Method For Processing Experimental Data In Physics Education

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ABSTRACT
This article examines the feasibility of using a MatLab application to approximate experimental data using the least-squares method. Examples are provided for linear, quadratic, and other relationships between physical quantities.

Keywords: least-squares method, physical experiment, reliability of results

Introduction

The least-squares method is the most widely used method for processing data from physics experiments. This method has come to be used for statistical processing of data from laboratory work in elementary and general physics, as well as pedagogical experiments. The calculation is typically programmed using various programming languages, with the subsequent output of calculation results in the form of tables or graphs [1-3, 6]. The diversity of calculation programs used and their different levels of compatibility with operating systems necessitate the development of a more universal MatLab application for this method. The authors of this paper have developed a MatLab application for the least-squares method for processing data from any physics experiment. Several examples of the application of the new technique to the calculation of specific experimental data are discussed.

If some physical quantity y depends on another quantity x, then this dependence can be investigated by measuring y at different values of *x*. The resulting measurements yield a series of values:

$$x_1, x_2, ..., x_i, , ... , x_n;$$
$$y_1, y_2, ..., y_i, , ... , y_n.$$

Using the data from such an experiment, a graph of the dependence y = ƒ(x) can be constructed. The resulting curve allows one to judge the form of the function ƒ(x). However, the values of the constant coefficients that comprise this function remain unknown. Using the least squares method makes it easy to determine their values. The experimental points, as a rule, do not lie exactly on the curve. The least squares method requires that the sum of the squared deviations of the experimental points from the curve, i.e., $[y_i - f(x_i)]^2$, be minimal.

In practice, this method is most often used in the case of a linear dependence, i.e., when

$$y = kx \quad \text{или} \quad y = a + bx.$$

Linear relationships are very common in physics problems. Even when the relationship is nonlinear, attempts are usually made to plot the graph so as to obtain a straight line. For example, if we assume that the refractive index of glass n is related to the wavelength λ of the light wave by the formula $n = a + b/\lambda^2$, then the graph plots n versus λ^{-2}.

Let's consider the case where the experimental points must satisfy the formula:

$$y = a + bx. \quad (1)$$

The problem is to find the most approximate values of a and b given a set of values x_i and y_i.

Let's formulate the mean square error χ^2, equal to the sum of the squares of the deviations of points x_i and y_i from the straight line (1).

$$\chi^2 = \sum_{i=1}^{n} (y_i - a - bx_i)^2 \quad (2)$$

and we find the values of a and b under conditions corresponding to the minimum of χ^2:

$$\frac{\partial \chi^2}{\partial a} = -2\sum_{i=1}^{n}(y_i - a - bx_i) = 0,$$

$$\frac{\partial \chi^2}{\partial b} = -2\sum_{i=1}^{n}x_i(y_i - a - bx_i) = 0.$$

By solving these equations together we can obtain:

$$b = \frac{\sum (x_i - \bar{x})y_i}{\sum (x_i - \bar{x})^2} \quad (3)$$

$$a = \bar{y} - b\bar{x} \quad (4)$$

where $\bar{y} = \sum_{i=1}^{n} y_i$, $\bar{x} = \sum_{i=1}^{n} x_i$

When processing measurement results using this method, it's convenient to combine all data into a MatLab table, which calculates all the physical quantities included in formulas (1)–(4). Furthermore, it's convenient to express the calculation results in graphical or diagrammatic form, which allows for a clear analysis of the physical mechanism underlying the relationship being studied.

MatLab makes it easy to determine unknown coefficients not only for linear functions, but also for quadratic and other more complex functions. The application of this method to data processing for various problems is demonstrated in the examples discussed below.

Example 1. In order to determine the temperature coefficient of surface tension, the surface tension of water was measured as a function of temperature and the following results were obtained: $\boldsymbol{\sigma}$ = 63 mN/m (at t = 0 °C); $\boldsymbol{\sigma}$ = 61 mN/m (at t = 20 °C); $\boldsymbol{\sigma}$ = 58 mN/m (at t = 40 °C); $\boldsymbol{\sigma}$ = 55 mN/m (at t = 60 °C); $\boldsymbol{\sigma}$ = 53 mN/m (at t = 80 °C). It is necessary to determine the temperature coefficient of surface tension of water.

Solution. Assume that the temperature coefficient of water's surface tension depends on temperature as follows:

$$\sigma = at + b$$

and find the unknown coefficients using the method described.

To determine a and b, denote σ as a function of y, and t as the argument x. Entering the initial experimental data in columns 2 and 3, we perform the calculation and obtain the results in column 4 (Table 1).

Table 1.

Experimental data y(σ) and calculation results y^t(σ^t) for different values x(t)

№	*x (t)*	*y (σ)*	y^t (σ^t)
1	0	63	63,20
2	20	61	60,60
3	40	58	58,00
4	60	55	55,40
5	80	53	52,80

The proposed method makes it easy to construct a diagram of the dependence of the surface tension coefficient on temperature (Fig. 1), where the points correspond to the experimental data and the line to the calculation results.

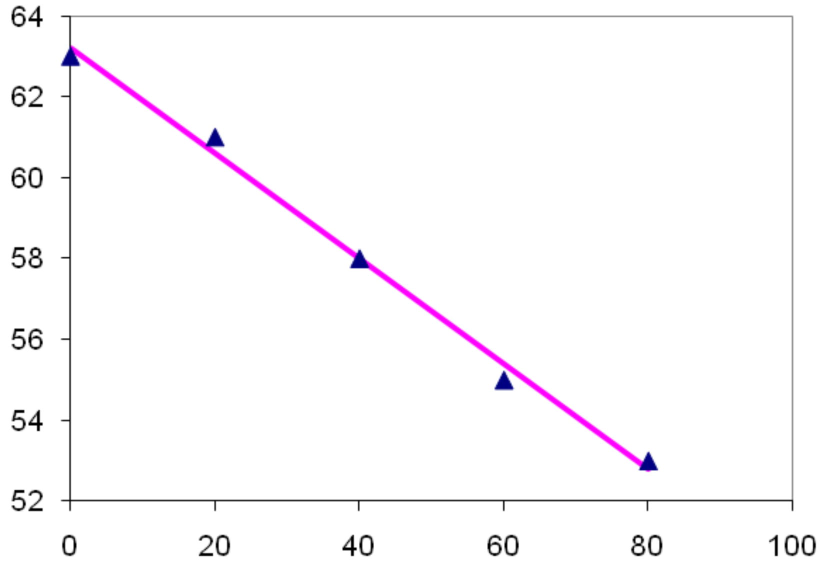

Fig. 1. Graphic diagram of the dependence of the surface tension coefficient on temperature: dots – experimental data; line – calculation results.

Consistently, the values of the desired coefficients can be determined by calculation: n = 5; b = 63.20; a = -0.13; χ^2 = 0.13.
The data in Table 1, the graph (Fig. 1), and the determined value of χ^2 indicate the reliability of the results.
Example 2. To determine the acceleration due to gravity, the fall time of a body with an initial velocity v_0, dropped from various heights, was measured, and the following results were obtained:

Table 2.

t, s	0	0,45	0,72	0,87	1,12	1,30
h, м	0	3	6	9	12	15

It is necessary to determine the acceleration due to gravity.
Solution: As is known, the height of a freely falling body is determined by the formula $h = v_0 t + gt^2/2$.
Let us denote $h \to y$, $t \to x$, then $y = ax^2 + bx + c$. Let us determine the unknown coefficients a, b and c under conditions corresponding to the minimum of χ^2, using the method described above (Table 3).

Table 3.
Experimental data *h(y)* and calculation results $y^t(h^t)$ for different values of t(x)

№	t (x)	h (y)	$y^t(h^t)$
1	0	0	-0,12
2	0,45	3	3,32
3	0,72	6	6,32
4	0,87	9	8,29
5	1,12	12	12,06
6	1,3	15	15,14

Taking advantage of the MatLab application, one can easily construct a graphical representation of the dependence of the fall height on the flight time (Fig. 2).

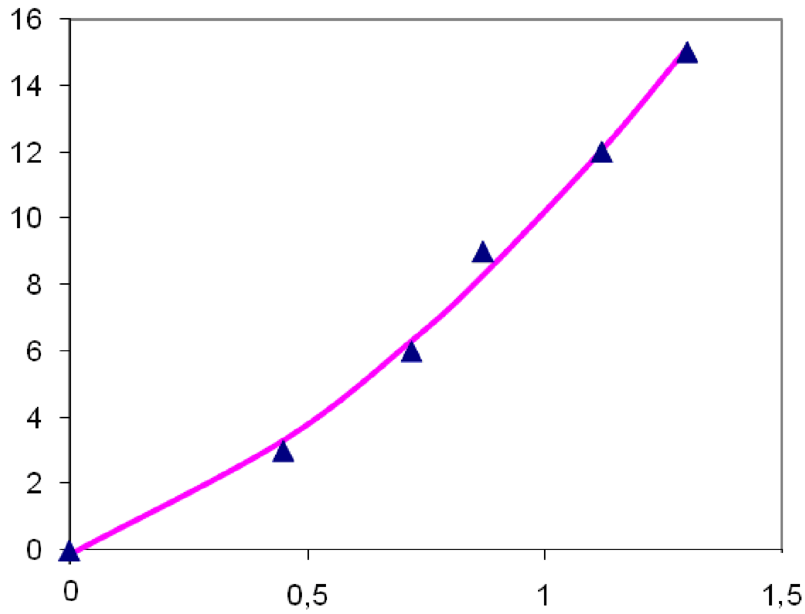

Fig. 2. Graphic diagram of the dependence of the fall height on the flight time: points are experimental data; line is the calculation results.

Consistently, the values of the required coefficients can be determined by calculation: n = 6; b = 5.48; a = 4.82; c = -0.12 and χ^2 = 0.15.
The data in Table 3, the graph (Fig. 2) and the determined values of χ^2 and g=2a=9.84 indicate the reliability of the results.

Conclusion
Thus, this paper proposes the use of the MatLab application for approximating experimental data using the least-squares method. Examples are provided for linear, quadratic, and other relationships between physical quantities. The reliability of the results demonstrates that the least-squares method can be used for any problem in experimental physics.

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