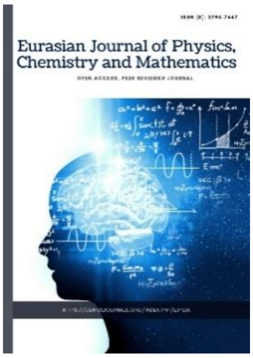

Asymptotic Properties Of Ordinal Statistics

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ABSTRACT

The asymptotic theory of ordinal statistics deals with the distribution of appropriately normalized (and centered) quantities $X_{r:n}$ at $n \to \infty$. At the first stage, it is usually assumed that $X_{r:n}$ is the rth ordinal statistic in a random sample of volume n from a distribution with a distribution function $P(x)$. However, as we will see later, many types of dependence between $X_1, X_{2,}, \ldots, X_n$. Cp do not violate the type of marginal distributions.

Keywords: ordinal statistics, extreme value, asymptotic properties, Asymptotic distribution, mathematical expectation, variance

The asymptotic theory of ordinal statistics deals with the distribution of appropriately normalized (and centered) quantities $X_{r:n}$ at $n \to \infty$. At the first stage, it is usually assumed that $X_{r:n}$ is the rth ordinal statistic in a random sample of volume n from a distribution with a distribution function $P(x)$. However, as we will see later, many types of dependence between $X_1, X_{2,}, \ldots, X_n$. Cp do not violate the type of marginal distributions. This feature makes the theory in question much more useful. If $r / n \to \lambda$ $n \to \infty$ then significantly different results are obtained for the cases $0<\lambda<1$; $\lambda=0$, $\lambda=1$. In the first case, $X_{1:n}$ is a sample quantile and (under certain regularity conditions) has an asymptotically normal distribution. The second case includes the extreme values of $X_{1:n}$, $X_{n:n}$ and, generally speaking, the mth extremes at a fixed t. These values have an abnormal marginal distribution.

We have often referred to asymptotic results in previous chapters. In particular, the asymptotic estimation of the limiting number of quantiles arises in the problems of "optimal choice of ordinal statistics". In the next section, we present a theory of distributions that justifies this application, and, following Mosteller (1946), establish the joint asymptotic normality of quantiles.

In the rest of the chapter, we deal with the theory of extreme values and the asymptotic distribution of linear functions of ordinal statistics, as well as their use in asymptotic estimation. Here, more than anywhere else in this book, we limit ourselves to a brief summary of the very extensive available literature, providing evidence for only some of the main results.

The most remarkable result of the theory of extreme values is now a classic one: if the value $X_{n:n}$, properly normalized, has a limiting distribution, then it must be a distribution of one of the three types given by the relation. There are many applications of extreme value distributions. For example, the simple assumption that a chain is no stronger than its weakest link leads us to interpret the value of $X_{1:n}$, , to the strength of the chain (consisting of n links), and hence to an impressive theory of tensile strength. Liblein (1954 b) traces this idea back to Chaplin's 1860 work. The most useful distribution describing tensile strength is the so-called Weibull distribution, which has a distribution function

$$F(y)=1-\exp\left[-\left(\frac{y-\gamma}{\delta}\right)^{\alpha}\right] \quad (\gamma<y<\infty;\ \delta>0;\ \alpha>0).$$

Here, γ can be interpreted as a guaranteed minimum strength, and δ is a scale factor. Obviously, $X=-(Y-\gamma)/\delta$ has a $\Lambda_2(x)$, the Weibull distribution is simply the second of the three types, but only for the smallest, not for the largest value of. In life expectancy tests, y can indicate the time until death. Further, the distribution of floods or other extreme meteorological events often has the form Λ_3. We refer the reader to Gumbel's book (1965), which contains other appendices and various references.

Gumbel also discusses in detail various methods for estimating parameters such as γ and δ in and it is assumed that the data represent a set of n (not necessarily large) observed maxima or minima. Graphical methods are widely used, especially probability diagrams is a distribution that depends (for a fixed a) on the shift and scale parameters, their estimation using ordinal statistics is also possible. In this regard, we can mention the work of Maritz and Munro (1967), in which all three parameters of the generalized distribution of extreme values are estimated using ordinal statistics:

$$F(y)=\exp\left\{-\left|\frac{1-(y-\gamma)}{\delta\beta}\right|^{\beta}\right\},$$

Here

$$\begin{aligned}&-\infty<y<\gamma+\delta\beta, \quad agar \quad \beta>0,\\ &\gamma+\delta\beta<y<\infty, \quad agar \quad \beta<0\end{aligned}$$

Putting $\gamma+\delta\beta=0$, we get Λ_1 for $\delta\beta=1$, $\beta=-\alpha$ and Λ_1 for $\delta\beta=1$, $\beta=\alpha$. For $x=(y-\gamma)\delta$ and $\beta\to\infty$, we get $\Lambda_3(x)$.

Other works complementing Gumbel's book are his article on the assessment of tensile strength in the book by Sarkhan and Greenberg (1970), as well as Gumbel's (1961) work on tensile strength and fatigue, Gumbel's (1963) work on drought forecasting, Pike's (1966) work on cancer, considered as a breakthrough of the weakest link. Barnett and Lewis (1967) on the probabilities of low temperatures, Epstein (1967) on the moments of bacterial extinction, and Mann (1968) on estimation procedures.

In the final section of this chapter, estimates are derived that are asymptotically optimal for a distribution that depends only on the scale and shift parameters. Closely related to this issue are methods for obtaining estimates, which, although not necessarily optimal for the most interesting distributions, have good properties over the entire selected set of distributions. Such robust estimates were discussed in section although mainly for small samples.

Asymptotic distribution of the extreme value

Asymptotic behavior of $X_{(n)}$ (the largest observation in a sample of size n from a distribution with a distribution function. $P_{(x)}$ was a task that challenged many major experts in mathematical statistics. The most notable contributions to this field were made by Dodd (1923), von Mises (1923, 1936), Frechet (1927), Fischer and Tippett (1928), de Finetti (1932), and Gu.mbel (since 1933) and ending with the final work of 1958) and, finally, Gnedenko (1943), who conducts the most complete and rigorous study of this issue. We can also mention the work of Barandoff Nielsen (1963), which briefly outlines these and related issues, and the work of Dwass (1964) and Lamperty (1964), which discusses an approach related to stochastic processes if there is a limit distribution, then this distribution must belong to one of three types

$$\Lambda_1(x)=\begin{cases} 0, & agar \quad x\le 0,\ \alpha>0 \\ \exp\left(-x^{\alpha}\right), & agar \quad x>0; \end{cases}$$

$$\Lambda_2(x)=\begin{cases} \exp\left[-(-x)^{\alpha}\right], & agar \quad x\le 0,\ \alpha>0 \\ 1, & agar \quad x>0; \end{cases}$$

$$\Lambda_3(x)=\exp(-\exp(-x)) \qquad (-\infty<x<\infty).$$

The above can be formulated in the form of the following theorem (Gnedenko): The class of propositions for $P^n(a_n x+b_n)$ where $a_n>0$ and b_n are appropriately selected constants, contains only laws of types $\Lambda_k(x)$ $(k=1,2,3)$.

We will not prove this theorem, but instead present an ingenious key idea already used earlier by Fischer and Tippett. Since the largest observation in a sample of volume $m\cdot n$ can be considered as the largest member in a sample of volume n, consisting of the maximum members of samples of volumes m , and since in the case of the existence of a limiting distribution $\Lambda(x)$, both of these distributions will tend to $\Lambda(x)$ at $m\to\infty$, then $\Lambda(x)$ must satisfy the ratio

$$\Lambda^n(a_n x+b_n)=\Lambda(x),$$

that is, the largest Observation in the sample of volume n from the distribution of the distribution function $\Lambda(x)$ should, after appropriate normalization, itself have a limiting distribution function L. Solving this functional equation with respect to $\Lambda(x)$ gives us all possible limiting types.

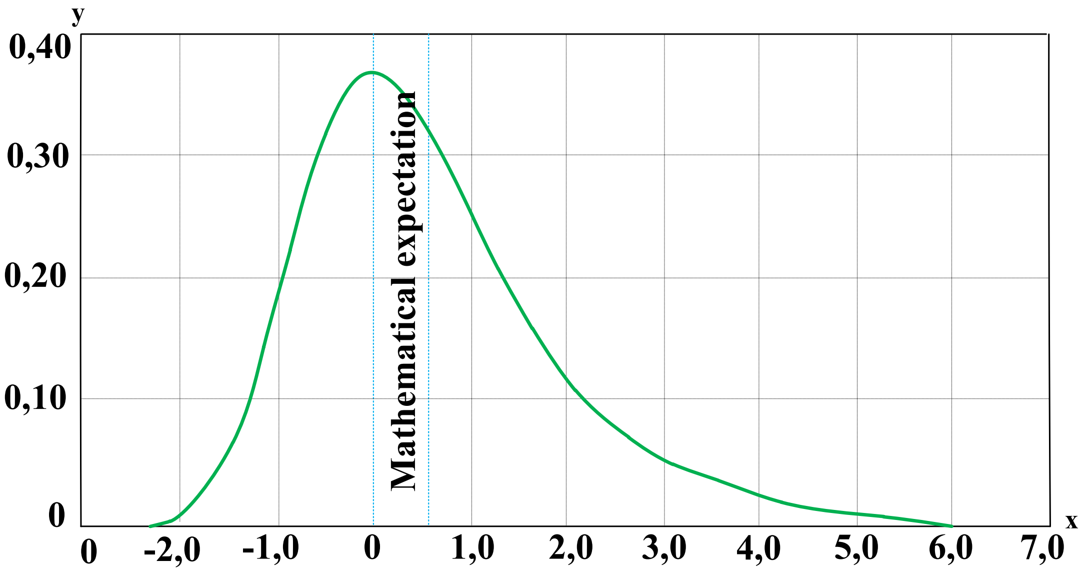
Further, if in $a_n \neq 1$, then denoting $x_0 = b_n/(1-a_n)$, we get $x_0 = a_n x_0 + b_n$ and therefore $\Lambda^n(x_0) = \Lambda(x_0)$, $\Lambda(x_0) = 0$ or 1. Provided that that $\Lambda(x)$ exists, x_0 must be a constant that can be set to zero without detracting from generality. Then, due to the fact that $x_0 = 0$ implies $b_n = 0$, the solutions fall into the following three classes:

(1) $\Lambda(x) = 0, \ agar \quad x \leq 0, \Lambda^n(a_n x) = \Lambda(x), \ agar \quad x > 0;$

(2) $\Lambda^n(a_n x) = \Lambda(x), \ agar \quad x \leq 0, \quad \Lambda(x) = 1, \quad agar \quad x > 0;$

(3) $\Lambda^n(x + b_n) = \Lambda(x),$

These classes obviously correspond to the cases $a_n > 1$, $a_n < 1$, and $a_n = 1$. It follows from standard mathematical reasoning that the only solutions to functional equations (1)-(3) are the expressions $\Lambda_1(x), \ \Lambda_2(x)$ *and* $\Lambda_3(x)$. respectively.



1-picture.

Now let's take a closer look at $\Lambda_3(x)$. Since it stands out from other types, it is often referred to as an extreme value distribution, although, of course, this term applies to all three types. It is easy to see that the maximum observation in a sample of volume n from the distribution $\Lambda_3(x)$. has a value that differs from $\Lambda_3(x)$. only by an offset of b_n Right, where b_n is determined by the equation

$$\exp(-ne^x) = \exp\left(-e^{-(x-b_n)}\right),$$

that is, $b_n = \log n$. example $A'_3(x) = \exp\{-x - e^x\}$ is shown in picture-1. Using our generating function of cumulants, it is easy to show that $\mu = \gamma$ (Euler's constant) $= 0{,}5772\ldots, \ \mu_2 = \sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6} = 1{,}6449\ldots, \ \beta_1 = 1{,}2986\ldots, \ \beta_2 = 5{,}4$

Gnedenko (1943) obtained the necessary and sufficient conditions for the distribution of $P(x)$ to belong to the "region of attraction" of each of the following two limiting laws:

(1) $P(x)$ belongs to the domain of attraction Ax(x) if and only if

$$\lim_{x\to\infty}\frac{1-\mathrm{P}(x)}{1-\mathrm{P}(kx)}=k^{\alpha}$$

for each $k>0$.

(2) $P(x)$ belongs to the domain of attraction, $\Lambda_2(x)$.) if and only if

(a) there exists X_0 such that

$$\mathrm{P}\left(x_0\right)=1, \quad \mathrm{P}\left(x_0-\varepsilon\right)<1 \quad \text{for optional } \varepsilon>0$$

(b) $$\lim_{x\to -0}\frac{1-\mathrm{P}\left(kx+x_0\right)}{1-\mathrm{P}\left(x+x_0\right)}=k^{\alpha}$$

for each $k>0$

It is easy to see that $P(x)$ is not bounded on the right in the first case and bounded in the second by). Gnedenko points out that $\Lambda_3(x)$. can be the limiting distribution in both cases.

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