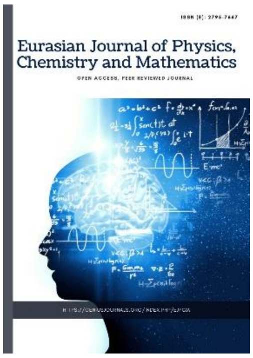

Limit calculation technology using linear substitutions

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ABSTRACT

The article discusses the issues of simplifying the calculation of some non-standard sequences and limits of functions using linear substitutions, lemmas, and theorems.

Keywords: Software and didactic complex, limit calculation methods, calculating limits using lemmas, calculation of limits, Lopital's rule.

Introduction

In higher educational institutions, the fundamental sciences of mathematics are built on the basis of program-didactic complexes, student-centered learning, contextual issues, problematic issues, large, medium and small modular technologies and teaching methods using computer mathematical programs in our country and abroad. Yunusova [1], Todzhiev M. [2], Ergashev Zh.B. [3], Ashurova D.N. [4], Goyibnazarova G.N. [6], Durdyev D.V. [7], Kuznetsova I.V. Research was carried out by J.I. Zaitseva [9], I.S. Novikova [10], Elizabeth Ackerman-Hicks [11] and other scientists and other scientists conducted research. In the works of A. Khakimov, S.X.Abjalilov, D. N. Ashurova and others, some innovative methods for calculating the limit of classes of sequences and functions were developed [13-17].

Discussion

This article presents theoretical and practical applications of calculating the limits of certain classes of sequences and functions using simple substitutions.

It is known that a number of studies have been carried out on the limits of sequences and functions, but the calculation of limits using lemmas and theorems is not entirely justified. The main purpose of this article is devoted to the use of lemmas and theorems that facilitate the calculation of the limits of the first and second miraculous limits. The article is devoted to the problems of increasing the effectiveness of teaching the subject “Calculation of sequences and limits of functions” in the process of training future mathematics teachers and improving teaching methods.

PROBLEM STATEMENT

Lemma. To the district point. $x = x_0$ if completed

$$\lim_{x\to x_0} f(x) = \lim_{x\to x_0} \varphi(x) = 0$$

then the following relation is suitable

$$\lim_{x\to x_0} \frac{a^{f(x)} - a^{\varphi(x)}}{x - x_0} = pq\,(a > 0, a \neq 1).$$

Here

$$p = \lim_{x\to x_0}\left(\frac{a^{f(x)-\varphi(x)} - 1}{f(x)-\varphi(x)}\right) = \ln a,\; q = \lim_{x\to x_0} \frac{f(x)-\varphi(x)}{g(x)}.$$

Proof. $\lim_{x\to x_0} \frac{a^{f(x)} - a^{\varphi(x)}}{x - x_0} = \lim_{x\to x_0} \frac{a^{\varphi(x)}\left(a^{f(x)-\varphi(x)} - 1\right)}{x - x_0} = \lim_{x\to x_0} \frac{a^{\varphi(x)}\left(a^{f(x)-\varphi(x)} - 1\right)}{f(x)-\varphi(x)} = \lim_{x\to x_0} \frac{f(x)-\varphi(x)}{x - x_0} = pq$

$$p = \lim_{x\to x_0} \frac{a^{\varphi(x)}\left(a^{f(x)-\varphi(x)} - 1\right)}{f(x)-\varphi(x)} = \ln a,\; q = \lim_{x\to x_0} \frac{f(x)-\varphi(x)}{x - x_0}.$$

Based on the above lemma, it is easy to prove the following theorems.

Theorem 1. For the function $f(x), \varphi(x), g(x)$ around the point $x = x_0$

$$\lim_{x\to x_0} f(x) = \lim_{x\to x_0} \varphi(x) = \lim_{x\to x_0} g(x) = 0, (a > 0,\ \mathrm{a} \neq 1)$$

If the conditions are met, then the following relation is appropriate

$$\lim_{x\to x_0} \frac{a^{f(x)} - a^{\varphi(x)}}{g(x)} = pq. \tag{1}$$

Proof.

$\lim_{x\to x_0} \frac{a^{f(x)} - a^{\varphi(x)}}{g(x)} = \lim_{x\to x_0} \frac{a^{\varphi(x)}(a^{f(x)-\varphi(x)} - 1)}{g(x)} = \lim_{x\to x_0} a^{\varphi(x)} \lim_{x\to x_0}\left(\frac{a^{f(x)-\varphi(x)} - 1}{f(x)-\varphi(x)}\right)\cdot\left(\frac{f(x)-\varphi(x)}{g(x)}\right) = pq,$ This one is on the ground

$$p = \lim_{x\to x_0} \frac{a^{f(x)-\varphi(x)} - 1}{f(x)-\varphi(x)} = \ln a,\quad q = \lim_{x\to x_0} \frac{f(x)-\varphi(x)}{g(x)}.$$

The theorem is proven.

Theorem 2. $x = x_0$ Around

$$\lim_{x\to x_0} f(x) = \lim_{x\to x_0} \varphi(x) = \lim_{x\to x_0} g(x) = 0 \text{ and } g'(x_0) \neq 0$$

If the relation is satisfied, then the equality $(b > 0, b \neq 1, a > 0, a \neq 1)$,

$$\lim_{x\to x_0} \frac{a^{f(x)} - b^{\varphi(x)}}{g(x)} = p\ln a - q\ln b.$$

Proof.

$$\lim_{x\to x_0} \frac{a^{f(x)} - b^{\varphi(x)}}{g(x)} = \lim_{x\to x_0} \frac{a^{f(x)} - 1}{g(x)} - \lim_{x\to x_0} \frac{b^{\varphi(x)} - 1}{g(x)} = \lim_{x\to x_0} \frac{a^{f(x)} - 1}{f(x)} \cdot \lim_{x\to x_0} \frac{f(x)}{g(x)} -$$

$$- \lim_{x\to x_0} \frac{b^{\varphi(x)} - 1}{g(x)} \cdot \lim_{x\to x_0} \frac{\varphi(x)}{g(x)} = p\ln a - q\ln b,$$

Here $p = \lim_{x\to x_0} \frac{f(x)}{g(x)};\ q = \lim_{x\to x_0} \frac{\varphi(x)}{g(x)}.$

Now let us look at questions related to the above topic:

$1^0. \lim_{x\to 0}\frac{a^x - a^{\sin x}}{x} = \lim_{x\to 0}\frac{a^{\sin x}(a^{x-\sin x}-1)}{x} = \lim_{x\to 0}\frac{a^{x-\sin x}-1}{x-\sin x}.$

$\lim_{x\to 0}\frac{x-\sin x}{x} = \ln a\cdot\lim_{x\to 0}\left(1-\frac{\sin x}{x}\right) = \ln a\cdot(1-1) = 0$

$2^0. \lim_{x\to 0}\frac{a^{\sin x}-a^{tgx}}{x^3} = \lim_{x\to 0}\frac{a^{tgx}(a^{\sin x - tgx}-1)}{x^3} = \lim_{x\to 0}\left(\frac{a^{\sin x-tgx}-1}{\sin x - tgx}\cdot\frac{\sin x - tgx}{x^3}\right) =$

$= \lim_{x\to 0}\frac{a^{\sin x - tgx}-1}{\sin x - tgx} = \ln a.$

$\lim_{x\to 0}\frac{\sin x - tgx}{x^3} = \lim_{x\to 0}\frac{\sin x(\cos x - 1)}{x^3\cos x} = -2\lim_{x\to 0}\frac{\sin x}{x}\cdot\lim_{x\to 0}\frac{\sin^2\frac{x}{2}}{\left(\frac{x}{2}\right)^2 4}\cdot\lim_{x\to 0}\frac{1}{\cos x} = -2\cdot 1\cdot\frac{1}{4}\cdot 1 = -\frac{1}{2}$

So, $\lim_{x\to 0}\frac{a^{\sin x}-a^{tgx}}{x^3} = -\frac{1}{2}\ln a.$

$3^0. \lim_{x\to 1}\frac{a^{\ln x}-b^{\sin(x-1)}}{x-1} = \lim_{x\to 1}\frac{a^{\ln x}-1}{x-1} - \lim_{x\to 1}\frac{b^{\sin(x-1)}-1}{x-1} =$

$= \lim_{x\to 1}\frac{a^{\ln x}-1}{\ln x}\cdot\lim_{x\to 1}\frac{\ln x}{x-1} - \lim_{x\to 1}\frac{b^{\sin(x-1)}-1}{\sin(x-1)}\cdot\lim_{x\to 1}\frac{\sin(x-1)}{x-1}\ln a\cdot 1 - \ln b\cdot 1 = \ln\frac{a}{b}.$

Let us prove the following theorem, which facilitates the process of solving practical problems and examples.

Theorem 3. If the following conditions are satisfied $\varphi'(x)\neq 0$ for functions $f(x), \varphi(x)$ around a point: $x = a$ and $\lim_{x\to a} f(x) = a,\ \lim_{x\to a}\varphi(x) = a$

In this case, the following relation is appropriate

$$\lim_{x\to a}\frac{a^{f(x)} - f(x)^a}{\varphi(x)} = a^a p\ln\frac{a}{e},\ (a>0,\ a\neq 1) \tag{2}$$

Proof. In the process of proving the theorem, we will make the following rather complicated substitution:

$$\frac{a^{f(x)} - f(x)^a}{\varphi(x)} = \frac{\left(a^{f(x)-a}-1\right)a^a}{\varphi(x)} - a^a\left(\left(1+\frac{f(a)-a}{a}\right)^a - 1\right).$$

To calculate the limit, subtract a^a from its image, then add and replace

$$\frac{a^{f(x)} - f(x)^a}{\varphi(x)} = \frac{(a^{f(x)-a}-1)a^a}{\varphi(x)} - \frac{a^a((1+\frac{f(x)-a)^a}{a})-1)}{\varphi(x)}.$$

Let's start calculating the limit here

$$\lim_{x\to a}\frac{a^{f(x)} - f(x)^a}{\varphi(x)} = a^a\lim_{x\to a}\frac{a^{f(x)-a}-1}{f(x)-a}\cdot\lim_{x\to a}\frac{f(x)-a}{\varphi(x)} - a^{a-1}\lim_{x\to a}\frac{\left(1+\frac{f(x)-a}{a}\right)^a - 1}{(f(x)-a)a}\cdot\lim_{x\to a}\frac{f(x)-a}{\varphi(x)} = \text{Here}$$

$$= a^a p\ln a - a^a p = a^a p(\ln a - \ln e) = a^a p\ln\frac{a}{e}.$$

$$p = \lim_{x\to a}\frac{f(x)-a}{\varphi(x)} \tag{3}$$

To prove the theorem, the following relations were used, given in [12]

$$\lim_{x\to a}\frac{(1+(f(x)-a))^{\mu}}{f(x)-a}=\mu,\ \lim_{x\to a}\frac{a^{f(x)-a}-1}{f(x)-a}=\ln a.$$

Let us apply formula (2) to a practical example.
If $\varphi'(x)\neq 0$ necessary if done then

$$\lim_{x\to e}\frac{e^{f(x)}-f(x)^{e}}{\varphi(x)}=0,$$

It follows that the relations corresponding to [13-14].

$$1^0.\ \lim_{x\to a}\frac{a^x-x^a}{\sin(x-a)}=\lim_{x\to a}\frac{a^a(a^{x-a}-1)}{\sin(x-a)}-\lim_{x\to a}\frac{a^a\left(\left(1+\frac{x-a}{a}\right)^a-1\right)}{\frac{x-a}{a}}=$$

$$=a^a\lim_{x\to a}\frac{a^{x-a}-1}{x-a}\cdot\lim_{x\to a}\frac{x-a}{\sin(x-a)}-a^a\lim_{x\to a}\frac{\left(1+\frac{x-a}{a}\right)^a-1}{\frac{x-a}{a}}.\lim_{x\to a}\frac{x-a}{a\sin(x-a)}=$$

$$=a^a\ln a-\frac{a^a\cdot a}{a}=a^a(\ln a-1)=a^a\ln\frac{a}{e} \qquad (4)$$

From (4) $a=e$ it follows that $\lim_{x\to e}\frac{e^x-x^e}{\sin(x-e)}=\lim_{x\to e}\frac{e^x-x^e}{tg(x-e)}=\lim_{x\to e}\frac{e^x-x^e}{\arcsin(x-e)}=\lim_{x\to e}\frac{e^x-x^e}{arctg(x-e)}=0$

$$2^0.\ \lim_{x\to 2}\frac{2^{2\sin\frac{\pi x}{4}}-\left(2\sin\frac{\pi x}{4}\right)^2}{x^2-4}=2^2p\ln\frac{2}{2} \qquad (5)$$

Let us use (3) to prove relation (5):

$$p=\lim_{x\to a}\frac{f(x)-f(a)}{\varphi(x)}=\lim_{x\to 2}\frac{2\sin\frac{\pi x}{4}-2}{x^2-4}=2\lim_{x\to 2}\frac{\sin\frac{\pi x}{4}-1}{x^2-4}=-4\lim_{x\to 2}\frac{\sin^2\left(\frac{\pi}{8}(x-2)\right)}{x^2-4}=$$

$$=-4\lim_{x\to 2}\frac{\sin^2\left(\frac{\pi}{8}(x-2)\right)}{\left(\frac{\pi}{8}(x-2)\right)^2}\cdot\frac{\left(\frac{\pi}{8}(x-2)\right)^2}{x-2}=-4\left(\frac{\pi}{4}\right)^2\lim_{x\to 2}(x-2)=0.$$

So,

$$\lim_{x\to 2}\frac{2^{2\sin\frac{\pi x}{4}}-\left(2\sin\frac{\pi x}{4}\right)^2}{x^2-4}=0$$

$3^0.\ \lim_{x\to x_0}f(x)=a,\ \lim_{x\to x_0}\varphi(x)=a,\ \lim_{x\to x_0}g(x)=0$ Then calculate the next limit.

$$\lim_{x\to x_0}\frac{f(x)^{\varphi(x)}-\varphi(x)^{f(x)}}{g(x)}=\lim_{x\to x_0}\frac{a^{f(x)\log_a\varphi(x)}-a^a}{g(x)}=$$

$$=a^a\left(\lim_{x\to x_0}\frac{a^{\varphi(x)\log_a f(x)-a}-1}{g(x)}-\lim_{x\to x_0}\frac{a^{f(x)\log_a\varphi(x)-a}-1}{g(x)}\right)=$$

$$=a^a\Big(\lim_{x\to x_0}\frac{a^{\varphi(x)\log_a f(x)-a}-1}{\varphi(x)\log_a f(x)-a}\cdot\frac{\varphi(x)\log_a f(x)-a}{g(x)}-\lim_{x\to x_0}\frac{a^{f(x)\log_a \varphi(x)-a}-1}{f(x)\log_a\varphi(x)-a}\cdot$$

$$\cdot\frac{f(x)\log_a\varphi(x)-a}{g(x)}\Big)=a^a\ln a\Big(\lim_{x\to x_0}\frac{\varphi(x)\log_a f(x)-a}{g(x)}\Big)-\lim_{x\to x_0}\frac{f(x)\log_a\varphi(x)-a}{g(x)}\Big)$$

4^0. Calculate. $\lim_{x\to a}\dfrac{(2x-a)^{\frac{x^2}{a}}-\left(\frac{x^2}{a}\right)^{2x-a}}{x^2-a^2}$. (6)

(6) The limit can be calculated in two ways. In this case, the desired result is obtained using Theorem 3, and the application of such theorems to problem solving creates some convenience for students.

Method 1: Calculate the limit using Lopital's rule

$$\lim_{x\to a}\frac{(2x-a)^{\frac{x^2}{a}}-\left(\frac{x^2}{a}\right)^{2x-a}}{x^2-a^2}=\lim_{x\to a}\frac{\left((2x-a)^{\frac{x^2}{a}}\right)'-\left(\left(\frac{x^2}{a}\right)^{2x-a}\right)'}{(x^2-a^2)'}=$$

$$\lim_{x\to a}\frac{\frac{2x}{a}(2x-a)^{\frac{x^2}{a}}\left(\ln(2x-a)+\frac{x}{2x-a}\right)-2\left(\frac{x^2}{a}\right)^{2x-a}\left(\ln\frac{x^2}{a}+\frac{2x-a}{x}\right)}{2x}=\frac{2a^a(\ln a+1)-2a^a(\ln a+1)}{2a}=0.$$

Method 2: Use Theorem 3 to calculate the limit. We use the formula

$$\lim_{x\to x_0}\frac{f(x)^{\varphi(x)}-\varphi(x)^{f(x)}}{g(x)}=a^a\ln a\Big(\lim_{x\to x_0}\frac{\varphi(x)\log_a f(x)-a}{g(x)}-\lim_{x\to x_0}\frac{f(x)\log_a\varphi(x)-a}{g(x)}\Big)$$

$$\ln a\cdot\lim_{x\to a}\frac{\frac{x^2}{a}\log_a(2x-a)-(2x-a)\log_a\frac{x^2}{a}}{x^2-a^2}=$$

$$a^a\ln a\cdot\left[\lim_{x\to a}\frac{\frac{x^2}{a}\log_a(2x-a)-a\log_a(2x-a)}{x^2-a^2}-\lim_{x\to a}\frac{(2x-a)\log_a\frac{x^2}{a}a\log_a\frac{x^2}{a}}{x^2-a^2}+\lim_{x\to a}\frac{a\log_a(2x-a)-a\log_a\frac{x^2}{a}}{x^2-a^2}\right].1)$$

$$\lim_{x\to a}\frac{\frac{x^2}{a}\log(2x-a)-a\log_a(2x-a)}{x^2-a^2}=\lim_{x\to a}\frac{\left(\frac{x^2}{a}-a\right)\log_a(2x-a)}{x^2-a^2}=\lim_{x\to a}\frac{\log_a(2x-a)}{a}=\frac{1}{a}$$

2) $$\lim_{x\to a}\frac{(2x-a)\log_a\frac{x^2}{a}-a\log_a\frac{x^2}{a}}{x^2-a^2}=\lim_{x\to a}\frac{2(x-a)\log_a\frac{x^2}{a}}{x^2-a^2}=\lim_{x\to a}\frac{2\log_a\frac{x^2}{a}}{x+a}=\frac{1}{a}$$

3) $$\lim_{x\to a}\frac{a\left(\log_a(2x-a)-\log_a\frac{x^2}{a}\right)}{x^2-a^2}=\lim_{x\to a}\frac{a\log_a\frac{(2x-a)a}{x^2}}{x^2-a^2}=a\lim_{x\to a}\frac{\log_a\left(\frac{2a}{x}-\frac{a^2}{x^2}-1+1\right)}{x^2-a^2}=$$

$$=a\lim_{x\to a}\frac{\log_a\left(1-\left(\frac{a}{x}-1\right)^2\right)}{x^2-a^2}=a\lim_{x\to a}\log_a\left(1-\left(\frac{x-a}{x}\right)^2\right)^{\frac{1}{x^2-a^2}}=$$

$$= a\log_a \lim_{x\to a}\left[\left(1-\frac{x-a}{x}\right)^{\frac{1}{x^2-a^2}}\cdot\left(1+\frac{x-a}{x}\right)^{\frac{1}{x^2-a^2}}\right] = a\log_a\left(e^{-\frac{1}{2a^2}}\cdot e^{\frac{1}{2a^2}}\right) = a\cdot 0 = 0.$$

Theorem 4. $\lim_{x\to x_0} u(x)=1, \lim_{x\to x_0} v(x)=\infty$ When equality holds

$$\lim_{x\to x_0} u^v = \exp\left\{\lim_{x\to x_0}(u-1)v\right\}$$

Proof. $\lim_{x\to x_0} u^v = \lim_{x\to x_0}\left(\left(1+(u-1)\right)^{\frac{1}{u-1}}\right)^{(u-1)v} = \exp\left\{\lim_{x\to x_0}(u-1)v\right\}$

Consider the following practical examples of theorem 4. [15]:

1. $\lim_{x\to\infty}\left(\frac{a+x}{b+x}\right)^{kx} = e^{k(a-b)}.$

$$\lim_{x\to\infty}\left(\frac{1+x}{2+x}\right) = \exp\left\{\lim_{x\to\infty} x\left(\frac{1+x}{2+x}-1\right)\right\} = \exp\left\{\lim_{x\to\infty}\frac{(-x)}{2+x}\right\} = e^{-1}.$$

2. $\lim_{x\to 0}\left(\frac{1+x}{1+2x}\right)^{\frac{1}{x}}\cdot \lim_{x\to 0} u(x)=1$ and $\lim_{x\to 0} v(x)=\infty$ when $\lim_{x\to x_0} u^v = \lim_{x\to x_0}\left(\left(1+(u-1)\right)^{\frac{1}{u-1}}\right)^{(u-1)v} =$

$= \exp\left\{\lim_{x\to x_0}(u-1)v\right\}$ we use the fact that

$$\lim_{x\to 0}\left(\frac{1+x}{1+2x}\right)^{\frac{1}{x}} = \exp\left\{\lim_{x\to 0}\frac{\frac{1+x}{1+2x}-1}{x}\right\} = \exp\left\{\lim_{x\to 0}\frac{(-1)}{1+2x}\right\} = e^{-1}.$$

Conclusion

Forming the dynamics of studying mathematics by students, analyzing the level of theoretical knowledge, practical skills and qualifications they have acquired, as well as studying the possibilities of mathematics in developing the level of mastery, increasing the efficiency of the educational process, the methodological basis of qualifications, independent learning of students is created.

This knowledge is used to develop skills for independent study of the “Theory of Limits” section and analyze the level of mastered theoretical, practical skills and qualifications, improve the level of skill, as well as to study the capabilities of the “Theory of Limits” section. The “Theory of Limits" creates a methodological basis for increasing the effectiveness of practical training.

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