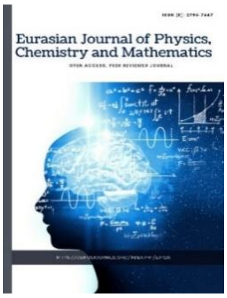

Development Of Logical Thinking Of Future Teachers Of Mathematics Using The Methods Of Newton And Chebishev.

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ABSTRACT

This article is devoted to the methodology of developing logical thinking of future mathematics teachers using the methods of Newton and Chebyshev, based on a related approach. Effective methods for developing logical competencies of future mathematics teachers are described

Keywords: monotone, convex, section, polynomial, root, logical thinking, development, ability to think, creativity.

α number $f(x)$ because it is a simple root of a polynomial $f'(a) \neq 0$ will be Therefore, $f''(a) \neq 0$ We accept this, otherwise the level of the problem $f(x)$ less than (otherwise) level $f''(x)$ were used to calculate the root of the polynomial.

Then (a,b) is the intersection $f(x)$ from α not just a root, but $f'(x)$ and also any root of the polynomial $f''(x)$ we assume that it does not contain the roots of the polynomial.

So, as we know from the course of mathematical analysis, $y = f(x)$ the curve (a, b) either increases monotonically or decreases monotonically on the section.

Therefore, at all points of this section it has a convexity either upward or downward. Consequently, at the location of the curve in section (a, b), four cases can occur, shown in Figures 1-4.

A and b are on which of the limits $f(x)$ hint at $f''(x)$ if it matches the sign a_0 we define through $f(a) va f(b)$ having a different sign, $f''(x)$ and since (a, b) has a sign at all points of the section, then in the cases shown in Figures 1-4 $a_0 = a$, and in two other cases $a_0 = b$ will $y = f(x)$ lines a_0 by abscissa there is bat a dead center, i.e. $(a_0, f(a_0))$ Let's try this line at the point where it is inclined to the coordinates and let's try this x We denote the abscissa of the point of intersection with the axis by d.

Drawings 1-4 d numbers α shows that it can be taken as an approximate value of the root.

Let's derive a formula by which we find the number d. It is known $y = f(x)$ to the curve $(a_0, f(a_0))$ equation of an experiment carried out at a point $y - f(a_0) = f'(a_0)(x - a_0)$

can be written as

$-f(a_0) = f'(a_0)(d - a_0)$

we form equality, hence $d = a_0 - \frac{f(a_0)}{f'(a_0)}$

comes from (2).

Example.

This method is as follows

$h(x) = x^5 + 2x^4 - 5x^3 + 8x^2 - 7x - 3$

We use it for many things.

As we know, this is a lot. $1 < \alpha_1 < 2$ located between the borders α_1 has a common root.

It is known $h'(x), h''(x), \ldots, h''''(x)$ derivatives $x = 1$ takes positive values when . $x = 1$ value $h'(x)$ So $h''(x)$ It follows that the upper limit function of positive roots for $h''(x)$ is positive everywhere in this interval and

$h(1) = -4,$

$h(2) = 39$

Because it is $a_0 = 2$ should be accepted.

$h'(2) = 102$ considering that from formula (2)

$d = 2 - \frac{39}{109} = \frac{179}{109} = 1{,}64,..$

We make an equation.

On the other hand, formula (1).

$c = \frac{2(-4) - 39}{-4 - 39} - \frac{47}{43} = 1{,}09\ldots$

gives equality and, therefore, α_1 get root rights for this version 1.09< α_1 <1.65

do not lie between the borders $h(x)$ for many of us and his α_1 Let's return to the root and note that all values of the following polynomials are calculated using Horner's method.

$h(1{,}3) = -0{,}13987,$

$h(1{,}31) = 0{,}0662923851$

Since 1.3< α_1 <1.31 which means we α_1 root meaning0.01Let's apply the linear interpolation method to these new limits:

$c = \frac{1.31(-0{,}13987) - 1{,}3 \cdot 0{,}0662923851}{0{,}13987 - 0.0662923851} = \frac{0{,}26940980063}{0{,}2061623851} = 1{,}30678\ldots$ Newton's method is applicable to the same boundaries, here $a_0 = 1{,}31$ should be taken as

$h'(1{,}31) = 20{,}92822405$

for part

$d = 1{,}31 - \frac{0{,}0662923851}{20{,}92822405} = \frac{27{,}3496811204}{20{,}92822405} = 1.30683\ldots$

So,

1,30678< α_1 <1.30684

and here's why α_1 If we take =1.30681, we will get an error less than 0.00003. Now we will prove the convergence of these methods for the Newton method.

$f(x)$ polynomial prime number α let the root be in the interval (a,b) satisfying Newton's method. From this, in particular, it follows that there exist positive numbers Ava B such that (a,b) are at all points of the segment $\begin{matrix} |f'(x)| > A, \\ |f''(x)| < B \end{matrix}$ (3)

will $c = \dfrac{B}{2A}$ let's introduce the notations and

s(ba)<1 (4)

let's assume that a_0 Newton's method should be applied within the limits of formula numbers a, b (2) mainly we. α as approximate values of the root (a, b) lying in the interval and with each other

$a_k = a_{k-1} - \dfrac{f(a_{k-1})}{f'(a_{k-1})},$ (5) are related by equations $a_1, a_2, \ldots, a_k, \ldots$ we form numbers in a row.

$k = 1,2,\ldots$

$\alpha = a_k + h_k,$

$k = 0,1,2,\ldots$

be in that case. $0 = f(a) = f(a_k) + h_k f'(a_k) + \dfrac{h_k{}^2}{2} f(a_k + \theta \cdot h_k)$ will be here $0 < \theta < 1$.(a,b) according to the condition imposed on the section $f'(a_k) \neq 0$ taking into account (5) and (6), we find:

$$-\frac{h^2{}_k}{2}\frac{f''(a_k + \theta \cdot h_k)}{f'(a_k)} = h_k + \frac{f(a_k)}{f'(a_k)} = \alpha - (a_k - \frac{f(a_k)}{f\ \ (a_k)}) = \alpha - a_{k+1} = h_{k+1}.$$

From this, $$|h_{k+1}| = h_k{}^2 \left| \frac{f''(a_k + \theta \cdot h_k)}{2 f'(a_k)} \right| < h_k{}^2 \frac{B}{2A} = Ch^2{}_k,$$

$k = 0,1,2,\ldots$

So, $|h_{k+1}| < Ch^2{}_k < C^3 h^4{}_{k-1} < C^7 h^3{}_{k-2} < \ldots < C^{2^{k+1}} h_0{}^{2^{k+1}}$

or $|h_0| = |\alpha - a_0| < b - a$ because .

$$|h_{k+1}| = C^{-1}[C(b-a)]^{2^{k+1}} \quad (7)$$

$k = 0,1,2,\ldots$

Therefore, according to condition (4), α with the root is generated by Newton's method sequentially a_k between the value of approx. h_k difference k tends to zero as it grows. This had to be proven.

Given to P.L. Chebyshev in 1838 $f(x)$ is an inverse function $g(y)$ proposes a method for constructing a higher-order iteration by describing a function using the Taylor formula.

Let's assume $f(x) = 0$ equations $x = \xi$ let the root lie in the interval [a,b] and $f(x)$ let the function and its derivatives of sufficiently high order be continuous. Also, all points in this interval $f'(x \neq 0)$ let it be

In this case $f'(x)$ maintains its position in this interval and $f(x)$ is a monotone function, $x - g(y)$ will have the opposite function.

Reverse function $g(y)$ $f(x)$ The domain of variation is defined in [c,d], and $f(x)$ no matter how many continuous derivatives it has, it has the same number of continuous derivatives according to the definition of the inverse function.

$$\begin{matrix} x \equiv g(f(x)) & (x \in [a,b]), \\ y \equiv f(g(y)) & (y \in [c,d]). \end{matrix} \quad (1)$$

So, $\xi = g(0),$ (2)

If $y \in [c,d]$ if , then by Taylor's formula

$$\xi = g(y) = g(y-y) = g(y) + \sum_{k=1}^{p-1} (-1)^k \frac{g^{(k)}(y)}{k!} y^k + (-1)^p \frac{g^{(p)}(\eta)}{p!} y^p \quad (3)$$

Here η number 0 and y lies between

Or y instead of $f(x)$ having put and $g(y) = x$ referring to

$$\xi = x + \sum_{k=1}^{p-1} (-1)^k \frac{g^{(k)}(f(x))}{k!} f^k(x) + (-1)^p \frac{g^{(p)}(\eta)}{p!} f^p(x). \quad (4)$$

we generate.

If

$$\varphi_p(x) = x + \sum_{k=1}^{p-1} (-1)^k \frac{g^{(k)}(f(x))}{k!} f^k(x)$$

if we define this, then

$$x = \varphi_p(x) \quad (5)$$

for the equation $x = \xi$ There will be a solution because

$$\varphi_p(\xi) = \xi + \sum_{k=1}^{p-1} (-1)^k \frac{g^{(k)}(f(\xi))}{k!} f^k(\xi) = \xi$$

From this $\varphi_p{}^{(j)}(\xi) = 0, \quad j = \overline{1, p-1}$

because it is

$$x_{n+1} = \varphi(x_n) \quad (n = 0,1,2,..,x_0 \in [a,b]) \quad (6)$$

the iterative process has p-order.

If $x_0 \xi$ is close to , it is determined by formula (6). $\{x_n\}$ subsequence ξ is approaching. Indeed, $\varphi_p'(\xi) = 0$ for what you ξ it was found that there is $|\varphi_p'(x)| \le q < 1$ will happen from this too. $x_0 \xi$ close enough to $\{x_n\}$ the convergence of the iterative sequence is obtained.

Now $\varphi_p(x)$ from $f(x)$ And φ we find the expression determined by the derivatives. To do this, we take successive derivatives from (1).

$$\begin{cases} g'(f(x))f'(x) = 1 \\ g''(f(x))f'^2(x) + g'(f(x))f'^4(x) = 0 \\ g'''(f(x))f'^3(x) + 3g''(f(x))f'(x)f''(x) + g'(f(x))f'^4(x) = 0 \\ ----------------------------------- \end{cases} \quad (7)$$

$p = 2$ When

$$\varphi_2(\alpha) = x - \frac{f(x)}{f'(x)} \text{ And } x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \quad (8)$$

This process partially coincides with Newton's process.

$p = 3$ from point (5), (7).

$$\begin{aligned} \varphi_3(\alpha) &= x - \frac{f(x)}{f'(x)} - \frac{f''(x)f^2(x)}{2[f'(x)]^3} \\ x_{n+1} &= x_n - \frac{f(x_n)}{f'(x_n)} - \frac{f''(x_n)f^2(x_n)}{2[f'(x_n)]^3} \end{aligned} \quad (9)$$

arises.

$p = 4$ For

$$\varphi_4(\alpha) = x - \frac{f(x)}{f'(x)} - \frac{f''(x) f^2(x)}{2[f'(x)]^3} - \frac{f^3(\alpha)}{12} \cdot \frac{3f''^2(x) - f'(x) f''(x)}{[f'(x)]^5} \quad (10)$$

And $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} - \frac{f''(x_n) f^2(x_n)}{2[f'(x_n)]^3} - \frac{f^3(x_n)}{12} \cdot \frac{3f''^2(x_n) - f'(x_n) f'^4(x_n)}{[f'(x_n)]^5}$.

we generate.

These iterative processes will be 2nd, 3rd and 4th order iterations respectively.

Now $\varepsilon_n = \xi - x_n$ To do this, in equation (4) we estimate the error rate tending to zero. $x = x_n$ Taking this into account and taking into account (6), we obtain the following.

$$\xi - x_{n+1} = \frac{(-1)^p g^{(p)} f(\tilde{x})}{p!} f^{(p)}(x_n) \quad (11)$$

Here $\bar{x}$ ξ With x_n lies between

$f(\xi) = 0$ because

$$f(x_n) = -[f(\xi) - f(x_n)] = -(\xi - x_n) f(\tilde{\tilde{x}}) \qquad (12)$$

($\tilde{\tilde{x}}$ too much ξ With x_n (12) in (11): $\varepsilon_{n+1} = \frac{g^{(p)}(f(\tilde{x}))}{p!}[f'(\tilde{\tilde{x}})]^p \varepsilon_n{}^p$ (13)

Following

$q = \max\limits_{a \le \tilde{x}, \tilde{\tilde{x}} \le b} \left| \frac{g^{(p)}(f(\tilde{x}))}{p!}[f'(\tilde{\tilde{x}})]^p \right|$ from (13), introducing the notations

$$|\varepsilon_{n+1}| \le q|\varepsilon_n|^p \qquad (14)$$

we obtain an inequality. Applying this inequality consistently, we obtain the following:

$$|\varepsilon_n| \le q^{1+p+\ldots+p^{n-1}} |\varepsilon_0|^p = (q|\varepsilon_0|)^{\frac{p^n-1}{p-1}} |\varepsilon_0|^{\frac{p^n(p-2)+1}{p-1}}.$$

If $|\varepsilon_0| < 1$ And $q|\varepsilon_0| = \omega < 1$ so be it $|\varepsilon_n| < \omega \frac{p^n - 1}{p-1}$ (15)

It turns out that iteration (6) is rapidly approaching.

Privately $\begin{matrix} \omega \le 10^{-1} \\ |\varepsilon_0| < 1 \end{matrix}$ then for iterations (8), (9), (10) above we have the following:

$p = 2$ For

$|\varepsilon_1| \le 10^{-1}$,

$|\varepsilon_2| \le 10^{-3}$,

$|\varepsilon_3| \le 10^{-7}$,

$|\varepsilon_4| \le 10^{-15}, \ldots$

$p = 3$ For

$|\varepsilon_1| \le 10^{-1}$,

$|\varepsilon_2| \le 10^{-4}$,

$|\varepsilon_3| \le 10^{-7}$,

$|\varepsilon_4| \le 10^{-40}, \ldots$

$p = 4$ For

$|\varepsilon_1| \le 10^{-1}, |\varepsilon_2| \le 10^{-5}, |\varepsilon_3| \le 10^{-18}, |\varepsilon_4| \le 10^{-85}, \ldots$

So, $\omega < 0{,}1$ when , only the third iteration gives us the required accuracy.

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