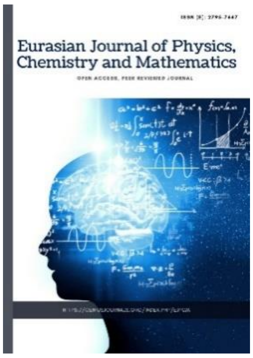

Applying The Theory Of Differential Equations In Economics

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ABSTRACT

In modern science, differential equations play an extremely important role. They have many applications in various fields, one of which is economics. It is extremely important for economics students to master this theory and be able to apply it in practice

Keywords:

In modern science, differential equations play an extremely important role. They have many applications in various fields, one of which is economics. It is extremely important for economics students to master this theory and be able to apply it in practice. Using problems with economic content when studying differential equations will allow you to:

- show the importance of differential equations for economics and business;
- use examples of problems from economics to explain the meaning of fundamental concepts from this section;
- to interest students in an in-depth study of differential equations for the purpose of application in research problems typical for many areas of economics.

Let us consider problems in economics that lead to differential equations with separable variables.

The general form of the nth order differential equation is:

$$F(x, y, y', \dots, y^n) = 0$$

where x is the independent variable $y(x)$ is the unknown function we are looking for. Accordingly, the first order equations have the form:

$$F(x, y, y') = 0$$

function

$$y = \varphi(x)$$

which turns the equation into an identity when replacing "y" and its derivatives with $\varphi(x)$ and its derivatives is called a partial solution of the differential equation. Function

$$y = \varphi(x, c_1, c_2, \dots, c_n)$$

which, when replacing the constants c_1, c_2, ..., c_n with specific numerical values, becomes a particular solution of the equation, is called the general solution of this equation.

Equations of the form:

$$y' = f(x) \cdot g(x)$$

is called a separable equation. Such equations are solved by the method of separation of variables in the form

$$\frac{dy}{g(y)} = f(x)dx$$

and integrate both parts. The general integral of the equation can be written as follows:

$$\int \frac{dy}{g(y)} = \int f(x)dx$$

By analyzing data from various sources of information, problems involving the use of differential equations in economic problems can be divided into several groups:

- performance tasks
- tasks for the amount of work
- study of functions in economics, functions of supply and demand, equilibrium price
- law of diminishing returns
- elasticity
- marginal cost problems and others.

Basic designation and economics

1. P – price;
2. Q – quantity;
3. D – demand;
4. S – offer;
5. Q_D – quantity demanded;
6. Q_s – supply quantity;
7. E_{DP} – coefficient of elasticity supply;
8. Productivity Challenges;
9. Tasks for the volume of work;
10. Study of functions in economics, functions of supply and demand, equilibrium price;
11. Law of Diminishing Returns;
12. Elasticity;
13. Marginal cost problems and others.

Let the function $V = V(t)$ express the amount of products produced over time. Let's find labor productivity at time t_0. Over the period of time t_0. At the quantity of products produced will change from the value $V_0 = V(t_0)$ to the value $V_0 + \Delta V - V(t_0 + \Delta t)$, then the average labor productivity for this period of time

$$(x + a)^n = \sum_{k=0}^{n} \binom{n}{k} x^k a^{n-k} \; y' = f(x) \cdot g(y)$$

$$E_x(y) = \lim_{\Delta x \to 0} \left(\frac{\Delta y}{y} : \frac{\Delta x}{x} \right) = \frac{x}{y} \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \frac{x}{y} \cdot y'$$

The elasticity of the function shows approximately how many percent $y = f(x)$ will change when the independent “x” changes by 1%.

- If $E_x(\mathrm{y}) = 0$, then the demand for a given product is called absolutely inelastic;
- If $0 \leq |E_x(\mathrm{y})| \leq 1$ then the demand for this product is called inelastic or relatively inelastic;
- If $E_x(\mathrm{y}) = 1$ then the product is said to have unit elasticity;
- If $|E_x(\mathrm{y})| \geq 1$, then the demand for this product is called elastic or relatively inelastic;
- If $E_x(\mathrm{y}) = \infty$, then the demand for a given product is called absolutely elastic.

Demand is the relationship between the unit price of a good and the quantity of the good that consumers are willing to buy at each possible price, over a given period of time, and other things being equal.

Profit is the difference between sales proceeds and production costs of a product.

Equilibrium price is the price that emerges on the market when the quantities of supply and demand are equal.

Marginal cost – the production of a good is the cost that is associated with producing an additional unit of output.

Let's consider problems involving the use of equations with separable variables in economic problems.

Problem 1. Let supply and demand for a product be determined by the relations

$$D(p) = 4p' - 2p + 59$$
$$S(p) = 44p' + 2p - 1$$

where $D(p)$ is demand, $S(p)$ is supply, p is the price of the product, p' is the tendency of price formation, the derivative of price over time. Also, let the unit price of the product “p” be equal to 1 unit at the initial time. Based on the requirement that demand matches supply, find the law of price changes depending on time.

Solution:

In order for demand to correspond to supply, the equality must be fulfilled:

$$4p' - 2p + 59 = 44p' + 2p - 1$$
$$40p' + 4p - 60 = 0,\ 10p' + p - 15 = 0$$
$$p' = \frac{dp}{dt},\quad \frac{-10dp}{dt} = p - 15$$

We separate the variables, we have an equation with separable variables, and by integrating we get

$$\int \frac{dp}{p-15} = -\int \frac{dt}{10} \qquad ln|p-15| = -\frac{1}{10}t + ln\,C$$

$$ln\left|\frac{p-15}{C}\right| = ln\,e^{-\frac{1}{10}t} \qquad \left|\frac{p-15}{C}\right| = e^{-\frac{1}{10}t} \qquad p = Ce^{-0.1t} + 15\,.$$

From $P_{t=0} = 1$ we find $1 = C + 15$ and $C = -14$. Thus

$$p = -14e^{-0.1t} + 15.$$

It follows from this that in order for equality to be maintained between supply and demand, the price must change in accordance with the resulting formula.

Problem 2. The function of dependence of the production of some raw materials on time in hours has the form

$$y(t) = 4 - 5e^{-5t}.$$

Find the dependence of the quantity of produced products $Q(t)$ on time, if it is known that

$$Q(6) = 30.$$

Solution:

Let $Q(t)$ be the amount of raw materials produced by time "t", then $y(t) = Q'(t)$ is the rate of production of raw materials, we have with separable variables $\frac{d(Q(t))}{dt} = y(t)$, integrating we get

$$\int \mathrm{d}\big(\mathrm{Q(t)}\big) = \int \mathrm{y(t)dt}$$
$$Q(t) = \int y(t)dt = \int (4 - 5e^{-5t})dt = \int 4dt - 5\int e^{-5t}dt = 4t + e^{-5t} + C$$

where $C = const$. The partial solution will look like:

$$Q = y(t, C_0) \ \text{где}\ C = C_0 \quad Q(6) = 30.$$
$$30 = 4 \cdot 6 + e^{-5\cdot 6} + C \quad C = 30 - 24 - e^{-30}$$
$$C = 6 - e^{-30}.$$

Hence, the dependence of the quantity of produced products $Q(t)$ on time will have the form:

$$Q(t) = 4 - 5e^{-5t} + 6 - e^{-30}.$$

Problem 3. At the textile enterprise "ORZU-TEKS", specializing in the production of knitted products, there is an "experimental workshop" in which modern models are created using creative ideas and current fashion trends in the production of products such as Polo shirts and others, the workshop produces and sells 2 thousand shirts per day costing 80 soums per shirt, 2% of the proceeds of sold shirts are used to expand production. Doubling the investment for expansion leads to an increase in sewing speed by one and a half times. How many shirts per day will the workshop produce by the end of the month?

Solution:

The dependence of shirt damage on t is given by the function y(t) (time is measured in days). Proceeds from the sale of shirts amount to (80y) soums, of which

$$\frac{(80y)\cdot 2\%}{100\%} = \frac{8}{5} = (1{,}6y)$$

is used to expand production. Since the sewing speed y' increases by 1.5 times, we have

$$\frac{1.5}{2}\cdot 1{,}6y = (1{,}2y).$$

We have an equation with separable variables

$$y' = 1{,}2y \quad \frac{dy}{dt} = 1{,}2y \quad \int\frac{dy}{y} = \int 1{,}2dt.$$

Let's integrate

$$ln\,y = 1,2t + ln\,C \quad ln\,y = ln\,e^{1,2t}\cdot C \quad y = e^{1,2t}\cdot C$$

general solution. Since we need to know the number of shirts produced per month, $t = 30$ and we get

$$\mathrm{y(30)} = 2000e^{1{,}2\cdot 30} = 2000e^{36}.$$

Problem 4. Find the demand function if elasticity is equal to $E_x = -\frac{1}{5}$ for any values of "P".

Solution:

Elasticity formula

$$E_x(P) = \lim_{\Delta x \to 0}\left(\frac{\Delta P}{P} : \frac{\Delta x}{x}\right) = \frac{x}{P}\lim_{\Delta x \to 0}\frac{\Delta P}{\Delta x} = \frac{x}{P}\cdot P'$$

$$E_x(P) = \frac{x}{P}\cdot P'.$$

Using the data we obtain differential equations with separable variables

$$\frac{Pdx}{xdp} = -\frac{1}{5} \qquad 5\frac{dx}{x} = -\frac{dp}{p} \qquad 5\,ln|x| = -\ln|p| + lnC$$

$$\ln|x^5| = ln\frac{C}{P} \qquad x^5 = \frac{C}{P} \qquad Px^5 = C.$$

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