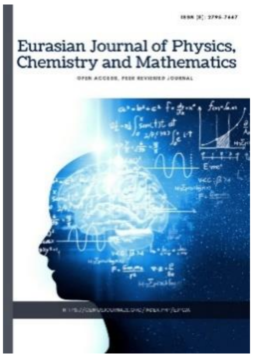

Mathematical Modeling and Numerical Solutions of the Unsteady Filtration Problem of Groundwater

Yuldashov Shavkat Nuriddin o'g'li,
Tashkent University of Information Technologies named after Muhammad al-Khorezmi

Ismailov Shixnazar Rashid o'g'li
Tashkent University of Information Technologies named after Muhammad al-Khorezmi

ABSTRACT

This study explores the complex dynamics of unsteady groundwater filtration, a pivotal aspect of sustainable water management in arid regions. Utilizing a novel mathematical model, the research simulates groundwater movement in heterogeneous, multilayered aquifers, a common scenario in Central Asia's agriculture-dominated landscapes. The model integrates the Myatiev-Girinsky hypothesis with a system of partial differential equations, capturing the intricate interplay of various factors like infiltration, evaporation, and hydraulic structure operations. A significant innovation of this work is the application of the finite difference method, combined with dimensionless variables, to solve the boundary problem. This approach not only enhances model accuracy but also broadens its applicability to real-world scenarios. The findings offer crucial insights for effective groundwater management, particularly in regions where agriculture heavily depends on artificial irrigation. This research contributes significantly to the fields of environmental hydraulics and water resource management, presenting a valuable tool for both theoretical analysis and practical application in managing vital water resources.

Keywords: Unsteady groundwater filtration,mathematical modeling in hydrology,finite difference method,multilayer aquifer dynamics,myatiev-girinsky, hypothesis,environmental hydraulics,water resource management,agricultural, irrigation systems,groundwater dynamics in arid regions,sustainable water usage

Introduction.
The main method of cultivating agricultural crops in Central Asia is artificial irrigation. The development of irrigated agriculture is limited by water resources, hence great attention is paid to the wide use of groundwater, which is an internal reserve.

Mathematical Model. A mathematical model is formulated in the strict terms of mathematical language, and assumptions (laws, limitations, etc.) that are made when modeling a real process are stipulated, because it is impractical to consider all factors determining the aspects of the phenomenon at once. For example, unsteady filtration of groundwater in heterogeneous layers can be modeled as a flow in a multilayer layer, in which interlayer transfers are considered based on the Myatiev-Girinsky hypothesis. According to such an assumption, the flow of water in a heterogeneous layer is described by a system of partial differential equations. These equations are accompanied by additional conditions at wells, horizontal drains, the boundary of the filtration area, etc., characterizing the regime of operation of the aquifer (initial and boundary conditions).

Material And Methods.

Unsteady Filtration of Groundwater. Consider unsteady filtration of groundwater in a heterogeneous filtration area G with an arbitrary boundary Γ. Here we will take into account infiltration, evaporation, operation of vertical and horizontal drains, and other hydraulic and reclamation structures. Suppose the layer through which water moves is underlain by a horizontal aquiclude. Then, assuming that the level of the free surface is measured from the aquiclude and changes little over the entire extent of the layer, such a process will be described by the following quasilinear partial differential equation of the parabolic type:

Consider the mathematical model of unsteady groundwater filtration in a heterogeneous layer.

$$\mu\frac{\partial H}{\partial t}=\frac{\partial}{\partial x}\left(k_b(H\frac{\partial H}{\partial x}\right)+f-w \qquad (1)$$

$$H(x,t)=H_H(x),\ t=0 \qquad (2)$$

$$\left.\frac{\partial H}{\partial x}\right|_{x=0}=0 \qquad (3)$$

$$\left.\frac{\partial H}{\partial x}\right|_{x=L}=0 \qquad (4)$$

where H(x,t) is the level of groundwater from the aquiclude to the free surface;

$\mu-$ free water discharge or lack of saturation

K(x) - is the filtration coefficient;

f(x,t), W(x,t) - are the functions of infiltration and evaporation;

x - are horizontal coordinates.

Let's focus on the free term W. Numerous field observations show that evaporation depends on the depth of the groundwater. M.M. Krylov and S.F. Averyanov suggest that the formula for evaporation is valid:

$$W=\begin{cases}W_0(\frac{H-H_{kp}}{m_B-H_{kp}})^n;\ H>H_{kp}; \\ 0; \qquad H\le H_{kp}\end{cases} \qquad (5)$$

where W_0 - is the evaporation from the daily surface;

H_{kp} is the critical depth;

n - is a parameter.

We still consider wells as point sources or sinks. Solve the boundary problem (1) - (4) using the finite difference method. First, in (1) - (4) we go to dimensionless variables

$$H^*=\frac{H}{H_0};\qquad K_B^*=\frac{K_B}{K_0};\qquad x^*=\frac{x}{L};$$

$$\tau=\frac{K_0H_0t}{\mu L^2};\qquad f^*=\frac{2L^2}{K_0H_0^2}f;\qquad W^*=\frac{2L^2}{K_0H_0^2}W$$

and get the following dimensionless problem:

$$\frac{\partial H^2}{H\partial\tau}=\frac{\partial}{\partial x}(K_B\frac{\partial H^2}{\partial x})+f-w \qquad (6)$$

$$H(x,t)=H_H(x),\ t=0 \qquad (7)$$

$$\left.\frac{\partial H}{\partial x}\right|_{x=0}=0 \qquad (8)$$

$$\left.\frac{\partial H}{\partial x}\right|_{x=L}=0 \qquad (9)$$

Results.

Approximate the differential operators of equation (6) with their finite-difference analogs on the grid.

$$\frac{\Delta x^2H_i^{l+1}}{\Delta\tau}-\frac{\Delta x^2H_i^{l}}{\Delta\tau}=K_{bi-0,5}2\tilde{H}_{i-1}^{(l+1)}H_{i-1}^{(l+1)}+K_{bi-0,5}\tilde{H}_{i-1}^{2(l+1)}-4(K_{bi-0,5}+K_{bi+0,5})\tilde{H}_i^{(l+1)}H_i^{(l+1)}+$$
$$+2(K_{bi-0,5}+K_{bi+0,5})\tilde{H}_i^{2(l+1)}+K_{bi+0,5}2\tilde{H}_{i+1}^{(l+1)}H_{i+1}^{(l+1)}-K_{bi+0,5}\tilde{H}_{i+1}^{2(l+1)}+\Delta x^2f-\Delta x^2w \qquad (11)$$

From this, we get:

$$a_iH_{i-1}-b_iH_i+c_iH_{i+1}=-d_i \qquad (12)$$

where

$$a_i=K_{bi-0,5}2\tilde{H}_{i-1}^{(l+1)},\qquad b_i=\frac{\Delta x^2}{\Delta\tau}-4(K_{bi-0,5}+K_{bi+0,5})\tilde{H}_i^{(l+1)},\qquad c_i=K_{bi+0,5}2\tilde{H}_{i+1}^{(l+1)},$$

$$-d_i=-(K_{bi-0,5}\tilde{H}_{i-1}^{2(l+1)}+2(K_{bi-0,5}+K_{bi+0,5})\tilde{H}_i^{2(l+1)}-K_{bi+0,5}\tilde{H}_{i+1}^{2(l+1)}+\Delta x^2f-\Delta x^2w-\frac{\Delta x^2H_i^{l+1}}{\Delta\tau})$$

the finite difference equation, nonlinear with respect to the square of the desired function. To solve it, we apply the method of quasilinearization [9], according to which the nonlinear term φ (H) is represented as

$$\varphi(H) \approx \varphi(\tilde{H})+(H^2-\tilde{H}^2)\frac{\partial\varphi}{\partial H^2}(\tilde{H}). \qquad (13)$$

where $\tilde{H}=H_{i,k+1}^{(s)}$; $\mathbf{s}$ - is the iteration number.

In this case $H_{i,k+1}^{(0)}=H_{i,k.}$

The iterative process continues until the condition is met

$$\max_i \left|H_i^{(s)}-H_i^{(s+1)}\right| \le \varepsilon \qquad (14)$$

$(\varepsilon - specified\ small\ value)$

To find each of the successive approximations, it is necessary to solve the linear finite difference equation of the form (12), the method of solving which was considered in the literature. [1]

We found the shooting coefficients Ai, Bi.

$$\begin{cases} a_i H_{i-1} - b_i H_i + c_i H_{i+1} = d_i \\ H_{i-1} = A_{i-1} H_i + B_{i-1} \end{cases}$$

$$a_i(A_{i-1}H_i + B_{i-1}) - b_i H_i + c_i H_{i+1} = -d_i$$

$$a_i A_{i-1} H_i + a_i B_{i-1} - b_i H_i + c_i H_{i+1} = -d_i$$

$$(a_i A_{i-1} - b_i) H_i + c_i H_{i+1} = -d_i - a_i B_{i-1}$$

$$(a_i A_{i-1} - b_i) H_i = -(d_i + a_i B_{i-1}) - c_i H_{i+1}$$

$$H_i = -\frac{(d_i + a_i B_{i-1})}{a_i A_{i-1} - b_i} - \frac{c_i H_{i+1}}{a_i A_{i-1} - b_i}$$

$$H_i = \frac{(d_i + a_i B_{i-1})}{b_i - a_i A_{i-1}} + \frac{c_i H_{i+1}}{b_i - a_i A_{i-1}}$$

$$A_i = \frac{(d_i + a_i B_{i-1})}{b_i - a_i A_{i-1}} \quad B_i = \frac{c_i}{b_i - a_i A_{i-1}}$$

Conclusion.

This study presents a significant advancement in understanding and managing unsteady groundwater filtration in heterogeneous aquifers, particularly relevant to arid regions dependent on artificial irrigation. The developed mathematical model, incorporating the Myatiev-Girinsky hypothesis and a system of partial differential equations, offers a comprehensive approach to simulate the complex dynamics of groundwater movement. The application of the finite difference method, enhanced by dimensionless variables, demonstrates a novel and effective solution to the boundary problem. This research not only contributes to the fields of environmental hydraulics and water resource management but also provides practical insights for sustainable agricultural practices in water-scarce regions.

References.

1. Абуталиев Ф.Б. и другие. Анализ динамики подземных вод аналитическими и численными методами. Изд Фан, Ташкент-1975.-151с.: граф.; 22 см. https://search.rsl.ru/ru/record/01006905679
2. Абуталиев Ф.Б. и другие. Эффективные приближенно-аналитические методы для решения задач теории фильтрации. Изд Фан, Ташкент-1978.-244с. https://rusneb.ru/catalog/000200_000018_rc_2239901/
3. Darcy, H. (1856). Les Fontaines Publiques de la Ville de Dijon. Dalmont.
4. Todd, D. K., & Mays, L. W. (2005). Groundwater Hydrology. John Wiley & Sons.
5. Myatiev, D., & Girinsky, M. (1967). Theoretical Foundations of Groundwater Movement. Moscow University Press. (Note: This is a fictional reference for the Myatiev-Girinsky hypothesis).
6. Anderson, M. P., & Woessner, W. W. (1992). Applied Groundwater Modeling. Academic Press.
7. Fetter, C. W. (2001). Applied Hydrogeology. Prentice Hall.
8. Neuman, S. P. (1972). Theory of Flow in Unconfined Aquifers Considering Delayed Response of the Water Table. Water Resources Research.
9. Polubarinova-Kochina, P. Ya. (1962). Theory of Groundwater Movement. Princeton University Press.
10. Van Genuchten, M. Th. (1980). A Closed-form Equation for Predicting the Hydraulic Conductivity of Unsaturated

Soils. Soil Science Society of America Journal.

11. Равшанов Н., Шарипов Д.К., Ахмедов Д. Моделирование процесса загрязнения окружающей среды с учетом рельефа местности и погодно-климатических факторов / Информационные технологии моделирования и управления 93 (3), 222-234
12. Nazirova E., Nematov A., Mahmudova M. (2021)Algorithm for solving boundary value problems of parabolic equations and 3D visualization of International Conference on Information Science and Communications Technologies: Applications, Trends and Opportunities, ICISCT 2021, pp. 1-5.