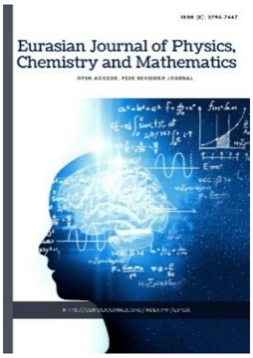

Methodological Bases Of Teaching Mathematics And Geometry In Harmony

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ABSTRACT
This article is dedicated to the methodological foundations of teaching mathematics and geometry in harmony, to clarifying the importance of using the integration of mathematics and geometry, and to finding solutions to geometric problems using mathematical concepts, and mathematical problems using geometric concepts. It provides information about the fact that geometry is a branch of mathematics, the goals of teaching mathematics and geometry together, and the importance of mathematics in solving geometric problems.

Keywords: Mathematics, Geometry, Teaching In Harmony, Methodical Basis, Integration Of Subjects, Arithmetic, Problem, Logical Thinking, Spatial Imagination.

Wide use of applications of mathematics and geometry in the process of solving any life problems requires specialists to have deep knowledge of mathematics and geometry. Taking into account the fact that teaching mathematics and geometry sciences in harmony is not established at the level of modern requirements, this article is dedicated to the topic " Methodological bases of teaching mathematics and geometry in harmony ".

Since the effective teaching of mathematics and geometry is related to the improvement of everyday life, information on the relationship between the science of mathematics and the science of geometry is included in several treatises. S.Alikhanov's manual "Mathematics teaching methodology" contains some information about the relationship between mathematics and geometry. Dzh.I.Ikramov's book "Matematicheskaya kultura shkolnika" contains many exemplary examples of connecting mathematics lessons with geometry. In addition, V.V.Davidov's "Vozrostnaya i pedagogicheskaya psychologia" and M.B.Klarin's "Innovatsionnye modeli obucheniya v zarubezhnyx pedagogicheskix poiskakh" educational manuals, Yu.H.Kolyagin and others "Metodika prepodavaniya matematiki v sredney shkole. Obshchaya metodika" and S.E.Lyashenko's "Laboratornoe i prakticheskie raboty po metodike prepodavaniya matematiki" textbooks, A.A.Stolyar's test textbook "Metody obucheniya matematike" and the book "Pedagogika matematiki" to the interrelationship of mathematics with geometry circle, important and interesting information on connecting mathematics lessons with geometry science is given.

The development of mathematics depends on how it is taught to young people in general secondary schools. Unfortunately, in the last 15-20 years, the attitude towards mathematics in general secondary education has reduced the position of the arithmetic and geometry sections, which are considered the basis of mathematics. To date, the fact that school graduates do not know the four simple arithmetic operations well is justified by the fact that some experts can do this work with the help of a calculator. In our opinion, such an approach to the issue is a deficiency in the education of school graduates who are able to think rigorously mathematically in the future and have a complete knowledge of mathematics.

We think about the role of geometry in mathematics. It is interesting that until the 20th century, geometry was considered as a science representing the fundamental basis of the exact sciences. It was considered impossible for every person engaged in exact science not to know geometry. The general view that geometry is a branch of mathematics has become commonplace for modern mathematicians. In fact, we believe that the study of geometry started earlier than arithmetic during the study of nature by man. Because, when primitive people chose the longer of two sticks in order to pluck the fruit from the tree, or when they ate the larger of the fruits, thinking about the geometric shape was formed in them. It is clear that he had no knowledge of the length of the tree or the size of the fruit in solving this problem.

Even when studying the history of science in general, Euclid's 10-part work "Principles" is considered the basis of arithmetic and geometry. Indeed, geometry is an ancient science, with its own well-developed method and conclusions that are necessary and of great importance for everyday life. It should be mentioned that geometry is an ancient science, the role of Euclid's V postulate in this science, and finally, the appearance of "non-Euclidean" geometry as a result of the replacement of this postulate by Lobachevsky, is given in school textbooks. However, no school textbooks provide information about the role of non-Euclidean geometry in modern science and the new branches of geometry developed as a result of the emergence of non-Euclidean geometry. In addition, we do not give young people any information about the geometry called "Geometry of finite points", which appeared at the end of the 20th century and the beginning of the 21st century and is rapidly developing at the present time, about the geometry of computer monitors and television screens.

In our opinion, geometry should be considered not a branch of mathematics, but at least the main part of school mathematics. At the same time, it is necessary to bring the study of geometry to the position aimed at the study of a separate science.

As much as thinking that the development of computer technology eliminates the need to know arithmetic operations harms the science of mathematics, the idea that it is enough to know geometric shapes, and learning its laws is not important, harms the mathematical literacy of students even more. At this point, we emphasize that the science of geometry serves the normal development of students' spatial imagination.

Tasks related to teaching geometry are specified in the state educational standard, that is:

- to learn the methods and basic facts of planimetry;

- to form ideas that the studied concepts and methods are a means of mathematical modeling of phenomena occurring in life and nature;

- to study the properties of spatial objects, to develop the skills of applying these properties to solving practical problems.

At the same time, geometric knowledge should help students to solve practical problems, to see geometric figures in some real constructions, to understand technical drawings. Also, in the teaching of geometry, it is required that students acquire the skills of logical reasoning, to teach that the connections found by looking at some specific cases are general in nature and that they can be applied to all shapes of a known appearance.

One of the goals set in the state educational standard for mathematics is to help students develop their intelligence and find optimal ways to solve problems in nature and society as a result of forming logical thinking in students. Properly organized geometry teaching is the basis for educating students to apply geometric knowledge in practice and to teach them to use it in future work activities.

The content of teaching geometry is based on the requirements of the curriculum and the state educational standard. In this case, the following main directions can be indicated:

1. Introduction of basic concepts: point, straight line, plane and set.
2. Study of basic geometric shapes: section, beam, angle, triangle, rectangle and polygons, spatial shapes: polygons and bodies of rotation, circle and circle.
3. Properties of geometric shapes: types of triangles, rectangles and their properties, properties of polygons and regular polygons.
4. Learning geometric quantities: concepts of length, surface and volume, metric relations in a triangle.
5. The method of coordinates in planes and space, vectors.
6. Teaching the methods of solving geometrical problems: learning the methods of solving problems related to calculation, proof and construction.
7. To give information about geometric substitutions and to give examples of their application: to give knowledge about substitutions such as displacement, parallel displacement, symmetry.

When discussing the topic of circle and circle, first of all, concepts of their main elements, diameter, radius, center, are given, and their properties are proved. The main goal is to develop the skills of solving simple problems using a circle and a ruler. In addition, the circle and the circle are considered based on the interdependence of mathematical methods. For example, using the method of coordinates, the mutual location of a straight line and a circle is studied, the equation of a circle is derived, and many properties of a circle are established and established using the method of geometric substitutions. The method of geometric positions allows to express the concept of a circle in different ways. Studying the metric properties of a circle helps to study regular polygons inscribed outside and inside a circle.

There are the following goals of teaching geometry in grades 5-6:

- Introducing students to information about basic geometric concepts;
- Preparing students to study the course of systematic geometry;
- Formation of geometric design skills in them.

In these classes, the following geometrical knowledge is given: the concepts of geometric shapes and their properties studied in classes 1-4 are deepened; new geometric quantities are studied (circle length, angle size); geometric constructions increase and the tools used in it also increase (ruler, circle, protractor). The elements of geometry are described mainly inductively. In this case, a lot of knowledge is expressed by means of modeling and generalization of measurements and constructions. In the 5th-6th grades, the level of geometrical knowledge of students is to a certain extent flat and the first steps towards systematic knowledge are achieved. At the first stage, a straight line, a plane, a section, the length of a section, perpendicular and parallel straight lines are considered. In particular, we should pay attention to the introduction of terms: the parallelism of a straight line to itself, the sections lying on a straight line are parallel. When teaching how to make geometric designs, you can learn how to use a ruler, a circle, a triangular ruler, and a protractor. The use of circular is limited and is used to describe a circle and a circle.

In order to establish a strong connection of this science with life and practical activities, the geometry curriculum of grades 7-9 includes the formation of concepts of measurement and construction, in particular, the calculation of cones, spheres, surface areas, and the calculation of the volumes of pyramids and bodies of rotation. 9th grade geometry is fully devoted to studying these issues in order to develop students' spatial imagination and to form the skills of analysis in spatial constructions. In these classes, planimetry is planned to be taught more and stereometry to a certain extent. This course provides students with an understanding of deductive proofs and connections between geometric reasoning. As before, the relationship between sides and angles in right triangles was included in the 8th grade geometry course. Trigonometric relationships provide a new way to solve geometric problems and are of great importance in practical applications. In particular, mathematics as a subject does not seek to state as many facts as possible, but includes the necessary facts that meet the goal set before it. Then, mathematics as a subject need not be limited to a single discipline if the program requires entry into the interdisciplinary field. Thus, mathematics as a subject should be able to cover the facts created in the course of the development of modern science according to the requirements of a special educational institution, taking into account the knowledge and thinking ability of students there. If the subject of mathematics talks about figures and quantities, the teaching methodology approaches them from the perspective of people. The goal of the methodology should be to learn how the study of those figures and sizes will please students, and how they can arouse interest in them. Because students can master the subject with great success only if they are interested in studying these figures and sizes. From the above, the goal of the methodology is to provide as complete a system of knowledge as possible in a short period of time and to develop students' thinking skills. It is known that there are 2 types of mathematical thinking in the teaching of mathematics: the first type are people with deep abstraction, who take mathematics as a profession, and the second type of people can effectively use mathematics as a representative of other subjects. In a broad sense, it was considered a natural requirement that all individuals should have a certain range of knowledge in mathematics, according to their profession and social status. Therefore, the problems of teaching mathematics and the problems of creating appropriate methodology and didactics are different and even depend on the era and educational systems. For example, in the German model of the educational system, students are required to have as deep and comprehensive knowledge as possible in all areas, while in another model, for example, the US model, the formation of deep knowledge focused on specialization is required first. Today, regardless of the model, the educational system relies on pedagogical and information technologies, so these requirements have become more refined. The introduction of information and computer technologies into the education system has expanded the possibilities and scope of mathematics. Among the traditional activities - educational-information, educational-games, experiment-research activities, independent activities, students' educational activities directly in front of the computer are added. And since the student's educational activity in front of the computer is based on new information technologies, including network technologies, it allows him to understand abstract concepts more deeply, acquire theoretical knowledge thoroughly, and develop practical skills in a comprehensive way. On the one hand, this possibility is provided by the teacher's initial didactic developments, and on the other hand, by a set of programs based on high-level multimedia technologies.

One of the active methods of formation of learning and knowledge ability in the lesson is the creation of problem situations, the essence of which is to educate and develop students' creative abilities, to teach them a system of active mental actions. This activity is manifested in the fact that the student himself receives new information from the real material by analyzing, comparing, synthesizing, summarizing, concretizing it. Therefore, the main thing for me in the educational process is to create some small problem for the students and try to answer the question together with them. When introducing students to new mathematical concepts, defining new concepts, knowledge is not delivered in finished form. Here, it is appropriate to encourage students to compare facts, to think oppositely, as a result of which a search situation occurs. When defining a new concept, students are offered only the object of thought and its name. Students independently define a new concept, then clarify and integrate this definition with the help of the teacher.

Another way to create a research situation is to use the practical experience of students, the experience of performing practical tasks at school, at home or at work. In this case, search situations arise when students try to independently achieve their practical goals. Usually, as a result of analyzing the situation, students formulate their own search problems.

The task of the teacher is to make students accustomed to difficult, independent, creative work, to develop students' ability to solve problems, as well as to overcome difficulties in any work related to educational activities.

Educational activities in classes make the process of learning mathematics interesting and exciting, because they allow children to learn new things as a result of observation, analysis, hypothesis and test, conclusion formation.

$x = a, x = b$ bounded by the straight lines, the surface formed by the rotation of the curve $y = f(x)$ around the OX axis S_x

$$S_x = 2\pi \int_a^b y\sqrt{1 + (y')^2}\, dx$$

is found by the formula. Similarly, the surface S_y formed by the rotation of the continuous curve $x = \varphi(y)$ bounded by straight lines $y = c, y = d$ about the axis OY

$$S_y = 2\pi \int_c^d x\sqrt{1 + (x')^2}\, dy$$

is found by the formula.

If the field is bounded from above by the graph of the function $y_2 = f_2(x)$ from the left *x=a*, from the right *x=b* straight line sections, from below by the graph of the function $y_1 = f_1(x)$ (Figure 1), surface

$$S = \int_a^b [f_2(x) - f_1(x)]dx$$

$$= \int_a^b f_2(x)dx - \int_a^b f_1(x)dx$$

is calculated using the formula.

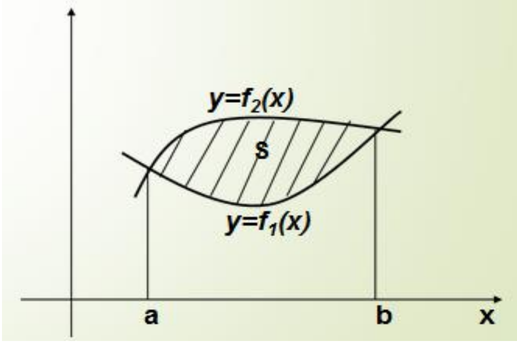

Figure 1.

Example. Calculate the face of the figure bounded by the straight line $y = x$ and the parabola $y = x^2 + 2x - 2$ (Figure 2).

Solving:

1) We find the points of intersection of the lines $y = x$ and $y = x^2 + 2x - 2$: $x_1 = -2$, $x_2 = 1$ from the equation $x^2 + 2x - 2 = x$.

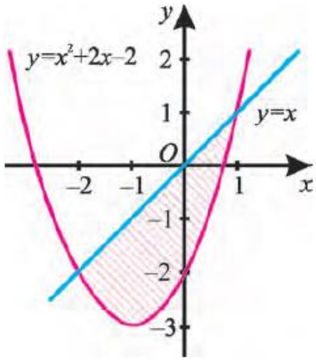

Figure 2.

So, the lines intersect at points $(1; 1)$, $(-2; -2)$. Clearly, in the interval $(-2; 1)$ the graph of the function $y = x$ lies above the graph of the function $y = x^2 + 2x - 2$.

Then $a = -2$, $b = 1$, $f_2(x) = x$, $f_1(x) = x^2 + 2x - 2$ in the formula $S = \int_a^b [f_2(x) - f_1(x)]dx = \int_a^b f_2(x)dx - \int_a^b f_1(x)dx$, according to the formula

$$S = \int_{-2}^{1} \left(x - (x^2 + 2x - 2)\right)dx =$$

$$\int_{-2}^{1} (-x^2 - x + 2)dx = \left(-\frac{x^3}{3} - \frac{x^2}{2} + 2x\right)\Big|_{-2}^{1}$$

$$= \frac{7}{6} - \left(-\frac{10}{3}\right) = 4{,}5.$$

Answer: $S = 4{,}5$ (sq. unit).

In recent years, there are a lot of questions about applying inequalities to solve some geometrical problems in mathematics competitions held at the regional, republican and even world level. It is important to study such issues. Solving problems is an important type of educational activity that develops students' theoretical and logical thinking and creative abilities. Developing the mathematical skills and creative potential of high school students includes the following: finding different situations of solving, identifying the strengths and weaknesses of each solution method in order to analyze the most rational of them. Different comparison and analysis of the same issue helps to make knowledge thorough and conscious. Solving the same problem in different ways is much more beneficial than solving similar exercises in a series. Non-standard and Olympiad problems are such problems that students do not have an algorithm to solve them, and it is necessary to independently search for the main idea. When solving non-standard geometric problems, mathematical imagination is formed, intelligence of the mind is trained, and the unity of mathematics is realized through explanation. Below we will try to solve some geometric problems using inequalities.

The problem. Prove the inequality:

$$\frac{1}{p-a}+\frac{1}{p-b}+\frac{1}{p-c}\geq 2\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$$

where a, b, c are the sides of the triangle, p is the half perimeter.

Solving: Before proving, let us establish the following inequality:

$$\frac{1}{m}+\frac{1}{n}\geq\frac{4}{m+n}$$

where $m>0$, $n>0$. Indeed, it seems $(m-n)^2\geq 0$ or $m^2-2mn+n^2\geq 0$. We change the form of this expression

$$m^2+2mn+n^2-4mn\geq 0 \quad (m+n)^2-4mn\geq 0 \text{ and } (m+n)^2\geq 4mn$$

$$\frac{m+n}{mn}\geq\frac{4}{m+n} \qquad \frac{1}{m}+\frac{1}{n}\geq\frac{4}{m+n}.$$

From these, we have the following inequality:

$$\frac{1}{p-a}+\frac{1}{p-b}\geq\frac{4}{(p-a)+(p-b)}=\frac{4}{c}.$$

It's similar

$$\frac{1}{p-b}+\frac{1}{p-c}\geq\frac{4}{a}, \qquad \frac{1}{p-a}+\frac{1}{p-c}\geq\frac{4}{b}.$$

Let's add the above one after the other,

$$\frac{2}{p-a}+\frac{2}{p-b}+\frac{2}{p-c}\geq\frac{4}{a}+\frac{4}{b}+\frac{4}{c}$$

or

$$\frac{1}{p-a}+\frac{1}{p-b}+\frac{1}{p-c}\geq 2\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right).$$

The inequality is proved.

The conclusion. Experience shows that solving geometric problems with the help of algebraic inequalities is important for the formation of deep knowledge and mathematical taste in students. It deepens students' creativity in optional lessons in classes that study mathematics in depth and helps them prepare for the Olympiads, prepares them for entrance exams to higher educational institutions where mathematics is highly demanded.

It is known that many problems encountered in school geometry and analytic geometry courses can be easily solved using vectors. Solving calculus problems using vectors is much more convenient than using a constructive approach, i.e. making additions and using elementary algebra and trigonometry tools. Effective solving of geometric problems using vectors requires not only the ability to apply the laws of vector algebra, but also the ability to convert a geometric problem into a vector "language", choose the correct method of solving the problem, and correctly plan its solution. Therefore, we believe that it is necessary to point out some basic cases.

1. If a distance or angle calculation is required, then using the scalar product of two vectors will give an efficient result.

2. There are two ways to use vectors to solve geometric problems:

a) correctly choosing the starting point for placing the given vectors;

b) depending on the drawing in the problem under consideration, to describe vectors using directed sections, and in doing so, they do not start from a single point.

3. If the issue concerns planimetry, then it is necessary to separate two non-collinear vectors and consider them as base vectors, and express the remaining vectors through them. If the problem under consideration concerns stereometry, three non-coplanar vectors are

taken as basis vectors. In these cases, it is not necessary to enter the starting point of the vector – to select it.

4. In many cases, for example, when solving problems related to polygonal angles, calculations are simplified if the unit vectors coming out from the ends of the polygonal angle are introduced.

Let's look at some problems that can be solved using vectors.

The problem. Calculate the obtuse angle formed by the intersection of the medians of the acute-angled ends of a right-angled equilateral triangle (Figure 3).

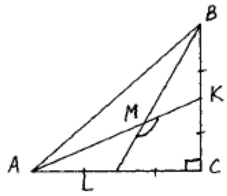

Figure 3.

Solving:

Let $\overrightarrow{CA} = \vec{a}$, $\overrightarrow{CB} = \vec{b}$. According to the condition $\vec{a} = \vec{b}$. The vector $\overrightarrow{AK}$ is equal to the difference of the vectors $\overrightarrow{CK}$ and $\overrightarrow{CA}$, i.e. $\overrightarrow{AK} = \overrightarrow{CK} - \overrightarrow{CA} = \frac{1}{2}\vec{b} - \vec{a}$ (because $\overrightarrow{CK} = \frac{1}{2}\overrightarrow{CB}$). Similarly $\overrightarrow{BL} = \frac{1}{2}\overrightarrow{CA} - \overrightarrow{CB} = \frac{1}{2}\vec{a} - \vec{b}$.

It is known that the angle between two vectors is found as

$$cos\alpha = \frac{(\overrightarrow{AK} \cdot \overrightarrow{BK})}{|\overrightarrow{AK}| \cdot |\overrightarrow{BL}|}$$

but

$$(\overrightarrow{AK} \cdot \overrightarrow{BL}) = \left(\frac{1}{2}\vec{b} - \vec{a}\right)\left(\frac{1}{2}\vec{a} - \vec{b}\right) = \frac{1}{4}|\vec{a}| \cdot |\vec{b}| \cos(\vec{a}\vec{b}) - \frac{1}{2}|\vec{b}|^2 - \frac{1}{2}|\vec{a}|^2 + |\vec{a}| \cdot |\vec{b}| \cos(\vec{a}\vec{b})$$

since

$$(\overrightarrow{AK} \cdot \overrightarrow{BL}) = -\frac{1}{2}|\vec{b}|^2 - \frac{1}{2}|\vec{a}|^2 = -\frac{1}{2}|\vec{a}|^2 - \frac{1}{2}|\vec{a}|^2 = -|\vec{a}|^2$$

will be, because $\vec{a} \perp \vec{b}$ so $\cos(\vec{a}\vec{b}) = 0$. The lengths of the vectors $\overrightarrow{AK}$ and $\overrightarrow{BL}$ are calculated according to the Pythagorean theorem.

$$\overrightarrow{BL} = \overrightarrow{AK} = \sqrt{AC^2 + CK^2} = \sqrt{|a|^2 + \frac{1}{4}|a|^2} = \frac{1}{2}|a|\sqrt{5}$$

so

$$cos\alpha = \frac{-|a|^2}{\frac{1}{4}|a|^2 \cdot 5} = -\frac{4}{5}.$$

Then we find $\alpha = \arccos\left(-\frac{4}{5}\right)$.

Answer: $\alpha = \arccos\left(-\frac{4}{5}\right)$.

Today requires us to develop new methods of education, to strengthen interdisciplinary communication, to educate young people who can think creatively and freely and in every way independently.

Since most of the algebraic problems are presented as problems of different level Olympiads in mathematics, we hope that solving algebraic problems in a geometrical way during the lesson, especially in mathematical circles, will be more effective if it is used to prepare students for mathematical Olympiads. Solving non-geometric problems in a geometric way allows students to deeply understand inter-discipline connections, forms their scientific thinking, worldview and creative work skills. We present the following types of issues:

1. Problems solved using the Cartesian coordinate system;
2. Problems solved using Pythagoras and cosine theorem;
3. Problems to be solved with the help of vectors.

Below, we will touch on each of these cases separately and provide solutions to the issues related to them.

Problems to be solved using Cartesian coordinate system

The problem. Solve the system of equations:

$$\begin{cases} x+y+z=3 \\ x^2+y^2+z^2=3 \end{cases}$$

Solving:

The plane given by the equation $x + y + z = 3$ intersects the axes of the rectangular Cartesian coordinate system at the following points (Figure 4): $A(3;0;0)$, $B(0;3;0)$, $C(0;0;3)$.

$x^2+y^2+z^2=3$ represents the equation of a sphere with center at point $O(0;0;0)$ and radius $r=\sqrt{3}$. We determine the distance from point O to the plane of triangle ABC. For this we look at the pyramid $OABC$.

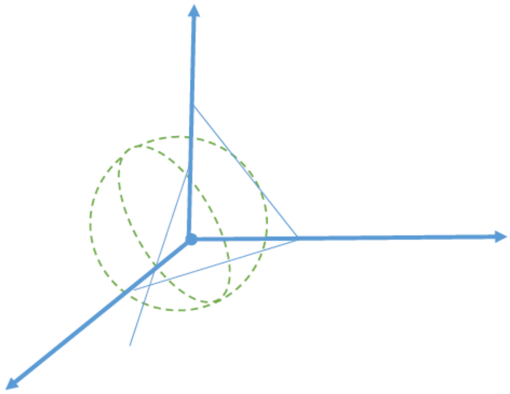

Figure 4.

It is known that the volume of the pyramid is found using the formula $V=\frac{1}{3}S_{\Delta ABC}\cdot H$. In this case, $H=OD$ (D is the center of triangle ABC). Accordingly, we find that the volume of the pyramid is

$$V=\frac{1}{3}\cdot\frac{(3\sqrt{2})^2\cdot\sqrt{3}}{4}\cdot H=\frac{3H\sqrt{3}}{2}.$$

On the other hand, the volume of this pyramid is

$$V=\frac{1}{3}S_{\Delta OAB}\cdot CO=\frac{1}{3}\cdot\frac{1}{2}\cdot 3^2\cdot 3=\frac{9}{2}.$$

From these two equations, we get the result

$$\frac{3H\sqrt{3}}{2}=\frac{9}{2},\quad H=\sqrt{3}.$$

From this we determine that the distance from point O to the plane of the triangle ABC is $OD=\sqrt{3}$, and the sphere touches the plane of the triangle ABC at the point D. Therefore, the given system of equations will have a unique root. This solution consists of the coordinates of the point $D(x;y;z)$. Since point D is the center of gravity of regular triangle ABC, $x=y=z=1$. In that case, the answer will be: $(1;1;1)$.

Problems solved using Pythagoras and cosine theorem

The problem. Solve the system of equations:

$$\begin{cases} x+y+z=60 \\ x^2+y^2=z^2 \\ \frac{xy}{z}=12 \end{cases}$$

Solving:

1) Let x,y,z be positive numbers. We can make a triangle ABC with legs x, y and hypotenuse z. The perimeter of this triangle is 60, and the height lowered to the hypotenuse is equal to 12 (Figure 5).

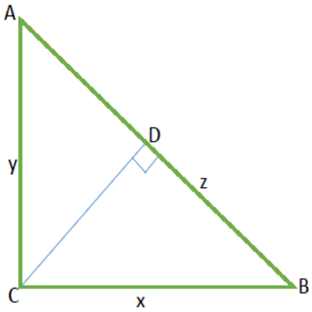

Figure 5.

From the first equation in the system $(x+y)^2=(60-z)^2$, and from the second and third equations of the system we create the equations

$$(x+y)^2=z^2+24z.$$

From the equality of the left sides of these two equations, we find that

$$(60-z)^2=z^2+24z \quad\Rightarrow\quad 144z=60^2 \quad\Rightarrow\quad z=25.$$

Then

$$\begin{cases} x+y=35 \\ xy=300 \end{cases}$$

and the values of these unknowns will be 15 and 20.

The solution of the system is (15;20;25) and (20;15;25).

2) The condition of the problem does not say anything about the sign of x,y,z. Let two of the unknowns be negative from the third equation in the system. We made sure above that $z>0$.

In that case, $x<0$ and $y<0$ must be. Since $x+y=35$, x and y cannot be negative. So the answer is: (15;20;25) and (20;15;25).

The problem. Find the smallest value of the function:

$$f(x)=\sqrt{x^2+4}+\sqrt{x^2-3\sqrt{3}x+9}.$$

Solving:

We will use the following drawing:

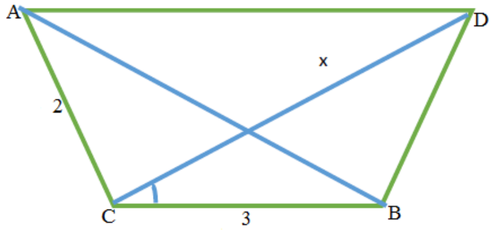

Figure 6.

According to the drawing, $AC = 2, CD = x, \angle ACD = 90°$ in the right-angled triangle ACD and $BC = 3$, $CD = x$, $\angle BCD = 30°$ in the triangle BCD are valid (Figure 6). Applying the Pythagorean theorem to triangle ACD, we get $AD = \sqrt{x^2 + 4}$, and if we apply the theorem of cosines to triangle BCD, we get the results $DB = \sqrt{x^2 - 3\sqrt{3}x + 9}$. Then the relationship

$$f(x) = \min(AD + DB) = AB$$

will be appropriate. Applying the theorem of cosines to triangle ABC, we find that

$$AB = \sqrt{2^2 + 3^2 - 2 \cdot 2 \cdot 3 \cdot cos120°} = \sqrt{19}.$$

Answer: $\sqrt{19}$.

Problems to be solved with the help of vectors

The problem. If it is known that $a + b + c = 1$ and $a \geq -\frac{1}{4}, b \geq -\frac{1}{4}, c \geq -\frac{1}{4}$, then prove the inequality $\sqrt{4a+1} + \sqrt{4b+1} + \sqrt{4c+1} \leq \sqrt{21}$.

Solving:

We solve this problem using scalar multiplication of vectors. For this, we first create 2 vectors:

$\vec{a}(\sqrt{4a+1}; \sqrt{4b+1}; \sqrt{4c+1})$ and $\vec{b}(1; 1; 1)$.

For these vectors $|\vec{a} \cdot \vec{b}| \leq |\vec{a}| \cdot |\vec{b}|$ if we apply the inequality, the following relationship will be appropriate:

$$|\vec{a} \cdot \vec{b}| = \sqrt{4a+1} + \sqrt{4b+1} + \sqrt{4c+1}$$

$$|\vec{a}| \cdot |\vec{b}| = \sqrt{\sqrt{4a+1}^2 + \sqrt{4b+1}^2 + \sqrt{4c+1}^2} \cdot \sqrt{3} = \sqrt{21}.$$

$$\sqrt{4a+1} + \sqrt{4b+1} + \sqrt{4c+1} \leq \sqrt{21}$$

according to $|\vec{a} \cdot \vec{b}| \leq |\vec{a}| \cdot |\vec{b}|$.

The inequality is proved.

Conclusion

In the process of preparing this article, I came to the following conclusions:

1. Solving algebraic problems in a geometrical way will be more effective if used during the lesson, especially in math clubs, to prepare students for math Olympiads. Solving non-geometric problems in a geometric way allows students to deeply understand inter-discipline connections, forms their scientific thinking, worldview and creative work skills.

2. Many problems encountered in a school geometry course can be easily solved using vectors. Solving calculus problems using vectors is much more convenient than using a constructive approach, i.e. making additions and using elementary algebra and trigonometry tools. Effective solving of geometric problems using vectors requires not only the ability to apply the laws of vector algebra, but also the ability to convert a geometric problem into a vector "language", choose the correct method of solving the problem, and correctly plan its solution.

3. There are two ways to use vectors to solve geometric problems:

a) correctly choosing the starting point for placing the given vectors;

b) depending on the drawing in the problem under consideration, to describe vectors using directed sections, and in doing so, do not start them from one point.

If the problem concerns planometry, then it is necessary to separate two non-collinear vectors and consider them as basis vectors, and express the remaining vectors through them. If the problem under consideration concerns stereometry, three non-coplanar vectors are taken as basis vectors. In this case, it is not necessary to enter and select the starting point of the vector.

In many cases, for example, when solving problems related to polygonal angles, calculations are simplified if unit vectors are introduced from the ends of the polygonal angle.

4. Experience shows that solving geometric problems with the help of algebraic inequalities is important for the formation of deep knowledge and mathematical taste in students. It deepens students' creativity in optional lessons in classes that study mathematics in depth and helps them prepare for the Olympiads.

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