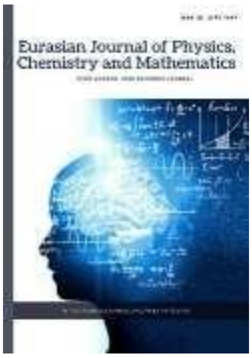

Some Result of Fuzzy Separation Axiom in Fuzzy Top Modules Spaces

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ABSTRACT

In this article, we study fuzzy separation axiom T_{F_i} ,i=0,1,2,3 in fuzzy top. module space.

Keywords: Fuzzy top. module space, Fuzzy T_{F_i} spaces,i=0,1,2,3

Introduction:

In 1965 [9], L. Zadeh established fuzziness. In 1968, C. Chang [1] studied fuzzy topology. Deb Ray, A [5, 6] studied fuzzy top. ring, fuzzy continuous function and studied left fuzzy top. ring. Basim Mohammed Melgat and Munir AL-Khafaji [3] (2019) gave some results of fuzzy separation axiom in fuzzy top. ring. In 2020 [7] Mohammed M. Ali, introduce fuzzy top. module space and fuzzy top. submodule space. In this work we study fuzzy separation axiom T_{F_i} ,i=0,1,2,3 in fuzzy top. module space and showing the relationship between them

To rich the article some notions of fuzzy set and point are giving. Let M be a set has at least one element and let J=[0,1]

Definition1.1[9]

fuzzy set A in a set M is a map $A:M\to J$ (J= [0,1]), that is, an element of J^M . let $E\in J^M$. $m\in M$, we symbolled by E(m) or m_α of the membership degree of m in E .

Definition 1.2[1]

A class $\tau_F\in J^M$ of fuzzy set is called a fuzzy top. for M if the following are hold:

$\emptyset ,M\in\tau_F$

$\forall A ,B\in\tau_F\to A\wedge B\in\tau_F$

$\forall (A_j)_{j\in J}\in\mu\to \vee_{j\in J} A_j\in\tau_F$

(M,τ_F) is called fuzzy top. space. The set E is fuzzy open if $E\in\tau_F$. and complement of E is a fuzzy closed.

Definition1.3 [7]

Let R be a ring and let M be a left R-module. A fuzzy set E in M is called a fuzzy left R-module if for each $m,n\in M$ and $r\in R$:

(1) $E(m+n) \geq \min\{E(m),E(n)\}$.

(2) $E(m) = E(m^{-1})$.

(3) $E(rm) \geq E(m)$.

(4) $E(0) = 1$.

Definition 1.4 [7]

Let R be a fuzzy top. ring, the set M is said to be left fuzzy top module on the fuzzy top ring R if:

(1) E left fuzzy module on R.

(2) E is a fuzzy top compatible with the stricture of fuzzy group on E and

satisfies the following axiom:

The mapping $R\times M\longrightarrow M$ defined by $(r,m)\longrightarrow r.m$,($r\in R$ and $m\in M$) is a fuzzy continuous

Definition1.5 [8]

A fuzzy top. space $(M,\tau_F)$is said to be fuzzy T_0-top. space iff $\forall m,n\in M, m\neq n, \exists\ U\in\tau_F$ such that either $U(m)= 1$ and $U(n)=0$ or $U(n)=1$ and $U(m)=0$.

Definition1.6 [8]

A fuzzy top. space $(M,\tau_F)$is said to be fuzzy T_1-top. space iff $\forall m,n\in M, m\neq n, \exists\ U,V\in\tau_F$ such that $U(m)= 1$, $U(n)=0$ and $V(n)=1$ and $V(m)=0$.

Proposition 1.7 [8]

A fuzzy top. space $(M,\tau_F)$is said to be fuzzy T_1- top. space iff the fuzzy point is a fuzzy closed set.

Definition 1.8 [8]

A fuzzy top. space (M,τ_F) is said to be fuzzy Hausdorff or fuzzy T_2-space iff for any two distinct fuzzy points $m,n\in M$, there exists disjoint fuzzy sets $U,V\in\tau_F$ with $U(m)=V(n)=1$

Definition 1.9 [8]

A fuzzy top. space (M,τ_F) will be called fuzzy regular if for each fuzzy point m_α and each fuzzy closed set H such that $H(m)=0$ there are fuzzy open sets U and V such that $U(m)>0$, $H\leq V$and $U\wedge V=\emptyset$.

Definition 1.10 [9]

A fuzzy top. space $(M,\tau_F)$is said to be fuzzy T_3-top. space iff it is fuzzy T_1- top. space and fuzzy regular.

Theorem 1.11[3]

For any fuzzy top. ring (R,τ_R) the following conditions are equivalent

1) (M,τ_{FM}) is fuzzy T_{F_2} top. ring space

2) $\{0_\alpha\}$ is fuzzy closed subset in M.

3) If $\{U_0\}$ is a basis of nbhd of 0_α, then $\bigcap_{V\in U_0} V=\{0_\alpha\}$

4) (M,τ_{FM}) is fuzzy T_{F_0} top. ring space

5) (M,τ_{FM}) is fuzzy T_{F_1} top. ring space.

6) (M,τ_{FM}) is fuzzy T_{F_3} top. ring space

Fuzzy top. R-module Separation Axioms

Definition 2.1

A fuzzy top. R-module space $(M,\tau_F)$is said to be fuzzy T_{F_0} top. R-module space iff $\forall m,n\in M, m\neq n, \exists\ U\in\tau_F$ such that either $U(m)=1$ and $U(n)=0$ or $U(n)=1$and $U(m)=0$.

Example 2.2

Let Z_2 (integers modulo 2) be Z-module. Define $l:Z_2\times Z\rightarrow Z_2$ by $l(n,m)=nm$ for all $n\in Z$ and $m\in Z_2$,i.e $l(n,m)=m+m+\cdots+m$ (n-times). Let a fuzzy set E_1,E_2 on Z_2 as

$E_1([0])=1, E_1([1])=0$,

$E_2([0])=0, E_2([1])=1$

for all$m\in Z_2$. Let $\tau_{FM}=\{\emptyset, Z_2\ E_1,E_2\}$ is a fuzzy top. R-module on Z_2, then (Z_2,τ_{FM}) is a fuzzy T_{F_0} top. R-module space

Definition 2.3

A fuzzy top. R-module space $(M,\tau_F)$is said to be fuzzy T_{F_1} top. R-module space iff $\forall m,n\in M, m\neq n, \exists\ U,V\in\tau_F$ such that $U(m)= 1$ and $U(n)=0$ and $U(n)=1$ and $U(m)=0$.

Example 2.4

Let Z_2 (integers modulo 2) be Z-module. Define $l:Z_2\times Z\rightarrow Z_2$ by $l(n,m)=nm$ for all $n\in Z$ and $m\in Z_2$,i.e $l(n,m)=m+m+\cdots+m$ (n-times). Let the fuzzy sets E_1,E_2 ,E_3 on Z_2as

$E_1([0])=0.25, E_1([1])=0$,

$E_2([0])=0, E_2([1])=0.25$

$E_3([0])=0.25$, $E_3([1])=0.25$

for all $m\in Z_2$. Let $\tau_{FM}=\{\emptyset,Z_2\ E_1,E_2\}$ is a fuzzy top. R-module on Z_2, then (Z_2,τ_{FM}) is a fuzzy T_{F_1} top. R-module space

Definition 2.5

A fuzzy top. R-module space (M,τ_F) is said to be fuzzy T_{F_2} top. R-module space iff for any two distinct fuzzy points $m,n\in M$, there exists disjoint fuzzy. sets $U,V\in\tau_F$ with $U(m)=V(n)=1$

Example 2.6

Let Z_4 (integers modulo 4) be Z-module. Define $l:Z_2\times Z\to Z_2$ by $l(n,m)=nm$ for all $n\in Z$ and $m\in Z_2$,i.e $l(n,m)=m+m+\cdots+m$ (n-times) with fuzzy discrete topology on it,

then (Z_4,τ_{FD}) is a fuzzy T_{F_2} top. R-module space

Definition 2.7

A fuzzy top. R-module space (M,τ_F) will be called fuzzy regular top. R-module space if for each fuzzy point $m\in M$ and each fuzzy closed set H such that $H(m)=0$ there are fuzzy open sets U and V such that $U(m)>0$, $H\le V$and $U\wedge V=\emptyset$

Definition 2.8

A fuzzy top R-module (M,τ_M) is said to be fuzzy T_{F_3} top. R-module space if (M,τ_{FM}) is fuzzy T_1 top R-module space and fuzzy regular top. R-module space.

Example 2.9

Let R be (real space) be Z-module. Define $l:R\times Z\to R$ by $l(n,r)=nr$ for all $n\in Z$ and $r\in R$,i.e $l(n,r)=r+r+\cdots+r$ (n-times) with fuzzy usual topology τ_{FU} on it. Then (R,τ_{FU}) is fuzzy T_{F_3} top. R-module space.

Theorem 2.10

For any fuzzy top. R-module space(M,τ_{FM}), the following conditions are equivalent

1) (M,τ_{FM}) is fuzzy T_{F_2} top. R-module space

2) $\{0_\alpha\}$ is fuzzy closed subset in M.

3) If $\{U_0\}$ is a basis of nbhd of 0_α, then $\bigcap_{V\in U_0} V=\{0_\alpha\}$

4) (M,τ_{FM}) is fuzzy T_{F_0} top. R-module space

5) (M,τ_{FM}) is fuzzy T_{F_1} top. R-module space.

6) (M,τ_{FM}) is fuzzy T_{F_3} top R-module space.

Proof

Is similarly to the proof of theorem 1.11

Remark 2.11

fuzzy regular top. R-module space need not to be fuzzy T_{F_2} top. R-module space. for example, the fuzzy indiscreet top R-module space is a fuzzy regular top. R-module space but it is not fuzzy T_2 top. R-module space.

Theorem 2.12

Every fuzzy subspace of fuzzy T_{F_1} top. R-module space is a fuzzy T_{F_1} top. R-module space

Proof: Clearly.

Theorem 2.13

Every fuzzy subspace of fuzzy T_i top. R-module space, i=1,2,3, is a fuzzy T_i-top. R-module space.

Proof: By using Theorem 2.10 and 2.12 we get the resulted.

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