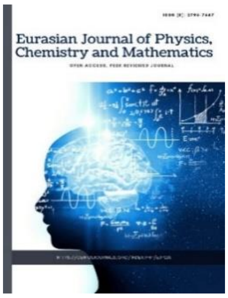

Solving linear differentiation equation system using fuzzy logic

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Abstract

In recent years, various interests in fuzzy set theory have emerged as a generalization of ideas from classical set theory. In this research, the classical mathematical equations of various types and their development were studied from the perspective of fuzzy logic, and the representation of the system of linear fuzzy equations was discussed and some methods of solving them were presented.

We have adopted a combative approach between the classical method and the fuzzy logic method in the scientific presentation of the material in this thesis in order to highlight the advantages of the general theory of the fuzzy group theory. It also promotes the theoretical aspects arising from many detailed examples. The demonstration shows that the results of fuzzy logic are consistent with the classical results, and that the fuzzy results are flexible and generalizable.

Keywords:

1. Introduction:

Differential equation is an essential axis in pure mathematics and applied mathematics, and it plays a prominent role in scientific, technical, economic and many other fields. The important feature of linear differential equations is the ease of dealing with them and the possibility of finding their analytical solutions. On the other hand, nonlinear differential equations can provide an accurate description of many natural properties that linear differential equations cannot describe[1].

Due to the importance of fuzzy kinetic systems and the emergence of the need for fuzzy differential equations (FDEs) that have functions that serve the problems of the current world in many fields, the fuzzy calculus has been developed steadily, and there have been many studies on this subject and many mathematical operations and formulas were presented to treat such as These systems, which made this topic achieve rapid development, and Professor of Mathematics at Peking University in China, Liu Baoding, was among those who worked in this field. I have studied fuzzy differential equations and developed a new formula for the difference between two fuzzy numbers that is a generalization of the Hausdorff difference. He also introduced the fuzzy stock model known as Liu's Stocks Model and defined the fuzzy process that the researcher relied on after in the definition of the Derivative of Fuzzy Process[2][3][4].

2. Goals research

1- Raising the question of the fuzzy primary value, and how to solve the linear differential equation first-placed, representing a system of fuzzy linear differential equations

2- Includes the question of the fuzzy primary value, and how to solve the linear differential equation that is second to the first place when the primary condition is a triangular fuzzy number.

3- Develop this issue when the initial requirement is a nearly skewed number. This chapter also addresses the question of representing a system of fuzzy linear differential equations.

3. Differential Equation:[5][6]

Suppose we have the following differential equation with constant coefficients of the first order:

$$\frac{dx(t)}{dt} = cx(t) + b$$

(1)

as is well known, the solution to this differential equation is as follows, see so x(0) is the initial value.

Let us now try to study the solution of the previous differential equation when the initial value $\tilde{x}(0)$ is a fuzzy number, and so on. The following theorem belongs to the researcher[1] and explains how to obtain the solution of the previous differential equation when the initial value is a triangular number, let it be $\tilde{x}(0)$ =(α, β, γ), since α, β and γ are real numbers representing the vertices of the triangle, α, β, γ. ∈R.

4. Fuzzy group:[1]

A fuzzy set is a fuzzy set whose boundaries are flexible, not Crisp, and whose boundaries cannot be defined by clearly and precisely, because the fuzzy is always there.

If X universal set, the fuzzy set A in X is the set of ordered Pairs and defined in the following form for each element x in X:

$A = \{(x, A(x)\}$, where A(x) is the membership function of the element x in A and A(x) ∈ [0,1].[1]

4.1 Membership functions

The concept of fuzzy logic is based on the fact that there is no complete affiliation of the elements in groups, or vice versa, there is a partial affiliation or affiliation

A level determined through cognitive processing, and these levels were called the term affiliation function or membership function through this function, the percentage of belonging to the group is determined, and the basic condition currently in force on this function is that, Its range lies between zero and one, and it has many forms[7][8]

1-Triangular function:

$$A(x) = \begin{cases} \dfrac{x-a}{b-a} & a \le x \le b \\ \dfrac{c-x}{c-b} & b \le x \le c \\ 0 & x \le a, c \le x \end{cases}$$

As a ,b ,c are parameters of the function represent the lists of the trigonometric function and that b represents the vertex of the triangle.

2-Function Trapezoidal:

$$A(x) = \begin{cases} \mathbf{0} & , \quad \boldsymbol{x \le a} \\ \dfrac{\boldsymbol{x-a}}{\boldsymbol{b-a}} & , \quad \boldsymbol{a \le x \le b} \\ \mathbf{1} & , \quad \boldsymbol{b \le x \le c} \\ \dfrac{\boldsymbol{d-x}}{\boldsymbol{d-c}} & , \quad \boldsymbol{c \le x \le d} \\ \mathbf{0} & , \quad \boldsymbol{d \le x} \end{cases}$$

where all of a,b,c,d are parameters of the function and a,d they represent the lower vertices of the figure while c,d They represent the heads supreme.

3- Gaussian function:

$$\boldsymbol{A(x) = e^{\frac{-(x-M)}{2}}}$$

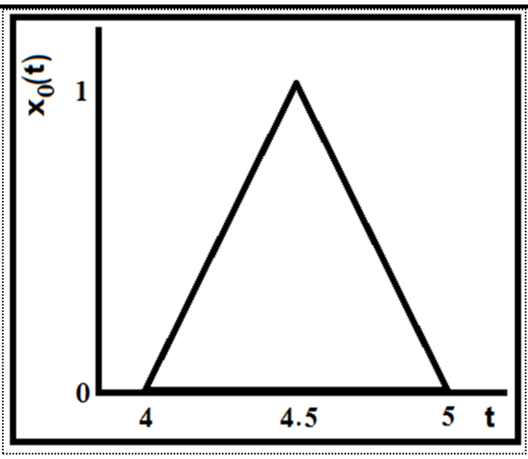
5. Solving fuzzy differential equations when the initial condition is trigonometric:[4][6]

Since a is a fixed quantity and x(0) represents the population in the base year. To find the value of the constant for the last model, we choose the base year to be the year 1947 (then the time index is t = 0). The value of the time index in 1957 is t = 1.

a = x(1)/x(0) = 1.3891.

Let the initial value of the fuzzy number $\tilde{x}(0)$ =4.5. If we assume that this number is triangular, so

$\tilde{x}(0)$ =(4,4.5,5) The affiliation function of this number is as follows:



The mathematical formula for the initial value affiliation function can easily be found to be:

$$x_0(t) = \begin{cases} 2(t-4); & t \in [4,4.5) \\ 2(5-t); i & t \in [4.5,5) \\ 0; & otherwise \end{cases}$$

Now we find the initial value

$$\tilde{x}t = a^t \tilde{x}_0; t = 1,2,...$$

$$x_1 = 1.3891 * (4,4.5,5) = (5.5564, 6.25095, 6.9455)$$

$$(\tilde{x}_2) = (1.3891)^2[(4,4.5,5)] = 1.9296(4,4.5,5) = (7.7$$

$\widetilde{x_3}$ =$(1.3891)^3$ [(4, 4.5, 5)] = 2.68041(4, 4.5, 5) = (10.7204, 12.061845, 13.40205).

$\widetilde{x_4}$ =$(1.3891)^4$ [(4, 4.5, 5)] = 3.72335(4, 4.5, 5) = (14.8934, 16.755075, 18.61675).

$\widetilde{x_5}$ =$(1.3891)^5$ [(4, 4.5, 5)] = 5.17211(4, 4.5, 5) = (20.68844, 23.27449, 25.86055).

and so…

Theorem(1)[1]

If we have the fuzzy differential equation (1), then one of the following two cases exists:

1- If c > 0 then:

$x(t) = e^{c(t-t0)} [x(0) + \int_{t0}^{t} b(k) . e^{-c(k-t0)} dk]$

It is a solution to the fuzzy differential equation (1) and x(t) is a strongly differentiable function of type (1)[9]

2- If a<0 and the –H difference is -H

$y_0 = \int_{x0}^{x} (-b(t)) . e{-}a(t{-}x0)\, dt$ exists, then:

$x(t) = e^{c(t-t0)} [x(0) + \int_{t0}^{t} -b(k) . e^{-c(k-t0)} dk]$

It is a solution to the fuzzy differential equation (1) and x(t) is a strongly differentiable function of type (2).

If the differential equation for the population is:

$$\frac{dx(t)}{dt} = cx(t).$$

The solution to the previous differential equation represents the mathematical model of the population:

$x(t) = x(0)e^{ct}$ (2)

Where as $x(0) = 4.564$ and c=0.3287

Matching (2) with the equation (3)

x'(t) = ax(t) + b(t) (3)

$x(0) = x_0$

and b(k)=0

According to the theorem(1)

$x(t) = e^{0.3287(t-t0)} [x(0) + \int_{t0}^{t}(b(k)) . e^{-c(k-t0)} dk]$

$= e^{0.3287\,t} [(4, 4.5, 5) + \int_0^t (0) . e^{-0.3287\,t} dk]$

$= e^{0.3287\,t} [(4, 4.5, 5) +1]$

$= e^{0.3287\,t} [(4, 4.5, 5) +1] = e^{0.3287\,t}[(5,5.5,6)]$

$= (5\, e^{0.3287\,t}, 5.5\, e^{0.3287\,t}, 6\, e^{0.3287\,t})$.

Example: If we have the following fuzzy differential equation:

$$\left.\begin{aligned} &x'(t) = 4x(t) + 16t \\ &x(0) = (5,6,7) \end{aligned}\right\} \quad (4)$$

We note that c = 4 >0 Therefore, according to the theorem (1) then

$x(t)= e^{4(t-t0)} [(5,6,7) + \int_{t0}^{t}(-b(k)) . e^{-4(kt-t0)} dk]$, when b(k)=0

$) . e^{-4(k-0)} dk] \int_0^t (0+ \quad = e^{4(t-0)} [(5,6,7)$

$= e^{4t}[(5,6,7) +1]$

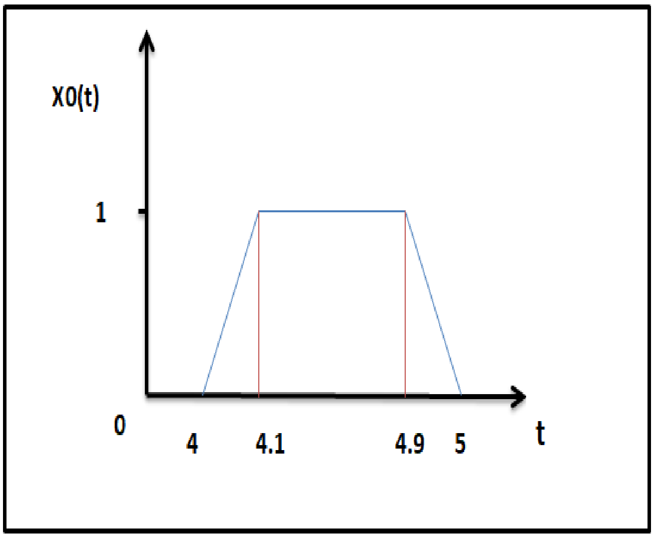
$= e^{4t}[(6,7,8)]$

$=[6\, e^{4t}, 7\, e^{4t}, 8\, e^{4t}]$.

In general, it can be concluded that if the initial value is a trigonometric number in the form $\widetilde{x_0} = (\alpha,\beta,\gamma)$

The solution to the fuzzy equation (1) is

$\tilde{x}(t) = (\alpha +1, \beta +1, \gamma +1)\, e^{0.3287\,t}$.



If the initial value is a fuzzy number of the form

$\tilde{x}(0) = e^{-\frac{1}{2\sigma^2}(x-\mu)^2}$ The solution to the differential equation for the population according to theorem(1) is as follows:

$x(t) = e^{0.3287(t-t0)} [x(0) + \int_{t0}^{t}(b(k)) \,.\, e^{-c(k-t0)}\, dk]$, when b(k)=0

$= e^{0.3287(t-0)} [e^{-\frac{1}{2\sigma^2}(t-\mu)^2} + \int_{0}^{t}(0) \,.\, e^{-c(k-0)}\, dk]$

$= e^{0.3287t} [e^{-\frac{1}{2\sigma^2}(t-\mu)^2} + 1]$

$= [e^{-\frac{0.3287t}{2\sigma^2}(t-\mu)^2}] + e^{0.3287t}$

6. Search objectives:

1. This research is dedicated to the study of fuzzy differential equations, and aims to address mathematical equations in general through the perspective of fuzzy logic.
2. Representing a system of fuzzy linear mathematical equations and showing how to solve them.
3. Resolving the fuzzy order.
4. Asking the problem of the initial fuzzy value, and how to solve a linear differential equation fuzzy from the first order fuzzy.
5. representation of a system of fuzzy linear differential equations

7. Conclusions Recommendations:

- Traditional groups enjoy extremism, while fuzzy groups enjoy transparency and ease
- This method makes fuzzy sets more flexible for representing real problems
- In fuzzy algebraic equations, changing the nature of the fuzzy numbers (even if they are of the same type) leads to a change in the result of the solution
- The solution to the fuzzy equation is of the same type as its coefficients. That is, if the coefficients of the equation are trigonometric fuzzy numbers, then the solution is also trigonometric, and if the coefficients are trapezoidal, then the solution is also trapezoidal…
- Develop fuzzy set theory to treat differential and differential equations when the variables are fuzzy (not the initial values or the coefficients).
- We recommend expanding its study and fuzzy logic in various fields of mathematics because of its importance and flexibility in dealing with it.

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