On the Self-Similar Solution of The Problem of Unsteady Movement of Groundwater Near a Reservoir in the Presence of Nonlinear Evaporation

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ABSTRACT

The problem of filtration near new channels and reservoirs, taking into account evaporation, is investigated. For one case of the dependence of evaporation on time, a self-similar solution of the problem with an unknown boundary has been found, which makes it possible to draw certain qualitative conclusions and which can be used as a test.

Keywords: self-similar solution, unsteady motion, groundwater, evaporation, nonlinear evaporation, water recovery coefficient, filtration coefficient, Boussinesq equation, unknown boundary.

The policy of intensification of agricultural production carried out in our country in accordance with the agrarian policy of the state has caused rapid development of hydraulic engineering construction. In particular, a large number of canals and reservoirs are being commissioned and built.

The construction of new channels and reservoirs radically changes the hydrogeological and reclamation conditions of coastal territories. An increase in the water horizon in hydraulic structures causes the surface of the ground flow in the territories adjacent to them and in some cases poses a threat of flooding of cities, settlements, as well as salinization and waterlogging of lands valuable for agriculture.

Due to the shallow occurrence near reservoirs, groundwater is intensively consumed for evaporation. If the soils and groundwater are saline, then the rising groundwater, dissolving the salts contained in the soils, transports them to the soil layer, which leads to its salinization.

Consequently, the study and analysis of the water regime of any territory cannot be carried out fully enough without taking into account evaporation. In this regard, the research of various mathematical models of the filtration process near new channels and reservoirs, taking into account evaporation, is undoubtedly relevant.

Consider the movement of groundwater near reservoirs, in which the water level instantly increases from the initial value h_0 $\left(h_0 < h_{kp}\right)$ up to the value $h^* = h_{kp} + h_0$, $0 < h_0 \le y_0$, $y_0 = h_m - h_{kp}$, h_{kp} – the critical level of groundwater standing, above which evaporation occurs, h_m – reservoir capacity.

Suppose the reservoir has a horizontal water barrier and there is no overflow from the underlying reservoir, and evaporation occurs from the surface of the groundwater flow, depending on the depth of groundwater and time according to the law

$$\varepsilon(h,t)=\begin{cases} 0,\ h\le h_{kp}, \\ \dfrac{\varepsilon_1(t)}{y_0{}^n}\left(h-h_{kp}\right)^n,\ h>h_{kp}, \end{cases}$$

where n – a parameter that can take values $0,1,2,3$.

By virtue of dependence $\varepsilon(h,t)$ от $h(x,t)$ the traffic area is divided into two zones with a movable interface $x=l(t)$, and in the area of $\psi_1(t)=h(0,t)>h(x,t)>h(l(t),t)=h_{kp}$ $(0<x<l(t))$ will have evaporation, and in the area of $h_0<h(x,t)\le h_{kp}$ $(x>l(t))$ be absent.

Within the limits of hydraulic theory, the groundwater level $h(x,t)$ satisfies the Boussinesq equation [1, c. 374]

$$\mu\frac{\partial h}{\partial t}=\frac{\partial}{\partial x}\left(k\cdot h\frac{\partial h}{\partial x}\right)+\varepsilon(h,t), \qquad (1)$$

where μ – water recovery coefficient (effective porosity), k – the filtration coefficient of the formation.

To simplify the study of the problem, equation (1) is usually considered in a linearized form and considered $\mu=const$, $k=const$. In filtration theory, two methods (methods) of linearization of the Boussinesq equation are known, the so-called first and second methods of linearization [1, p. 412]. In this paper, in contrast to these methods, equation (1) is linearized separately.

Let 's put:

$$a^2(x)=\begin{cases} a_1{}^2=\dfrac{k\cdot\overline{h_1}}{\mu}, 0<x<l(t); \\ a_2{}^2=\dfrac{k\cdot\overline{h_2}}{\mu}, \qquad x>l(t), \end{cases}$$

where $\overline{h_1}$ и $\overline{h_2}$ some average value $h(x,t)$ respectively from the intervals $[h_{kp}, h_m]$ и $[h_0, h_{kp}]$.

Problem statement: Find a free surface $h(x,t)$ $\left(h(x,t)=\begin{cases} h_1(x,t), & 0<x<l(t), \\ h_2(x,t), & x>l(t). \end{cases}\right)$ и curve $x=l(t)$, $l(t_0)=0$ from the following conditions:

$$\frac{\partial h_1}{\partial t}=a_1{}^2\frac{\partial^2 h_1}{\partial x^2}-\frac{\varepsilon_1(t)}{\mu\cdot y_0{}^n}\left(h-h_{kp}\right)^n,$$

$$0<x<l(t), \quad (2)$$

$$\frac{\partial h_2}{\partial t}=a_2{}^2\frac{\partial^2 h_2}{\partial x^2}, \quad l(t)<x<\infty, \qquad (3)$$

$$h_1(x,t)\big|_{x=0}=h_{kp}+h_0,\ h_1(x,t)\big|_{x=l(t)-0}=h_{kp},$$

$$t>t_0, \quad (4)$$

$$h_2(x,t)\big|_{x=l(t)+0}=h_{kp};$$

$$h_2(x,t)\big|_{x\to+\infty}=h_2(x,t)\big|_{t=t_0}=h_0, \qquad (5)$$

$$a_1{}^2\left.\frac{\partial h_1}{\partial x}\right|_{x=l(t)-0}=a_2{}^2\left.\frac{\partial h_2}{\partial x}\right|_{x=l(t)+0}, \quad t>t_0, \qquad (6)$$

where y_0 – critical depth of groundwater standing, t_0 – the time at which the water level reaches the value h_{kp}.

Let the intensity of evaporation from the soil surface $\varepsilon_1(t)$ changes by law

$$\varepsilon_1(t)=\frac{\varepsilon_0{}^*}{\mu_0\cdot(t-t_0)+1}$$

then for the time value $t>t^*$, where t^* – a sufficiently large time value; you can take $\mu_0\cdot(t-t_0)>>1$, т.е.

$$\varepsilon_1(t)=\frac{\varepsilon_0}{t-t_0}, \quad \varepsilon_0=\frac{\varepsilon_0{}^*}{\mu_0}=const. \qquad (7)$$

In the work of K.Zhamuratova and H.Umarova [2, p. 144] proves the uniqueness of the generalized solution of the problem (2) – (6), as it is known, the class of generalized solutions contains many classical solutions of the problem (2) – (6).

We show that with the law of evaporation from the soil surface (7), the problem (2) – (6) (movement) becomes self-similar.

So, assuming, $t>t^*$ let's move on to self-similar variables. Indeed, assuming

$$\xi = \frac{x}{l(t)},\ h_1 - h_{kp} = h_0 \cdot u_1(\xi),$$

$$h_2 - h_0 = u_2(\xi) \qquad (8)$$

in place (2) and (3) we have

$$a_1^2 u_1''(\xi) + l'(t)\cdot l(t)\xi \cdot u_1'(\xi) + \varepsilon_1(t)\cdot l^2(t)\frac{h_0^{n-1}}{\mu \cdot y_0^n} u_1^n(\xi) = 0$$

(9)

$$a_2^2 u_2''(\xi) + l'(t)\cdot l(t)\xi \cdot u_2'(\xi) = 0. \qquad (10)$$

Obviously, in order for the movement to be self-similar, the following conditions must be met

$$l'(t)\cdot l(t) = const\ ,\ \ \varepsilon_1(t)\cdot l^2(t) = const.$$

(11)

Hence it is clear that when $l(t_0) = 0$, $l(t)$ you need to search in the form

$$l(t) = \alpha \cdot \sqrt{t - t_0}\ , \qquad (12)$$

where α – some constant.

If formula (7) holds for $\mu_0 \cdot (t^* - t_0) >> 1$, the equality (11) and (12) will simultaneously be valid for $t > t^*$.

Taking into account (8), (12), the problem (2) – (6) will take the form without a change:

$$u_1''(\xi) + \frac{\alpha^2}{2a_1^2}\cdot \xi \cdot u_1'(\xi) - b_n \cdot \alpha^2 u_1^n(\xi) = 0,$$

$$\xi \in (0,1),\ (13)$$

$$u_1(0) = 1, \quad u_1(1) = 0. \qquad (14)$$

$$u_2''(\xi) + \frac{\alpha^2}{2a_2^2}\cdot \xi \cdot u_2'(\xi) = 0, \quad 1 < \xi < +\infty,$$

(15)

$$h_0 \cdot a_1^2 u_1'(\xi)\Big|_{\xi=1-0} = a_2^2 u_2'(\xi)\Big|_{\xi=1+0} \qquad (16)$$

$$u_2\Big|_{\xi=1+0} = h_{kp} - h_0 = \psi_0\ ,\quad u_2\Big|_{\xi\to+\infty} = 0\ ,$$

(17)

where $b_n = \varepsilon_0 h_0^{n-1} \Big/ \mu a_1^2 y_0^n$.

The solution of the boundary value problem (15) – (17) is sought in the form

$$u_2(\xi) = B_2 + A_2 \int_1^{\xi} e^{-\frac{\alpha^2\lambda^2}{4a_2^2}} d\lambda.$$

Satisfying condition (17) we find

$$B_2 = \psi_0,\quad A_2 = -\psi_0 \Big/ \sqrt{\pi} a_2 erfc(\alpha/2a_2).$$

Then the solution of the problem (15) – (17) has the form

$$u_2(\xi) = \psi_0\left(1 - \frac{erf(\alpha/2a_2 \cdot \xi) - erf(\alpha/2a_2)}{erfc(\alpha/2a_2)}\right),$$

(18)

where $erf(z) = \frac{2}{\sqrt{\pi}}\int_0^z e^{-\alpha^2} d\alpha$, $erfc(z) = 1 - erf(z)$.

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