Optimal stability of linear stochastic systems in the presence of information constraints

Asaad Naser Hussein Mzedawee
Asaad.nasir@qu.edu.iq
University of Al-Qadisiyah – College of Administration & Economics, Iraq

ABSTRACT

This paper is devoted to the problem of optimal stability of a semi-linear stochastic system with controlled parameters. Systems of this type are described by linear random differential equations with multiplicative perturbations, the matrices of which are generally nonlinear and control-dependent. The quality standard is a modification of the classical quadratic management quality standard. The task is to reduce the norm on the set of acceptable control operations. This formulation of the problem is interesting because it allows us to study a wide range of optimization issues of linear systems with multiplicative perturbations, including: optimization of system design parameters, problems of optimal stabilization in the presence of constraints on the coefficients of the linear regulator in the form of inequalities, problems of optimal stabilization of linear stochastic systems in the presence of information constraints. The main result of the work is the necessary conditions for optimizing the parameter vector in the problem of optimal stability of a semi-linear random system with controlled parameters. A gradient-type procedure is proposed for the synthesis of the optimal parameter stabilization vector. In addition, based on the optimal conditions obtained, an algorithm is created for the synthesis of suboptimal program control in the problem under consideration. The result of the proposed algorithm is a continuous multi-definition control, which gives the value of the criterion guaranteed to be no worse than the optimal stabilization vector. The resulting algorithm is simple and allows calculation in real time. The obtained results are applied to the problem of optimal stability with information constraints, where the necessary optimal conditions are also obtained and a procedure for the synthesis of Gradient-Type control is proposed. The use of the results obtained is illustrated by a model example.

Keywords: Continuous stochastic systems, optimal stabilization, continuous stochastic systems, optimal stabilization, information constraints)

1- Introduction

The most important scientific approach to studying and constructing the largest variety of objects and phenomena in the real world is mathematical modeling. The essence of this methodology is the construction of mathematical models of the objects or phenomena under consideration and their further study [1]. A common example of a mathematical model is a dynamical system [2]. Dynamic systems act as a mathematical description of a wide variety of objects and phenomena that develop over time in the real world. Controlled dynamic systems are understood as dynamic systems that have controlled parameters. These parameters simulate the magnitude of control disturbances affecting the simulated object. The law by which the values of the controlled parameters are determined at each moment of time is called the control or control function. A controlled dynamic system with control is a mathematical model of a control system [3]. Control systems are the main object of study of control theory, which plays a key role in such areas as the management of industrial and chemical processes, reactors, main Power Systems, Aerospace Engineering, and The Theory of quantum systems and the theory of computer systems [4]. The control function's values can be chosen based on specific parameters from the mathematical model. These parameters determine the minimum requirements for certain predefined functions. The quality of control achieved through this function is referred to as functional control quality, and tasks aiming to optimize control are known as optimal control tasks [3]. In practice, there are often situations [7] when it is not possible to accurately determine any parameters of the control object at the modeling stage. For example, such a situation arises if the control system cannot be actually implemented with the required accuracy, or if it is necessary to design a control device for an already existing object, and its exact parameters cannot be determined. Therefore, the solution of the problem of synthesis of control of dynamic systems in the presence of uncertainties is of particular importance [4]. An example of this type of uncertainty is the uncertainty of uncontrollable external disturbances that affect the control object. Although there is no complete information about the existing perturbations in the system, sometimes there is a priori auxiliary information about their properties, for example: perturbations are limited in magnitude by some constant, are continuous functions of time, described by some ordinary differential equation [5], etc. Examples of dynamical systems in which nonspecific external disturbances are random are well known [6]. It is natural to think that the more information about the characteristics of uncontrolled external influences is taken into account in modeling, the better the result of mathematical modeling. Among dynamical systems with continuous trajectories, there is a rich subclass of mathematical models described by ordinary differential equations. This allows you to apply. In the problems of analysis and synthesis of controls for mathematical models of this type, there is a rich apparatus of the theory of ordinary differential equations. However, within the framework of the theory of ordinary differential equations, in general, a strict mathematical description of continuous dynamical systems in the presence of random perturbations cannot be given [7, 8]. K. I managed to overcome this problem. It is ERMO, who in his works [8-10] introduced and justified the concept of stochastic integration and the related concept of stochastic differential. The construction of stochastic integration allowed us to describe the evolution of dynamical systems, which can be formally represented as ordinary differential equations containing random differentials of stochastic processes. Such relations are called stochastic differential equations, and the above dynamical systems are called stochastic. The models described by stochastic differential equations have found wide application in economics, physics, biology, sociology, aviation, rocket and Space Technology [4, 11, 12]. At the same time, various problems of control synthesis of systems of this type and, in particular, installation tasks are of normal importance.

2- Formulation of the optimal stability problem for nonlinear stochastic systems with controlled parameters

Semi-linear stochastic systems with controlled parameters are described by stochastic differential equations of the following model

$$d_x(t) = A^{(0)}\big(u(t)\big)x(t)dt + \sum_{i=1}^{b} A^{(i)}\big(u(t)\big)x(t)d_{\beta i}(t) \quad , \; x(0) = x_0 \quad (1)$$

Where $t \geq 0$ is time ; x A random operation with values in R^n ; β The standard Wiener process with values in R^b ; $u: R_+ \to R^m$ Software management ; $A^{(i)}: R^m \to R^{n\times n}$, $i = 0, b$ Continuously differentiable Matrix-valued functions on R^m ; x_0 A random vector that does not depend on $\beta(t), t \geq 0$ and Satisfies the condition $E\|x_0\| < +\infty$.

Denoted by D_{x_0} The set of valid control operations $z = (x, u)$, which are pairs of random operations x and control functions u, such as

1. The function U is bounded by a continuous defined manifold in every finite time interval.
2. A continuous stochastic process x is the solution of equation (1) with the given x_0 and .
3. The condition is met

$$E\int_0^{+\infty}\|x(s)\|^2 ds < +\infty \qquad (2)$$

Definition 1. Management , in which there is an acceptable management process $z=(x,u)\in D_{x_0}$, we will call acceptable.

Definition 2 . Management , We will call it a stabilizer if it is valid for any x_0 , $E\|x_0\|^2 < +\infty$

Note 1. The u -control stabilizes if and only if the system is closed in u is asymptotically stable in the mean square. On a set x_0 we define a functional $J: D_{x_0}\to R$

$$J(z) = E\int_0^{+\infty} x(s)^T L\big(u(s)\big)x(s)ds \qquad (3)$$

Where $L: R^m \to S^n$ - Is differentiable R^m by a matrix-valued function such that $L(v)\geq 0$, $v\in R^m$.

The task is to find the management process $\bar{z}=(\bar{x},\bar{u})\in D_{x_0}$ Which reduces the criterion (1.3) for example D_{x_0}

$$J(\bar{z}) = \min_{z\in D_{x_0}} J(z) \qquad (4)$$

3- LaGrange-Krutov functional

Following the Lyapunov-LaGrange method [12], we build an additional control quality function for this task. To do this, we fix some control operations $z=(x,u)\in D_{x_0}$. It is known [4, theorem 4.2.1] that for every function $(t,y)\to\varphi(t,y): R_+\times R^n\to R$, the presence of continuous partial derivatives $\frac{\partial\varphi}{\partial t}$, $\frac{\partial^2\varphi}{\partial y_i \partial y_j}$, $i,j=1,\ldots\ldots n$, the formula is correct

$$\varphi(t) = \varphi(0,x_0) + \int_0^t\Big(\frac{\partial\varphi}{\partial t}\big(s,x(s)\big) + \nabla_y\varphi\big(s,x(s)\big)^T A^{(0)}\big(u(s)\big)x(s)$$

$$+ \frac{1}{2}\sum_{i=1}^b x(s)^T A^{(i)}\big(u(t)\big)^T H_x^{\varphi}\big(s,x(s)\big)A^{(i)}\big(u(s)x(s)\big)ds$$

$$+\sum_{i=0}^b\int_0^t \nabla_y\varphi\big(s,x(s)\big)^T A^{(i)}\big(u(s)\big)x(s)\,d\beta_i(s) \quad t\geq 0 \qquad (5)$$

Where $\nabla_y\varphi(t,y)$ - the gradient of the function , $\varphi(t,.)$, $\nabla_y\varphi := \left(\frac{\partial\varphi}{\partial y_1},\ldots\ldots\ldots\ldots,\frac{\partial\varphi}{\partial y_n}\right)^T$; $H_y^{\varphi}(t,.)$ - the Hess matrix of the function , $\varphi(t,y)$, $\left(H_y^{\varphi}\right)_{ij} = \frac{\partial^2\varphi}{\partial y_i\partial y_j}$, $i,j=1,\ldots\ldots n$ − applying this formula to $\varphi(t,y)=y^T M_y$ $(t,y)\in R_+\times R^n$. Where $M\in S^n$, We get the Equality

$$x(t)^T M_x(t) = x_0^T M_{x_0} + \int_0^t x(s)^T \left(MA^{(0)}\big(u(s)\big) + A^{(0)}(u(s))^T \mathrm{M} + \sum_{i=1}^b A^{(i)}u(s)^T MA^{(i)}\big(u(s))x(s)\big)ds\right.$$

$$+2\sum_{i=0}^b\int_0^t x(s)^T MA^{(i)}\big(u(s)\big)x(s)\,d\beta_i(s) \ , t\geq 0$$

Let's take the mathematical expectation from the left and right sides of this equality. Then, taking into account the properties of the stochastic integral of [4, theorem 3.2.1], we will have

$$E(x(t)^T M_x(t)) = E(x_0^T M_{x_0}) + E\int_0^t x(s)^T \left(MA^{(0)}\big(u(s)\big) + A^{(0)}(u(s))^T \mathrm{M} + \sum_{i=1}^b A^{(i)}u(s)^T MA^{(i)}\big(u(s))x(s)\big)ds\right. \ , \ t\geq 0$$

Aiming t at Infinity, taking into account (1.2), we get

$$E(x_0^T M_{x_0}) + E\int_0^t x(s)^T \left(MA^{(0)}\big(u(s)\big) + A^{(0)}(u(s))^T \mathrm{M} + \sum_{i=1}^b A^{(i)}u(s)^T MA^{(i)}\big(u(s))x(s)\big)ds\right. = 0 \qquad (6)$$

Now let's take a look at the auxiliary management quality function $G: D_{x_0}\to R$

$$G(z) := E(x_0^T M_{x_0}) + E\int_0^t x(s)^T \left(MA^{(0)}\big(u(s)\big) + A^{(0)}(u(s))^T \mathrm{M} + \sum_{i=1}^b A^{(i)}u(s)^T MA^{(i)}\big(u(s))x(s)\big)ds\right.$$

It is easy to see that by virtue of equality (6) and the arbitrariness of the choice of the control operation Z , the following important property is satisfied

$$G(z)\equiv J(z) \quad , \quad z\in D_{x_0} \qquad (7)$$

Which does not depend on the choice of the Matrix $M \in S^n$. Let 's introduce the mapping into consideration $H: R^m \times R^{n\times n} \to R^{n\times n}$

$H(v, M) := MA^{(0)}(v) + A^{(0)}(v)^T M$
$+ \sum_{i=1}^{b} A^{(i)}(v)^T MA^{(i)}(v)) + L(v)$
(8)

With the help of H -mapping, it is possible to rewrite the functional G in a more compact form

$G(z) = tr[MP_0] +$
$E \int_0^{+\infty} x(s)^T H(u(s), M)x(s)ds$
(9)

Where $P_0 \in R^{n\times n}$ - The matrix of the first-second moments of the vector x_0

4- Stability of vector parameters

In this section and the next, continuous time management strategies will be considered $u(t) \equiv v \in R^n$. At the same time, we will determine the vector of parameters t and the control strategy corresponding to this vector $u(t) \equiv v$ and writing $z = (x, u) \in D_{x_0}$. If the program control corresponding to the vector is valid or stable, then we will also call the vector acceptable or stable, respectively. We denote the set of all stabilization vectors $V \in R^m$, and the set of corresponding control operations $D_{x_0}^V$,

$D_{x_0}^V := \{(x, v): (x, v) \in D_{x_0}, v \in V\}$

Later we will need the following result concerning stabilizing vectors.

Lemma 1. If the vector v is stabilizing, then there is a non-negatively defined matrix $M \in S^n$, which is the only solution to the equation

$MA^{(0)}(v) + A^{(0)}(v)^T M$
$+ \sum_{i=1}^{b} A^{(i)}(v)^T MA^{(i)}(v)) = -L(v)$
(10)

The linear Matrix equation (10) is called the generalized Lyapunov equation, and it plays a key role in the analysis of the stability of equation (1). A detailed study of the properties of this equation is available in the monograph [14]. The proof of Lima 1 is given in [122, P. 68]. A direct consequence of this Lemma is equality (7), (8), (9) is the following statement.

Statement 1. Let there be a management process $z = (x, u) \in D_{x_0}$. The value of the criterion $J(z)$ can be calculated by the formula

$J(z) = tr[MP_0]$,
(11)

Where is a matrix that is not negatively defined $M \in S^n$, - The only solution of the equation (10).

Now we show that in the group $D_{x_0}^V$ functional G can be represented as a function $\hat{G}$ from the variable v is differentiable by . Let there be a management process $= (x, u) \in D_{x_0}$. It is known (See, for example, [14, P.9]) that the matrix of the second initial moments $P(t)$ of a random variable $x(t)$ is described by a linear ordinary matrix differential equation.

$\dot{P} = A^{(0)}(u(t))P(t) + P(t)A^{(0)}(u(t))^T$
$+ \sum_{i=1}^{b} A^{(i)}(u(t))P(t)A^{(i)}(u(t))^T$
$t \geq 0, P(0) = P_0$ (12)

Suppose that $(t) \equiv v \in R^m$. Let's integrate equation (12) on the interval $[0, +\infty)$. At the same time, we will take into account that as follows from (2), limit $\|P(t)\|$ at $t \to +\infty$ equal to zero. We obtain a linear Matrix equation

$-P_0 = \hat{P}A^{(0)}(v) + \hat{P}A^{(0)}v)^T$
$+ \sum_{i=1}^{b} A^{(i)}(v)\hat{P}A^{(i)}(v)^T$
(13)

Where $\hat{P} := \int_0^{+\infty} P(s)ds$.

Using the symmetric Kronecker product and the symmetric vectorization operator, equations (12) and (13) can be rewritten as linear vector equations with respect to $svec$ $[P(t)]$ and svec $[\hat{P}]$

$svec\,[P\hat{}(t)]^0 = \omega(v)svec[P(t)]$, $t \geq 0$
(14)

$svec[P(0)] = svec[P_0]$
$\omega(v)svec[\hat{P}] =$ - $svec[P_0]$
(15)

$\omega(v) := [2A^{(0)}(v) \times I + \sum_{i=0}^{b} A^{(i)}(v)A^{(i)}(v)$

Where $I \in S^n$ - the unit matrix. The following statements are true.

Statement 2. The vector v is stabilizing if and only if the real parts of the eigenvalues of the matrix $\omega(v)$ are strictly less than zero.

Proof : As noted in Lemma 1, the vector v is stabilizing if and only if the closed system is asymptotically stable in the mean square. It is known (see, for example, [14, pp. 11-13]) that the system (1) is asymptotically stable in the mean quadratic if and only if the matrix differential equation (12) or the equivalent system of linear differential equations (14) is asymptotically stable. It is well known from the stability theory of deterministic systems that a linear system with constant coefficients (14) is asymptotically stable if and only if the spectrum of the matrix of the system $\omega(v)$ belongs to the left open half-plane.

Statement 3. The set of V is open.

Proof : From the properties of the symmetric Kronecker product and the differentiability of maps $A^{(i)}, i = 1, \dots\dots.., m$ by v on R^m it follows, that the mapping $\omega: R^m \to R^{\frac{n^2+n}{2} \times \frac{n^2+n}{2}}$ is also differentiable by v on R^m . In particular, it is continuous. It follows from statement 2 that when mapping ω, the set V is a complete prototype of the set of asymptotically stable matrices, which is open .Thus, the set V, as a prototype of an open set under continuous mapping, is open.

Statement 4. If $v \in V$ then there is a matrix that is not negatively defined $\hat{P} \in S^n$, which is the only solution to equation (13). In this case, $\hat{P}$ can be considered as a differentiable function of v.

Proof: Consider the equivalent (13) linear vector equation with constant coefficients (15). From statement 2 it follows that the matrix of the system is non-degenerate and, therefore, there is a unique solution to this equation

$$svec[\hat{P}] = -\omega(v)^{-1} svec[P_0] \quad (16)$$

The obtained Matrix $\hat{P}$ It will satisfy equation (13) and the construction is non negatively indeterminate. Since in which $v \in V$ the corresponding matrix is defined $\hat{P}$, we will further assume that there is a mapping $\hat{P}: V \to R^{n\times n}$, determined by equality (16). Moreover, the function $\hat{P}$ is a differentiable function on R^m , the values of the partial derivatives $\frac{\partial}{\partial v_i}\hat{P}(v)$, $i = 1, \dots\dots.., m$,they can be found from equality (13) or (15) as derivatives of an implicit function. Differentiating , for example, (15) by vi and we get that

$$\left[\frac{\partial}{\partial v_i} A(v)\right] \left[svec\left[\hat{P}(v)\right] + \omega(v) svec\left[\frac{\partial}{\partial v_i}\hat{P}(v)\right]\right] = 0$$

From where

$$svec\left[\frac{\partial}{\partial v_i}\left[\hat{P}(v)\right]\right] = -\omega(v)^{-1}\frac{\partial}{\partial v_i}\omega(v) svec\left[\hat{P}(v)\right] = -\omega(v)^{-1}\frac{\partial}{\partial v_i}\omega(v)\omega(v)^{-1} svec[P_0] \quad (17)$$

In the problem statement it is indicated that the assignments $A^{(i)}, i = 1, \dots\dots.., b$, is continuously differentiable by R^m , and from equality (17) it follows that the partial derivatives $\frac{\partial}{\partial v_i}\hat{P}(v)$, $i = 1, \dots\dots.., m$ are also continuous functions on the set V. Thus, sufficient conditions for differential mapping are satisfied $\hat{P}$ on the set .

Let's $M \in S^m$ - Some Matrix. Using the Integral of the matrix of the second moments $\hat{P}(v)$, we can offer the function G on the set $D_{x_0}^V$ in the following form

$$G(z) = tr[MP_0] + E\int_0^{+\infty} x(s)^T H(v, M) x(s) ds$$
$$= tr[MP_0] + \int_0^{+\infty} Etr[H(v, M) x(s) x(s)^T] ds$$
$$= tr[MP_0] + tr[H(v, M)\int_0^{+\infty} P(s) ds]$$
$$= tr[MP_0] + tr\left[H(v, M)\hat{P}(v)\right] =: \hat{G}(v), \quad z = (x, v)$$

Thus, on the set $D_{x_0}^V$ functional , let's imagine in the form of a function $\hat{G}: R^m \to R$, differentiated by v on V. Also note that if v is a valid vector, then the value $\hat{G}(v)$ it does not depend on the choice of the Matrix $\in S^m$.

5- Necessary conditions for optimizing the stability vector of parameters

The following necessary conditions are obtained in the task of finding the control process $\bar{z} = (\bar{x}, \bar{v}) \in D_{x_0}^V$, which reduces the criterion (3) to a narrow range of acceptable control operations $D_{x_0}^V$,

$$J(\bar{z}) = \min_{z \in D_{x_0}^V} J(z) \quad (18)$$

Theory 1. If the control process $z = (x, u) \in D_{x_0}^V$ optimal set $D_{x_0}^V$ in the equations (1)–(3), (18), then the following .The conditions are met

$$tr[(M\frac{\partial}{\partial v_i}A^{(0)}(v) + \sum_{j=1}^{b}A^{(j)}(v)^T M\frac{\partial}{\partial v_i}A^{(j)}(v)) + \frac{1}{2}L(v)\bar{P}] = 0 \quad , \quad i = 1, \dots\dots, m \qquad (19)$$

Where are the negatively indeterminate matrices $M \in S^n$ and $\bar{P} \in S^n$ - The only solutions of equations

$$MA^{(0)}(v) + A^{(0)}(v)^T\mathrm{M} + \sum_{i=1}^{b}A^{(i)}(v)^T MA^{(i)}(v)) = -L(v)$$
$$\bar{P}A^{(0)}(v)^T + \bar{P}A^{(0)}(v) + \sum_{i=1}^{b}A^{(i)}(v)\bar{P}A^{(i)}(v)^T = -P_0 \qquad (20)$$

Proof : There should be no valid control operation $z = (x, u) \in D_{x_0}^V$, from the lemma 1 implies that there is a unique non-negatively defined matrix $M \in S^n$satisfying the first equality in (20). Let's fix this matrix . By definition, the mapping H will be executed

$$H(v, M) = 0 \qquad (21)$$

It was previously shown that the functional G on the set $D_{x_0}^V$ it can be represented as differentiable by V functions $\hat{G}$. Taking into account (21), we obtain the values of the partial derivatives of the function $\hat{G}$ at this point $v \in V$

$$\frac{\partial}{\partial v_i}\hat{G}(v) = \frac{\partial}{\partial v_i}tr[H(v, M)\hat{P}(v)] = tr[\hat{P}(v)\frac{\partial}{\partial v_i}H(v, M) + H(v, M)\frac{\partial}{\partial v_i}\hat{P}(v)] = tr[\hat{P}(v)\frac{\partial}{\partial v_i}H(v, M)] \qquad (22)$$
$$= 2tr[(M\frac{\partial}{\partial v_i}A^{(0)}(v) + \sum_{j=1}^{b}A^{(j)}(v)^T M\frac{\partial}{\partial v_i}A^{(j)}(v)) + \frac{1}{2}\frac{\partial}{\partial v_i}L(v)\hat{P}(v)] \quad , i = 1, \dots, m$$

Where $\hat{G}(v)$ by definition, it is a decision on $\bar{P}$ the second equation of (20).

Management process $z = (x, u) \in D_{x_0}^V$ is the minimum functional point G on the set $D_{x_0}^V$ if and only if v the minimum point of the function $\hat{G}$ on the set . Considering that $\hat{G}$ is differentiable by V, and V is open, the necessary conditions for a first-order vector v are satisfied, namely

$$\frac{\partial}{\partial v_i}\hat{P}(v) = 0 \quad , i = 1, \dots\dots\dots, m$$

Submit an appointment $\bar{P} = \hat{P}(v)$, in total (22) we get the condition at (19).

6- Numerical method for the synthesis of optimal stability vectors

Expression (22) for partial derivatives of the function $\hat{G}$ can be applied to construct the following gradient-type procedure. Let there be a management process $z^{(l)} = (x^{(l)}, u^{(l)}) \in D_{x_0}^V$. Then we can improve the functional quality value by choosing a new vector $v^{(l+1)}$ according to the following formula

$$v_i^{(l+1)} = v_i^{(l)} - \theta\frac{\partial}{\partial v_i}\hat{G}(v^{(l)}), \quad i = 1, \dots\dots m \qquad (23)$$

Where $\theta > 0$– a small enough step.

Based on the procedure (23), the following Gradient-Type algorithm is proposed for synthesizing the optimal parameter stabilization vector:

Step 1. Collection $\theta > 0$ -step by step gradient method , $\varepsilon_1 > 0$ and $\varepsilon_2 > 0$ – required approximation errors , $v^{(0)}$ – initial approximation (stabilizing vector), and put the iteration number $k = 0$, number of successful iterations $i = 0$.

Step 2. Numerically solve the system of matrix equations (20) by putting in them $v = v^{(k)}$.

Step 3. Numerically check that the resulting vector is stabilizing. If $k = 0$ and vector $v^{(k)}$is not stabilizing, then finish the calculation. If $k > 0$ and vector $v^{(k)}$ is not stabilizing, then put $i = 0$, reduce θ by half , reduce k by one and go to step 7 .

Step 4. Calculate the value $J(z^{(k)})$ according to the formula (11). If = 0 , then go to step 6. If $J(z^{(k)}) > J(z^{(k-1)})$,then put $i = 0$, reduce θ by half , reduce k by one and go to step 7 . Otherwise go to step 5.

Step 5. If = 2 , double θ, put $i = 0$.

Step 6. Calculate $\frac{\partial \hat{G}(v^k)}{\partial v_i}$, $i = 1, \dots\dots m$ according to the formula (22) .

Step 7. Calculate the value $\|\nabla\hat{G}(v^{(k)})\|$ and check the fulfillment of the conditions

$\|\nabla\hat{G}(v^{(k)})\| < \varepsilon_1$, $\theta < \varepsilon_2$: if any of the conditions are met, the desired value $\bar{v}$ put equal $v^{(k)}$ and finish the calculation, otherwise calculate $v^{(k+1)}$by the formula (23) and go to step 7.

Step 8. Increase k by one and go to step 2.

7- Suboptimal software management

Finding the optimal program control in the problem (1)– (4) involves a number of difficulties, and the exact numerical algorithm for its solution is not known. Therefore, approximate algorithms for the synthesis of suboptimal software controls are attractive. Note the fact that the conditions obtained above in Theorem 1 contain the matrix $P_0$the second initial moments of the vector x_0 . This fact leads to the idea of an algorithm for the synthesis of piecewise constant program control based on the recurrent calculation of a stabilizing vector satisfying the conditions of Theorem 1. An example of such an algorithm is the following :

Step 1. Set the split $0 = t_0 < t_1 < \dots\dots\dots\dots < t_q < +\infty$ interval $[0, +\infty)$; $v^{(0)}$ -initial approximation (stabilizing vector satisfying the conditions of Theorem 1); $\varepsilon > 0$ - the parameter of the algorithm responsible for the stop condition. Put the iteration number $k = 0$.

Step 2. Calculate $P(t_{k+1})$ solving the Cauchy problem (12) on the time interval $[t_k, t_{k+1}]$ for $u(t) \equiv v^{(k)}$ with an initial condition $P(t_k)$, $P(t_0) = P_0$.

Step 3. Using the above procedure, calculate the stabilizing vector $v^{(k+1)}$satisfying the equations of theorem 1 for $P_0 = P(t_{k+1})$. For the initial approximation of the algorithm , take $v^{(k)}$.

Step 4. Check the fulfillment of the conditions $\|P(t_{k+1})\| < \varepsilon$ and $k + 1 < q$. If not done, then increase k by one and proceed to step 2. Otherwise, put the desired strategy $u(t)$ equal to a function of the form

$$u(t) = \begin{cases} v^{(0)} & if \quad 0 \le t < t_1 \\ v^{(1)} & if \quad t_1 \le t < t_2 \\ \dots\dots.. & \\ v^{(k+1)} & if \quad t_{k+1} \le t < +\infty \end{cases}$$

A useful property of this algorithm is that it is non-degrading, that is, whatever the partition of the time interval, the value of the criterion corresponding to the found control u will be no worse than the corresponding vector $v^{(0)}$. In addition, this algorithm is characterized by the relative simplicity of the calculations performed and is recurrent, that is, it is possible to perform calculations in real time. Among the disadvantages, it is worth noting that the result of the work can significantly depend on the choice of splitting the time interval. At the same time, the question of choosing the best partition remains open.

8- Model example

We will demonstrate the application of the obtained results on a model example. Let there be the following optimal stabilization problem

$$\begin{cases} dx_1(t) = 2x_2(t)dt + \frac{1}{2}u(t)x_1 d\beta(t), \\ dx_2(t) = -(x_1(t) + u(t)x_2)dt \ , \end{cases}$$

$$J(\bar{z}) = E\int_0^{+\infty}(1 + u(s)^2 x_1^2(s) + x_2^2(s))ds$$

$$J(\bar{z}) = \min_{z \in D_{x_0}^{v}} J(z)$$

This problem is a special case of problem (1)–(4). If we limit ourselves to considering only permanent management strategies $u(t) \equiv v \in R$ then we get the task (1)–(3),(18), for which, in section 5 , optimality conditions and a gradient procedure for the synthesis of an optimal regulator are obtained. The constant control u found using the numerical method satisfying the conditions of Theorem 1 and the corresponding value of the criterion are equal to

$$u \approx 0.598 \qquad , \qquad J = 5.997$$

Using the algorithm for the synthesis of optimal program control proposed in Section 7, the program piecewise constant control function is found for the switchover at the time $t_1 = 1.3950$ and the value of the criterion

$$u(t) = \begin{cases} 0.598 & 0 \le t < t_1 \\ 1.195 & t_1 \le t < +\infty \end{cases} \quad J \approx 5.682$$

Thus, even using a fixed multi-definition control strategy with one key, it was possible to significantly improve the value of the criterion.

9- Results

The optimal stability problem for nonlinear stochastic systems with controlled parameters is formulated and the following results are obtained:

1) The necessary optimal conditions are obtained and a numerical method is proposed for synthesizing the stabilization vector in the problem of optimal stabilization of semi-linear stochastic systems with controlled parameters.

2) A numerical method for synthesizing optimal program control was proposed in the problem of optimal installation of semi-linear stochastic systems with controlled parameters.

Reference

1. Klamka J. Controllability of dynamical systems. A survey // Bulletin of the Polish Academy of Sciences: Technical Sciences.–– 2013.–– Vol. 61, no. 2.–– P. 335–342
2. Bakule L. Decentralized control: An overview // Annual reviews in control.–– 2008.–– Vol. 32, no. 1.–– P. 87–98.
3. Hofer E., Sagirow P. Optimal systems depending on parameters // AIAA Journal.–– 1968.–– Vol. 6, no. 5.–– P. 953–956.
4. Dorato P. A historical review of robust control // IEEE Control Systems Magazine.–– 1987.–– Vol. 7, no. 2.–– P. 44–47.
5. Johnson C. Further study of the linear regulator with disturbances – The case of vector disturbances satisfying a linear differential equation //
6. IEEE Transactions on Automatic Control.–– 1970.–– Vol. 15, no. 2.––P. 222–228.
7. Øksendal B. Stochastic differential equations: an introduction with applications.–– Springer, 2002.
8. Ito^ K. Stochastic integral // Proceedings of the Imperial Academy.–– 1944.–– Vol. 20, no. 8.–– P. 519–524.
9. 14. It^o K. On a stochastic integral equation // Proceedings of the Japan Academy.–– 1946.–– Vol. 22, no. 2.–– P. 32–35.
10. It^o K. Stochastic differential equations in a differentiable manifold //Nagoya Mathematical Journal.–– 1950.–– Vol. 1.–– P. 35–47.
11. Sagirow P. Stochastic methods in the dynamics of satellites.–– Springer,1970.
12. Allen E. Modeling with Ito^ stochastic differential equations.–– Springer Science & Business Media, 2007.–– Vol. 22.
13. Beneˇs V. E. Existence of optimal strategies based on specified information, for a class of stochastic decision problems // SIAM Journal on control.–– 1970.–– Vol. 8, no. 2.–– P. 179–188.
14. Damm T. Rational matrix equations in stochastic control.–– Springer Berlin Heidelberg, 2004.–– ISBN: 3540205160.
15. Kleinman D. L. Optimal stationary control of linear systems with control-dependent noise // IEEE Transactions on Automatic Control.–– 1969.–– Vol. 14, no. 6.–– P. 673–677.
16. Jankunas A., Khasminskii R. Z. Estimation of parameters of linear ho mogeneous stochastic differential equations // Stochastic processes and their applications.–– 1997.–– Vol. 72, no. 2.–– P. 205–219.