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Sets That Exist in A Topological Space and Have the Qualities of Being Ideal and Pre-Semi Open

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ABSTRACT

The idea of an ideal topological space, a less robust variation of a topological space, is to be introduced in this essay. The introduction of new kinds of sets by means of the application of a local map is one of the many studies on ideal topological spaces. The paper also investigates various relationships of these sets. The study also concentrates on ideal semi-preopen sets in a topological space. We define pre-semi -interior, pre-semi -closure, and pre-semi -preopen sets and study the fact that most topological properties are unchanged when viewed from an ideal viewpoint. The different kinds of maps in a perfect topological space are also looked into, and their relationships are researched. It is investigated how these ideas relate to other pertinent ideas in ideal topological spaces. The introduction and detailed analysis of the fundamental properties of continuous maps in an ideal topological space.

Keywords: PSO set, IPSO set, IPS-continuous mapping, IPSO mapping.

1. Introduction and Preliminaries

The examination of topological ideals has been of great significance within the realm of topology for a considerable time period. Researchers such as Newcomb, Rancin, Samuels, Hamlet, and Jankovic have contributed noteworthy insights to the foundation of utilizing topological ideals to generalize fundamental properties in general topology [1-7]. The concept of ideals in a topological space was initially explored by Kuratowski and Vaidyanathaswamy in the 1930s [8]. Kuratowski, in 1933, first introduced the notion of an ideal on a nonempty set. Vaidyanathaswamy subsequently introduced the concept of a local map, denoted by $(\ )^*:\wp(W) \to \wp(W)$ in 1945 [9]. This local map has proven to be significant in the development of different versions of open sets.

In 1963, Levine's seminal work marked the introduction of concepts such as semi-open sets, semi-closed sets, and the semi-continuity of maps [10]. These concepts serve as a foundation upon which further investigations into topological ideals have been conducted. Jankovic and Hamlett conducted further research into topological ideals in 1990 [10], thereby contributing to the generalization of the field of general topology. The utilization of topological ideals to describe topological notions has been an intriguing subject of study for a number of years.

Notably, the notion of pre-open sets, as introduced by Mashhour et al., has received extensive attention from various topologists. Pre-open sets are a proper subset of open sets that can be used to generalize certain topological properties. Recent research has also focused on the application of topological ideals in the study of convergence. More specifically, the concept of an ideal limit point has been explored as a generalization of limit points. This notion has proven to be useful in the study of different types of convergence in topological spaces.

Furthermore, the study of topological ideals has been applied to various areas of mathematics, including algebraic geometry and mapal analysis. Topological ideals can be used to describe the structure of certain algebraic objects, such as rings and modules. In mapal analysis, the concept of an ideal of operators has been explored as a way to generalize the properties of closed, densely defined operators.

Definition 1.1 An ideal on a non-empty set W is defined as the set of subsets satisfying two conditions: (I) If $\mathcal{A}$ belongs to an ideal and $\mathcal{A} \subseteq \mathcal{A}$, then $\mathcal{A}$ also belongs to the ideal (this is known as inheritance); and (II) If $\mathcal{A}$ and $\mathcal{B}$ both belong to the ideal, then $\mathcal{A}$ and $\mathcal{B}$ also belong to the ideal [10]. A topological space (W, T) with an ideal I defined at a W point $w \in W$ where the space (W, T) is referred to as an ideal topological space, denoted (W, T, I), and the open neighborhood system of w is referred to as N(w) = $\{G \in T: w \in G\}$. Think about an ideal I on W and a topological space (X, T). The following is the definition of the set operator $(\ )^*: p(W) \to p(W)$as a local map of I on T.

$\mathcal{A}^*$ (I, T) represents the set of elements in W such that the intersection of H with any G in $T(w)$ does not belong to I, where $T(k) = \{G \in T: w \in G\}$. $\mathcal{A}^*$ can be used to denote $\mathcal{A}^*$ (I, T) in order to simplify the notation [9,11].

In addition, we define the Kuratowski closure operator for the topology T^*, which is defined as $CL^*(\mathcal{A}) = \mathcal{A} \cup \mathcal{A}^*$, and has a finer structure than the original topology T. Since $CL^*(W - \mathcal{A}) = W - \mathcal{A}$., we can define T^*as the set of subsets that belong to W.

We introduce a mapping $\Psi: p(W) \to p(W)$ stands for the power set of X, and $p(W)$ is defined as $\Psi(\mathcal{A}) = \mathcal{A}$ union $\mathcal{A}^*$ for all $\mathcal{A}$ belongs to $p(W)$. The map complies with the Kuratowski closure axioms [9, 11]:

i. $\Psi(\phi) = \phi$.
ii. If $\mathcal{A} \subseteq \mathcal{B}$, then $\Psi(\mathcal{A}) \subseteq \Psi(\mathcal{B})$.
iii. If $\mathcal{A}, \mathcal{B} \subseteq$ W, then $\Psi(\mathcal{A} \cup \mathcal{B}) = \Psi(\mathcal{A}) \cup \Psi(\mathcal{B})$.
iv. If $\mathcal{A} \subseteq W$, then $\Psi(\Psi(\mathcal{A})) = \Psi(\mathcal{A})$.

The foundation for the topology T^* is the collection $Q(I, T) = \{\mathcal{A} - \mathcal{B}: \mathcal{A} \in T \text{ and } \mathcal{B} \in I\}$. It is crucial to emphasize that even though $Q(I, T)$ acts as a basis, it is not always a topology [11].

If the ideal I in an ideal topological space (W, T, I) is defined as I = ϕ, then the topology T in that space is equivalent to the closure T^*.

Furthermore, according to [14]:

- $T = T^*$, if I = ϕ in (W, T, I).
- An ideal topological space (W, T, I) subset $\mathcal{A}$ is T^*-closed if and only if $\mathcal{A}^* \subseteq \mathcal{A}$.
- As T^*-open sets, the components of $T^*$are known. $\mathcal{A}$ is referred to as T^*-closed in the topology space (K, T^*) and is regarded as closed if $\mathcal{A}$ - W is a T^*-open set.
- $\mathcal{A}$ interior is denoted by the symbol $Int^*(\mathcal{A})$, and $\mathcal{A}$ closure in (W, T^*) is denoted by the symbol $Cl^*(\mathcal{A})$. T^*, open sets are the name given to the components of T^*. $\mathcal{A}$ is referred to as T^*, -closed if W - $\mathcal{A}$ is a T^*-open set.
- If $Cl^*(\mathcal{A}) = W$, then a subset $\mathcal{A}$ of an ideal topological space (W, T, I) is said to be T^*dense.

Assume K is a set that is not empty. The following families are regarded as ideal families in K [12,13] in the context of ideal theory.

- $I_\emptyset$: The trivial ideals on K, denoted by I = ϕ and $I = \mathbb{p}(W)$,
- Any set's closure in the topological space (W, T) remains closed when the topological space (W, T^*) is taken into account.
- Any set in the topological space (W, T) that is open in the topological space (K, T^*) is also an open set.
- The ideal subset of W that can be counted is called I_C.
- I_A: $I_{\mathcal{A}} = \mathbb{p}(\mathcal{A}) = \{\mathcal{B} \subseteq W: \mathcal{B} \subseteq \mathcal{A}\}$, where $\mathbb{p}(\mathcal{A})$ denotes the power set of $\mathcal{A}$. The

principal ideal produced by any set $\mathcal{A}$ from the topological space (W, T).

- IN: The ideal of nowhere dense sets, wherein I = {H ⊆ W: Int(cl(H)) = ϕ } wherein Int(cl(H)) denotes the interior of the closure of set H.
- If: The ideal set of all W's finite subsets.

Proposition 1.2 [15] Suppose that (W, T) be a topological space and let $\mathcal{A}, \mathcal{B} \subseteq W$. Then the below are holds:

1. $\mathcal{A} \subseteq$ PCL $(\mathcal{A})$.
2. if $\mathcal{A} \subseteq \mathcal{B}$, then PCL $(\mathcal{A}) \subseteq$ PCL $(\mathcal{B})$.
3. PCL $(\emptyset) = \emptyset$, and PCL (W) = W.
4. PCL $(\mathcal{A} \cap \mathcal{B}) \subseteq$ PCL $(\mathcal{A}) \cap$ PCL $(\mathcal{B})$.
5. PCL $(\mathcal{A}) \cup$ PCL $(\mathcal{B}) \subseteq$ PCL $(\mathcal{A} \cup \mathcal{B})$.
6. $\cup_{j\in\wedge}$ PCl $(\mathcal{A}_j) \subseteq$ PCl $(\cup_{j\in\wedge} \mathcal{A}_j)$.

Definition 1.3 [11] Suppose that $h: (W, T) \to (Y, \Im)$ be a mapping. Then the below are holds:

1. If h^{-1}(G) is an open set in W for each open set G in Y, a mapping $h: (W, T) \to (Y, \Im)$ is said to be continuous.
2. If $h(G)$ in T, ∀ G in T, a map $h: (W, T) \to (Y, \Im)$ is said to be open.
3. If $h(G)$ is a closed set in Y, ∀ G is a closed set in W, then the map $h: (W, T) \to (Y, \Im)$ is said to be closed.

2.-Concept of Ideal Pre-Semi Open Set

Definition 2.1 [15] For a topological space (W, T) and a mathematical subset (H, W). The union of all pre-open sets in H is what we refer to as the pre-interior of H and is denoted by the notation SInt (H). The intersection of all pre-closed sets that contain H is known as the semi-closure of H and is denoted by the symbol SCl (H).

Definition 2.2 [16] Make (W, T) a topological space, with $H \subseteq$ W. H is said to be pre-semi-open iff a semi open set G ⊆ W exists such that G ⊆ H ⊆ Int (G), where SInt(G) denotes the pre-closure of G. PSO(W) stands for the collection of every W pre-semi -open set. The collection of all pre -semi- -closed sets in W is denoted by PSC(W), and the complement of a pre-semi-open set is known as a pre-semi-closed set.

REMARK 2.3 [15] It can be asserted that there is a correlation between open sets, preopen sets and pre-semi-open sets. This correlation implies that every open set is a pre-semi-open set, and similarly, every semi open set is a pre-semi-open set.

Definition 2.4 [17] If there is a semi open set G such that H – Int (G) ∈I and G - H ∈ I, then a subset H ⊆W called to be ideal pre-semi open with (abbreviated as IPSO) iff W - H is I- pre-semi open, a subset H ⊆W is said to be ideal pre-semi-closed with (abbreviated as IPSC).

Proposition 2.7 Each PSO(W) set is IPSO(W) set.

Proof: Suppose $H \in$ PSO(W) for that ∃ U ∈ SO(W) s.t $G \subseteq H \subseteq$ SInt (G), so G − H = ∅ ∈ I and H − PCL (G) = ∅ ∈ I. Therefore, $H \in$ IPSO(W).

In general, the directions that contradict proposition 2.7 are incorrect, for instance:

Example 2.8 Suppose $W = \{\xi, \nu, \lambda\}, T = \{W, \{\xi\}, \{\xi, \nu\}, \emptyset\}, I = \{\{\lambda\}, \emptyset\}.\ Then\ SO(W) = PSO(W) = \{W, \{\xi\}, \{\nu\}, \{\xi, \nu\}, \{\xi, \lambda\}, \{\nu, \lambda\}\}, PC(K) = \{W, \{\xi\}, \{\nu\}, \{\lambda\}, \{\nu, \lambda\}, \{\xi, \lambda\}\}$, $IPSO(W) = \{W, \{\xi\}, \{\lambda\}, \{\xi, \nu\}, \{\nu, \lambda\}, \emptyset\}$. Thus, $\{\lambda\}$ ∈ IPSO(W) but $\{\lambda\}$ ∉ PSO(W).

Proposition 2.9 Each PSC(W) set is IPSC(W) set.

PROOF: Assume $N \in$ PSC(W), thus $N^c \in$ PSO(W) and by Proposition 2.7 we obtain $\mathcal{B}^c \in$ IPSO(W).

REMARK 2.10: It is simple to see that H is I pre -semi-open if H ∈ I. In addition, for any ideal I on W, every open set H is IPSO(W) and every PSO(W) set is IPSO(W), but the opposite is not true by.

Example 2.11: Suppose $K = \{\xi, \nu, \lambda\}$, $T = \{W, \{\xi, \lambda\}, \emptyset\}, I = \{\emptyset, \{\xi, \nu\}\}. Hence, SO(W) = PSO(W) = \{\emptyset, \{\xi\}, \{\lambda\}, \{\xi, \lambda\}, \{\xi, \nu\}, \{\nu, \lambda\}, W\}$ and IPSO(W)) $= p(W)$, then $\{\nu\} \notin$ SO(W) we see that $\{\lambda\}$ is an IPSO set but $\{\lambda\} \notin$ I.

The connections between the different ideas are shown in the diagram below.

Open set (W) ⟶ $SO(W)$ ⟶ $PSO(W)$ ⟶ $IPSO(W)$

Closed set (W) → $SC(W)$ → $PSC(W)$ → $IPSC(W)$

Proposition 2.12:
1. If H and N are both IPSO(W), then their union, H and N is also IPSO(W).
2. If both H and N are IPSC(W) then $H \cap N$ are also true.

Proof (1): Suppose H and N ∈ IPSO(W) sets in (W, T, I) thus, ∃ G_1, G_2 ∈ SO(KW) s.t G_1-H ∈ I and H-PCL (G_1) ∈ I, G_2-N ∈ I and N-SCL (G_2), now G_1-H ∈ I and G_2-N ∈ I. Hence, $(G_1$-$H) \cup (G_2$-$N)$ ∈ I, it is clear that $(G_1 \cup G_2)$-$(H \cup N) \subseteq (G_1$-$H) \cup (G_2$-$N)$, and because $(G_1$-$H) \cup (G_2$-$N)$ ∈ I, thus $(G_1 \cup G_2)$-$(H \cup N)$ ∈ I let $G_1 \cup G_2 = G_3$, then G_3- $(H \cup N)$ ∈ I, such that G_3 ∈ SO(W). Now, $((H$-SCl $(G_1)) \cup (N$-SCl $(G_2)))$,
hence $(H - N)$-(SCl$(G_1) \cup$ SCl$(G_2)) \subseteq (H$-SCl$(G_1)) \cup (N$- SCl$(G_2))$, since $(H$- SCl$(G_1)) \cup (N$-SCl$(G_2))$ ∈I, thus $(H \cup N)$-(SCl$(G_1) \cup$ SCl$(G_2))$ ∈ I, then $(H \cup N)$-(PCL$(G_1 \cup G_2))$ ∈ I. Therefore, $(H \cup N)$-SCl (G_3) ∈ I
(2): Suppose H and N ∈ IPSC(W), so H^c *and* N^c ∈ IPSO(W). Thus, $H^c \cup N^c$ ∈ IPSO(W) by (1). $(H \cap N)^c$ ∈ IPSO(W) by De Morgan's law. Therefore, $H \cap N$ ∈ IPSC(W).

Remark 2.13:
1. If H and N are two sets that satisfy IPSO(W), then H and N intersection need not satisfy IPSO(W).
2. If H and N are both IPSC(W), then H and N union do not necessarily have to be IPSC(W).

As an illustration:

Example 2.14 Suppose $K = \{\xi, \nu, \lambda\}$, $T = \{W, \{\xi, \lambda\}, \emptyset\}$, $I = \{\emptyset, \{\xi, \nu\}\}$, so IPSO(W)= {W, $\{\xi\}, \{\lambda\}, \{\xi, \nu\}, \{\xi, \lambda\}, \{\nu, \lambda\}$ }. Therefore, $\{\xi, \nu\} \cap \{\nu, \lambda\} = \{\nu\} \notin$ IPSO(W) and IPSC(W)= $\{W, \{\nu, \lambda\}, \{\xi, \nu\}, \{\lambda\}, \{\nu\}, \{\xi\}, \emptyset\}$, such that $\{\xi\} \cup \{\lambda\} = \{\xi, \lambda\} \notin$ IPSC(W).

3. **Many Different IPSO Mappings**

This section examines some types of map in relation to an ideal and the relationships among these types.

Definition 3.1: Assume that mapping h defined between the ideal topological spaces (W, T, I) and $(Q, \mathbb{T}, \mathbb{I})$,where h defined as: $h: (W, T, I) \rightarrow (Q, \mathbb{T}, \mathbb{I})$. This is how we classify h:
1. IPSO map if h(G) in IPSO(Q) for each G in IPSO(W).
2. If the I^*PSO(W) map is true for each G contained in T, then h(G) is equal to IPSO(Q).
3. When G is contained in IPSO(W), then h(G) is contained in $\mathbb{T}$ by the I^{**}PSO(W) map.

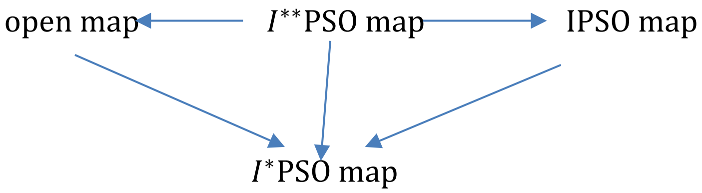
Proposition 3.2: Suppose that $h: (W, T, I) \rightarrow (Q, \mathbb{T}, \mathbb{I})$ be a mapping then:
1. A map that is I^*PSO(Q) is always open in $\mathbb{T}$.
2. IPSO(Q) maps include all I^{**}PSO(W) maps.
3. h is an I^*PSO(Q) map if h is an IPSO(Q) map.
4. I^{**}PSO(Q) maps are all open maps.
5. I^{**}PSO(W) map is an I^*PSO(W) map.

Proof (i): Let W have G as an open set. We have G is IPSO(W) because every open set is an ISPSO set. Since h is an I^*PSO(Q) map and every open set is an IPSO set, it follows that $h(G)$ IPSO(Q). We also deduce that $h(G) \in \mathbb{T}$ because every open set is an IPSO(Q) set. h is an open map as a result.
(ii): Assume G ∈ IPSO(W). $h(G) \in \mathbb{T}$ because h is an I^{**}PSO(W) map. Additionally, given that every open set is an IPSO(W) set, so $h(G)$ ∈ IPSO(Q). h is an IPSO(Q) map as a result.
(iii): Let Q be an open set in W and assume that h is an IPSO(Q) map. We have G ∈ IPSO(W) because every open set is an IPSO(W)_set. It follows that $h(G)$ is an IPSO(Q) set because h is an IPSO(Q) map. h is therefore an I^*PSO(Q) map.
(iv): Let G be an open set in W and h be an I^{**}PSO(Q) map. Every open set is an IPSO(Q) set, so we have G IPSO(W). As a result, G is also an IPSO(W) set. We conclude that $h(G)$ is an open set in Q because h is an I^{**}PSO(Q) map. h is an open map as a result.
(v): let h be an I^{**}PSO(W) map and let G is open set in W, and every open set is an IPSO(W) set, this implies G be an IPSO(W) set, since h is an I^{**}PSO(W) map. hence, $h(G)$ is open set in Q, thus h is an I^*PSO(W) map.
For instance, the opposite of the statements need not be true:

Example 3.3 Assume $h:(W,T,I)\ \rightarrow\ (Q,\mathbb{T},\mathbb{I})$ such that $W=\{\xi,\nu,\lambda\}$, T= $\{W,\{i,c\},\emptyset\}$ I= $\{\emptyset,\{\xi,\eta\},\{\xi\},\{\nu\}\}$, $\mathbb{I}=\ \emptyset$, $SO(W)=PSO(W)=\{\emptyset,\{\xi\},\{\lambda\},\{\xi,\lambda\},\{\xi,\eta\},\{\eta,\lambda\},X\}$, and IPSO(W)= $\mathbb{P}(W)$, SO(Q)= PSO(Q)= $\mathbb{I}$PSO(Y)= $\{\emptyset,\{\xi\},\},\{\lambda\},\{\xi,\lambda\},\{\xi,\nu\}\},\{\nu,\lambda\},W\}$, set $h(\xi)=\{\nu\}$, $h(\nu)=\{\xi\}$, and $h(\{\lambda\})=\{\lambda\}$,it is clear that h is I^{*}PSO(W) map since it is not open map and $\{\xi,\lambda\}\in$ T, where $h(\{\xi,\lambda\})=\{\xi,\nu\}\ \notin\ T$, and h is not IPSO map, since $\{\nu\}\in$ I-PSO(W) but $h(\{\xi\})=\{\nu\}\notin IPSO(Q)$ and h is not an I^{**}PSO(W) map, because $\{\xi\}\in$ IPSO(W) and $h(\{\xi\})=\{\nu\}\notin T$.

EXAMPLE 3.4 Assume $h:(W,T,I)\ \rightarrow\ (Q,\mathbb{T},\mathbb{I})$ such that $W=\{\xi,\nu,\lambda\}$, $\mathrm{T}=\{W,\{i,c\},\emptyset\}$, $I=\emptyset$, $\mathbb{I}=\{\emptyset,\{\xi,\nu\},\{\xi\},\{\nu\}\}$, SO(W)= PSO(W) = IPSO(W) = $\{\emptyset,\{\xi\},\},\{\lambda\},\{\xi,\nu\},\{\xi,\lambda\},\{\nu,\lambda\},W\}$, and SO(W)= PSO(W)= $\{\emptyset,\{\xi\},\{\lambda\},\{\xi,\nu\},\{\xi,\lambda\},\{\nu,\lambda\},W\}$, IPSO(W)= $\mathbb{P}$(W). We define $h(\xi)=\{\xi\}$, $h(\nu)=\{\nu\}$, $h(\lambda)=\{\lambda\}$. It is clear that h is IPSO map, but not I^{**}PSO map because $\{\xi\}\in IPSO(W)$ and $h(\nu)=\{\nu\}\notin$ T, also h is open map that is not I^{**}PSO map.

The relationships between the various ideas presented in definition 3.1 are explained in the diagram below.



4. Certain Types of Continuous Mappings:

The exploratory study of the connections between these ideas are covered in the sections that follow.

Definition 4.1: The map $h:(W,T,I)\rightarrow(Q,\mathbb{T},\mathbb{I})$ is called;

1. Whenever O is in $\mathbb{T}$, and h^{-1}(O) is in IPSO(W), the map is IPS-continuous.
2. If h^{-1}(O) in T for all O ∈ IPSO(Q), then the map is strongly IPS continuous.
3. If h^{-1}(O) ∈ IPSO(W) for every, O ∈ IPSO(Q), the map is IPS-irresolute.

Proposition 4.2 Take the following map, $h:(W,T,I)\ \rightarrow\ (Q,\mathbb{T},\mathbb{I})$. Then

1. The IPS-continuous map h is a continuous map if h is one.
2. H is a continuous map if and only if it is strongly IPS-continuous.
3. The map h is an IPS-irresolute map if it is strongly IPS-continuous.
4. H is an IPS-continuous map if h is an IPS-irresolute map.
5. H is an IPS-continuous map if the map h is strongly IPS-continuous.

Proof (i): Now, let's talk h^{-1}(G) ∈ T, because h is a continuous map. Furthermore, we have h^{-1}(G) ∈ IPSO(W) because every open set is an IPSO set. h is therefore an IPS continuous map.

(ii): Assume G ∈ $\mathbb{T}$. Since every open set is also an IPSO set, we have G ∈ IPSO(Q). We conclude that h^{-1}(G) ∈ T since h is a strongly IPS-continuous map. h is therefore a continuous map.

(iii): Suppose G ∈ IPSO(Q). So, h^{-1}(G) ∈ T because h is a strongly I-SP-continuous map. Furthermore, we discover h^{-1}(O) ∈ IPSO(W) because every open set is an IPSO set. Consequently, h is an IPS-irresolute map.

(iv): Assume G ∈ $\mathbb{T}$. We have G ∈ IPSO(Q) because every open set in Q is an IPSO set. We can infer that h^{-1}(G) ∈ IPSO(W) because h is an IPS-irresolute map. h is therefore an IPS continuous map.

(v): Assume h be a strongly IPS continuous map. Let G belong to T in order to demonstrate that h is an IPS continuous map. G must be an IPSO(Q), according to this. h^{-1}(G) belongs to T because h is a strongly IPS-continuous map. As a result, h^{-1}(G) is an IPSO set.

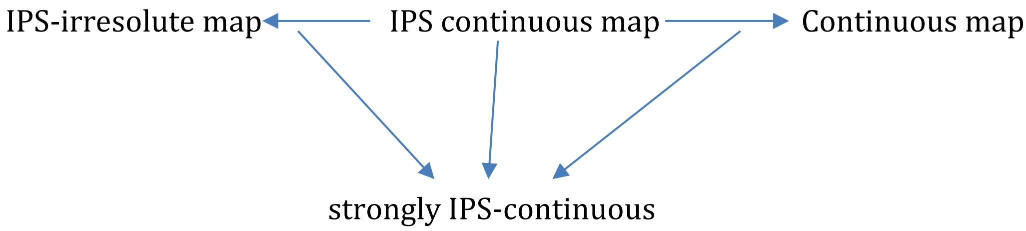
The converse of this statement might not always be true, it should be noted.

Example 4.4 Assume $W=Q=\{\xi,\nu\}$, $\mathrm{T}=\mathbb{T}=\{\emptyset,\{\xi\},\mathrm{W}\}$, $\mathrm{I}=\{\emptyset,\{\nu\}\}$ and $\mathbb{I}=\{\emptyset,\{\xi\}\}$ be defined. A map $h:(W,T,I)\ \rightarrow\ (Q,\mathbb{T},\mathbb{I})$ defined as: $h(\eta)=\eta$ makes it evident that h is both continuous and IPS-continuous; however, since

$\{\nu\}\in$ IPSO(Q) and $h^{-1}(\{\nu\})$=$\{\nu\}$is not IPSO(W) and not an open set, it is not strongly IPS-continuous and hence it is not an IPS-irresolute map.

Example 4.5. Assume $W = Q = \{\xi,\nu,\lambda\}$, $\mathrm{T} = \mathbb{T} = \{\emptyset,\{\xi,\lambda\},\{\lambda\},\mathrm{W}\}$, $\mathrm{I} = \{\emptyset,\{\nu,\lambda\}\}$ and $\mathbb{I}$ =$\{\emptyset,\{\xi,\lambda\}\}$ be defined. A map $h:(W,T,I) \rightarrow (Q,\mathbb{T},\mathbb{I})$ defined as: $h(\eta) = \eta$, makes it evident that h is an IPS-continuous map and an IPS-irresolute map. However, it is not a continuous map and not strongly IPS-continuous. However, since $\{\xi,\lambda\} \in$ IPSO(Q) and $h^{-1}(\{\xi,\lambda\})$= $\{\xi,\lambda\}$ is not IPSO(W).

The relationships between the various ideas in definition 4.1 are illustrated in the diagram below.



Conclusion

In this article, we introduced a new class named IPSO map, $\boldsymbol{I}^{*}$PSO map, $\boldsymbol{I}^{**}$PSO map, IPS-irresolute map, IPS continuous map and strongly IPS-continuous. The study investigates the relationships among various kinds of maps in a ideal topological space.

The essay also looks at how these ideas relate to other important ideas in the field of ideal topological spaces. In this article, the basic characteristics of continuous maps in a perfect topological space are introduced and thoroughly examined.

The essay concludes with a summary of the idea of ideal topological spaces and the creation of new sets using local maps. It examines these sets' characteristics and connections, concentrating on ideal semi-preopen sets. The essay as a thorough examination of the fundamental features of continuous maps in a perfect topological space.

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