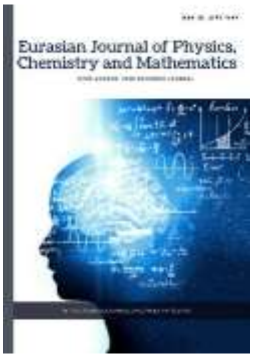

Mathematical Model Of Precious Securities Published On The Market

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ABSTRACT

Despite the fact that the stock market in Uzbekistan is not fully developed, the shares of many joint-stock companies are registered on the stock exchange and are being traded. Profits from stocks are realized in the form of dividends or in exchange for their secondary sale at a higher price. We use mathematical modules to decide which asset to invest in or not to invest in. We try to mathematically model some of these actions using linear stochastic differential equations (SDE) and the Euler-Maruyama scheme.

Keywords: Stock price forecasting, system of linear stochastic differential equations, model, volatility coefficients, simulation, forecasting, Euler-Maruyama scheme

Introduction

For years, investment management was studied as an activity rather than by mathematical models. However, as a result of the development of imported technologies, the expansion of the universe of investable assets creates the need to study mathematical models. That is, it requires a mathematical approach to asset price forecasting rather than mere speculation. This justifies the need for a scientific approach to investment management.

Analysis of literature

We $\Delta t = t_{i+1} - t_i \quad i = 1..30$ during the day, the daily closing price (price) of the share may increase by one unit [+1], remain unchanged [0] or decrease by one unit [-1]. We recorded the probability of each change and formed a system of linear stochastic differential equations while calculating the expectation (shift) and covariance (variability) of the change [4]. To fit the data to the model, we studied stock price movements over a 30-day average. Expectation (shift) and covariance (variability) coefficients for SDE were determined and used to simulate 30-day stock prices for the multivariate Euler-Maruyama scheme for the system of stochastic differential equations [8]. We compared the simulated prices with the observed prices and observed that the simulated prices are reasonably close to the observed prices.

The main purpose of this work is to form a stochastic model for the dynamics of the price of selected shares and to use new models for the analysis of the prices of selected shares [4]

Research methodology

A mathematical model of the dynamics of three share price changes. We consider the shares belonging to the joint-stock companies "Kvarts", "Kokan Mechanical Plant" and "Jizzakh Plastics", which entered the list of primary shares (IPO) of enterprises on the Tashkent "Stock Exchange"[3]. We will mark their value as S_1, S_2 and S_3, respectively. During the time interval Δt, the share price can fall by one unit [-1] or stay the same [0] or rise by one unit [+1] . Here there are 3^3 = 27 possibilities, according to which the share prices of S_1, S_2 and S_3 differ. Possible outcomes and probabilities of 3 stock price changes are presented in Table 1.

1-table

№	Changes in stock prices $\Delta S = \{\Delta S_1, \Delta S_2, \Delta S_3\}^T$	Probabilities
1	[-1 0 0]	$p_1 = d_1 S_1$
2	[1 0 0]	$p_2 = b_1 S_1$
3	[0 1 0]	$p_3 = b_1 S_2$
4	[0 -1 0]	$p_4 = d_1 S_2$
5	[0 0 -1]	$p_5 = d_1 S_3$
6	[0 0 1]	$p_6 = b_1 S_3$
7	[-1 1 0]	$p_7 = \alpha_{12} S_1 S_2$
8	[-1 -1 0]	$p_8 = \alpha_{11} S_1 S_2$
9	[1 1 0]	$p_9 = \alpha_{22} S_1 S_2$
10	[1 -1 0]	$p_{10} = \alpha_{21} S_1 S_2$
11	[-1 0 1]	$p_{11} = \beta_{12} S_1 S_3$
12	[-1 0 -1]	$p_{12} = \beta_{11} S_1 S_3$
13	[1 0 1]	$p_{13} = \beta_{22} S_1 S_3$
14	[1 0 -1]	$p_{14} = \beta_{21} S_1 S_3$
15	[0 1 1]	$p_{15} = \gamma_{22} S_2 S_3$
16	[0 -1 -1]	$p_{16} = \gamma_{11} S_2 S_3$
17	[0 -1 1]	$p_{17} = \gamma_{12} S_2 S_3$
18	[0 1 -1]	$p_{18} = \gamma_{21} S_2 S_3$
19	[-1 1 1]	$p_{19} = \delta_{122} S_1 S_2 S_3$
20	[-1 1 -1]	$p_{20} = \delta_{121} S_1 S_2 S_3$
21	[-1 -1 -1]	$p_{21} = \delta_{111} S_1 S_2 S_3$
22	[-1 -1 1]	$p_{22} = \delta_{112} S_1 S_2 S_3$
23	[1 -1 -1]	$p_{23} = \delta_{211} S_1 S_2 S_3$
24	[1 1 -1]	$p_{24} = \delta_{221} S_1 S_2 S_3$
25	[1 -1 1]	$p_{25} = \delta_{212} S_1 S_2 S_3$
26	[1 1 1]	$p_{26} = \delta_{222} S_1 S_2 S_3$
27	[0 0 0]	$p_{27} = 1 - \sum_{j=1}^{26} p_i$

Here, ΔS represents the change in stock price. For example, ΔS=[1 0 0] S1 means that the share price will increase by 1 unit, S_2 and S_3. shares will remain unchanged. ΔS=[1 -1 1] simultaneously means that S_1 and S_3 will increase by one unit and S_2 means that the stock will decrease by -1 unit. For example, a_1,2 represents a change involving two stocks S_1 S_2, where S_2 gains and S_1 loses. Also b_(j.k) represents changes involving two stocks S1 S3, where S3 gains and S1 loses. This means three stocks change at the same time [4]. All cases a subscript of 1 or 2 means a decrease of one unit or an increase of one unit, respectively. It should be noted that $\sum_{j=1}^{27} p_i = 1$ =1. Using the above representations for p_i and ΔS_ii, the expectation vector is derived as follows [4]:

$$E(\Delta S) = \sum_{j=1}^{27} p_i \Delta S_i = \begin{bmatrix} f_1 \\ f_2 \\ f_3 \end{bmatrix} \quad (1)$$

$f_1 = (d_1 + b_1)S_1$ $+(\alpha_{12} + \alpha_{11} + \alpha_{22} + \alpha_{21})$ S_1S_2+$(\beta_{12} + \beta_{21} + \beta_{22} + \beta_{21})S_1S_3$+$(\delta_{122} + \delta_{121} + \delta_{111} + \delta_{112} + \delta_{211} + \delta_{221} + \delta_{212} + \delta_{222})S_1S_2S_3$ (2)

f_1 where S_1 represents the full probability of change associated with shares.

$f_2 = (d_1 + b_1)S_2$ $+(\alpha_{12} + \alpha_{11} + \alpha_{22} + \alpha_{21})$ S_1S_2+$(\gamma_{12} + \gamma_{21} + \gamma_{22} + \gamma_{21})S_2S_3$+$(\delta_{122} + \delta_{121} + \delta_{111} + \delta_{112} + \delta_{211} + \delta_{221} + \delta_{212} + \delta_{222})S_1S_2S_3$ (3)

f_1 where S_1 represents the full probability of change associated with shares.

$f_3 = (d_1 + b_1)S_3 + (\beta_{12} + \beta_{21} + \beta_{22} + \beta_{21})S_1S_3$+$(\gamma_{12} + \gamma_{21} + \gamma_{22} + \gamma_{21})S_2S_3$+$(\delta_{122} + \delta_{121} + \delta_{111} + \delta_{112} + \delta_{211} + \delta_{221} + \delta_{212} + \delta_{222})S_1S_2S_3$ (4)

f_1 where S_1 represents the full probability of change associated with shares.

The covariance matrix is then derived as:

$dS_1 = (d_1 + b_1)S_1$ (5) $dS_2 = (d_1 + b_1)S_2$ (6)

$dS_3 = (d_1 + b_1)S_3$ (7) dS_1S_2 =$(\alpha_{12} + \alpha_{11} + \alpha_{22} + \alpha_{21})$ S_1S_2 (8)

$dS_1S_3 = (\beta_{12} + \beta_{21} + \beta_{22} + \beta_{21})S_1S_3$ (9)

$dS_2S_3 = (\gamma_{12} + \gamma_{21} + \gamma_{22} + \gamma_{21})S_2S_3$ (10)

$dS_1S_2S_3 = (\delta_{122} + \delta_{121} + \delta_{111} + \delta_{112} + \delta_{211} + \delta_{221} + \delta_{212} + \delta_{222})S_1S_2S_3$ (11)

$$E(\Delta S(\Delta S))^T = \begin{bmatrix} dS_1 & dS_1S_2 & dS_1S_3 \\ dS_2S_1 & dS_2 & dS_2S_3 \\ dS_3S_1 & dS_3S_2 & dS_3 \end{bmatrix} \quad (12)$$

The covariance matrix represents the coefficient of variation of the SDE. The covariance matrix is positive definite symmetric and has a root [7].

$$dS = \mu(t, S, S_1, S_2) + B(t, S, S_1, S_2)dW(t)$$

Here:

$$\mu(t, S, S_1, S_2) = \begin{bmatrix} f_1 \\ f_2 \\ f_3 \end{bmatrix} \text{ va } B(t, S, S_1, S_2) = \begin{bmatrix} dS_1 & dS_1S_2 & dS_1S_3 \\ dS_2S_1 & dS_2 & dS_2S_3 \\ dS_3S_1 & dS_3S_2 & dS_3 \end{bmatrix}^{\frac{1}{2}}$$

Analysis and results.

Stock prices are published daily. Daily stock prices of three selected stocks were observed for 30 days (09.01.2023-20.02.2023) to describe the expectation and covariance of the coefficients of stochastic differential equations. The table below shows the 30-day prices of the selected ones [3].

2-table

Day	1	2	3	4	5	6	7	8	9	10
S_1	3400	3540	3573	3500	3500	3500	3495	3495	3525	3500
S_2	889.3	920	821	826	900	880	880	879.8	924	900
S_3	3399.9	3400	3385	3399	3380	3380	3380	3399.9	3350	3350

Day	11	12	13	14	15	16	17	18	19	20
S_1	3449	3450	3400	3410	3490	3450	3498	3400	3400	3360
S_2	850	850	880	892	890	892	840	827	840	800
S_3	2800	3340	3339	3390	3390	3350	3339	3300	3030	3000
Day	21	22	23	24	25	26	27	28	29	30
S_1	3350	3378	3400	3445	3201	3299	3251	3300	3190	3200
S_2	807	925	811	919.4	917.9	816	907	899	906	900
S_3	3065	3200	3300	3150	3000	3380	2704	3110	3271	3250

Table 2 above shows how the price of each share has changed compared to the previous day. For example, in the header column [0 0 0], the price of S_1,S_2 and S_3 has not changed: [-1 1 0] this means that S_1 has decreased, S_2 has increased, S_3 has remained unchanged, etc. The probability of each event was determined from the above (Table 1). For example, the event that the stock price change is [0-1 0] is 1/27 times =0.0370. Probabilities for other events have been determined.

3- table

№	Changes in stock prices $\Delta S = [\Delta S_1, \Delta S_2, \Delta S_3]$	Number of events	Probabilities
1	[-1 0 0]	1	0.0370
2	[1 0 0]	0	0.0000
3	[0 1 0]	0	0.0000
4	[0 -1 0]	1	0.0370
5	[0 0 -1]	0	0.0000
6	[0 0 1]	0	0.0000
7	[-1 1 0]	0	0.0000
8	[-1 -1 0]	2	0.0470
9	[1 1 0]	0	0.0000
10	[1 -1 0]	1	0.0370
11	[-1 0 1]	0	0.0000
12	[-1 0 -1]	0	0.0000
13	[1 0 1]	1	0.0370
14	[1 0 -1]	0	0.0000
15	[0 1 1]	1	0.0370
16	[0 -1 -1]	0	0.0000
17	[0 -1 1]	0	0.0000
18	[0 1 -1]	1	0.0370
19	[-1 1 1]	4	0.1481
20	[-1 1 -1]	2	0.0741
21	[-1 -1 -1]	3	0.1111
22	[-1 -1 1]	0	0.0000
23	[1 -1 -1]	2	0.0470
24	[1 1 -1]	3	0.1111
25	[1 -1 1]	2	0.0740
26	[1 1 1]	3	0.1111
27	[0 0 0]	0	0.0000

Using formulas (2)-(12) from Table 3, the variables were calculated as follows.

$f_1 = (d_1 + b_1)S_1$ +$(\alpha_{12} + \alpha_{11} + \alpha_{22} + \alpha_{21})$ S_1S_2+$(\beta_{12} + \beta_{21} + \beta_{22} + \beta_{21})S_1S_3$+$(\delta_{122} + \delta_{121} + \delta_{111} + \delta_{112} + \delta_{211} + \delta_{221} + \delta_{212} + \delta_{222})S_1S_2S_3$

=0.0370+0.0470+0.0370+0.0370+0.1481+0.0741+0.1111+0.0470+0.1111+0.0740+ 0.1111 = 0.8345

$f_2 = (d_1 + b_1)S_2$ +$(\alpha_{12} + \alpha_{11} + \alpha_{22} + \alpha_{21})$ S_1S_2+$(\gamma_{12} + \gamma_{21} + \gamma_{22} + \gamma_{21})S_2S_3$+$(\delta_{122} + \delta_{121} + \delta_{111} + \delta_{112} + \delta_{211} + \delta_{221} + \delta_{212} + \delta_{222})S_1S_2S_3$ =0.0370+ 0.0470 + 0.0370 +0.0370+ 0.0370+0.1481 + 0.0741 + 0.1111 + 0.0470 + 0.1111 +0.0740+0.1111= 0.8716

$f_3 = (d_1 + b_1)S_3$ + $(\beta_{12} + \beta_{21} + \beta_{22} + \beta_{21})S_1S_3$+$(\gamma_{12} + \gamma_{21} + \gamma_{22} + \gamma_{21})S_2S_3$+$(\delta_{122} + \delta_{121} + \delta_{111} + \delta_{112} + \delta_{211} + \delta_{221} + \delta_{212} + \delta_{222})S_1S_2S_3$

= 0.0370+0.0370+0.1481+0.0741+0.1111 + 0.0470 + 0.1111 +0.0740+0.1111=0.7036

Hence, the expectation vector $E(\Delta S) = \sum_{j=1}^{27} p_i \Delta S_i = \begin{bmatrix} f_1 \\ f_2 \\ f_3 \end{bmatrix} = \begin{pmatrix} 0.8345 \\ 0.8716 \\ 0.7036 \end{pmatrix}$

$dS_1 = (d_1 + b_1)S_1$ = 0.0370 , $dS_2 = (d_1 + b_1)S_2$ = 0.0370, $dS_3 = (d_1 + b_1)S_3$ = 0

dS_1S_2 =$(\alpha_{12} + \alpha_{11} + \alpha_{22} + \alpha_{21})$ = 0.0840

$dS_1S_3 = (\beta_{12} + \beta_{21} + \beta_{22} + \beta_{21})S_1S_3$ = 0.0370

$dS_2S_3 = (\gamma_{12} + \gamma_{21} + \gamma_{22} + \gamma_{21})S_2S_3$ = 0.0740

$dS_1S_2S_3 = (\delta_{122} + \delta_{121} + \delta_{111} + \delta_{112} + \delta_{211} + \delta_{221} + \delta_{212} + \delta_{222})S_1S_2S_3$= 0.6765

Hence, the covariance matrix

$$E(\Delta S(\Delta S))^T = \begin{bmatrix} dS_1 & dS_1S_2 & dS_1S_3 \\ dS_2S_1 & dS_2 & dS_2S_3 \\ dS_3S_1 & dS_3S_2 & dS_3 \end{bmatrix} = \begin{bmatrix} 0.0370 & 0.0840 & 0.0370 \\ 0.0840 & 0.0370 & 0.0740 \\ 0.0370 & 0.0740 & 0.0000 \end{bmatrix}$$

For the resulting stochastic differential equation

$$dS = \mu(t, S, S_1, S_2) + B(t, S, S_1, S_2)dW(t) dS = \mu \begin{pmatrix} 0.8345 \\ 0.8716 \\ 0.7036 \end{pmatrix} dt + \begin{bmatrix} 0.0370 & 0.0840 & 0.0370 \\ 0.0840 & 0.0370 & 0.0740 \\ 0.0370 & 0.0740 & 0.0000 \end{bmatrix} dW(t)$$

The resulting SDE was solved using the multivariate Euler-Maruyama scheme. The algorithm is implemented through a MatLab script, and the simulation result is shown in the following graph:

1-rasm.

Figure 1 above shows three stock charts showing growth. Close to observed prices and therefore can be used for forecasting [7]. Below is a 30-day comparison of observed and simulated prices.

2-rasm.

Conclusion and suggestions

Comparison of Average Observed Price and Average Simulated Price When the observed stock prices are compared with the simulated prices, it is found that the result is close enough to the analyzed prices. We can forecast stock price changes for the next day using stochastic differential equation and Euler-Maruyama scheme.

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