The Resolution of Two Rows Weyl Module in the Cases of (6,4) and (6,4)/ (1,0)

[1]Njood Abd Hatim [1]Department of Mathematics, College of Basic Education, Misan University
najudi.a.h@uomisan.edu.iq,

,[2]Nuha Farhan Mansour [2]Ministry of Education \ The General of Directorate for Education of Diyala
nuhaf.995@gmail.com

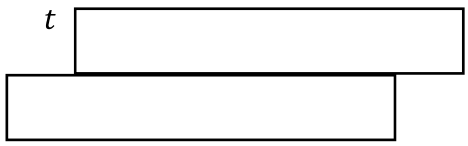
ABSTRACT

The intention of this work is to reconnaissance the resolution of two rows Weyl module in the partition (6, 4) and skew- shape (6,4)/(1,0) depending on the contracting homotopy.

Keywords: Resolution of Weyl module, place polarization, Skew-shape, mapping Cone.

1. Introduction

Let F be a free R-module and D_mF be divided power of degree m, the author [2] describe us the Weyl module $K_{\lambda/\mu} F$,where $K_{\lambda/\mu} F$ = Im($d'_{\lambda/\mu}$), $d'_{\lambda/\mu}$:DF $\to \wedge F$ (Weyl map) that take the image of:



$$\sum D_{p+k} \otimes D_{q-k} \xrightarrow{\ \square\ } D_p \otimes D_q \xrightarrow{d'_{\lambda/\mu}} K_{\lambda/\mu} \longrightarrow 0$$

And using us letter place, we will get:

$$\begin{pmatrix} w \\ w' \end{pmatrix}\begin{vmatrix} 1^{(p+k)} \\ 2^{(q-k)} \end{vmatrix} \xrightarrow{\partial_{21}^{(k)}} \begin{pmatrix} w \\ w' \end{pmatrix}\begin{vmatrix} 1^{(p)}2^{(k)} \\ 2^{(q-k)} \end{vmatrix} \longrightarrow \sum_w \begin{pmatrix} w_{(1)} \\ w'w_{(2)} \end{pmatrix}\begin{vmatrix} (t+1)'(t+2)'\dots(p+t)' \\ 1'2'3'\dots q' \end{vmatrix}$$

Where

$$w \otimes w' \in D_{p+k} \otimes D_{q-k} \ , \quad \square = \sum_{k=t+1}^{q} \partial_{21}^{(k)}$$

And

$$d'_{\lambda/\mu} = \partial_{q'2} \dots \partial_{1'2} \partial_{(p+t)'1} \dots \partial_{(t+1)'1}$$

Is the installation of place polarization, from positive places {1,2} to negative places {1', 2', … , (p+t)'}.
In particular, $\square$ moves an element $x \otimes y$ of $D_{p+k} \otimes D_{q-k}$ to $\sum x_p \otimes x'_k\, y$, $\sum x_p \otimes x'_k$ is the element of the diagonal of x in $D_p \otimes D_k$.

Let Z_{21} be the free generator of D(Z_{21}) [divided power algebra] , then the D($Z_{21}^{(k)}$) be the divided power algebra of the free generator $Z_{21}^{(k)}$ of degree k works on $D_{p+k} \otimes D_{q-k}$ by place polarization of degree ƙ from place 1 to place 2.

The graded algebra $D(Z_{21})$ works on the graded module $M = \sum D_{p+k} \otimes D_{q-k} = \sum M_{q-k}$, M is a graded left A-module, where $w = Z_{21}^{(k)} \in A$ and $v \in D_{\beta_1} \otimes D_{\beta_2}$, at our disposal:

$$w(v) = Z_{21}^{(k)}(v) = \partial_{21}^{(k)}(v)$$

$M_\bullet : 0 \to M_{q-t} \xrightarrow{\partial_s} \dots \to M_l \xrightarrow{\partial_s} \dots M_1 \xrightarrow{\partial_s} M_0$, of the normalized bar complex Bar(M, A; S, •), and S = {x}.

$$\sum_{k_1 \geq 0} Z_{21}^{(t+k_1)} x Z_{21}^{(k_2)} x \dots x Z_{21}^{(k_l)} x D_{p+t+|k|} \otimes D_{q-t-|k|} \xrightarrow{d_l}$$

$$\sum_{k_1 \geq 0} Z_{21}^{(t+k_1)} x Z_{21}^{(k_2)} x \dots x Z_{21}^{(k_l-1)} x D_{p+t+|k|} \otimes D_{q-t-|k|} \xrightarrow{d_{l-1}}$$

$$\dots \xrightarrow{d_1} \sum_{k_i \geq 0} Z_{21}^{(t+k)} x D_{p+t+|k|} \otimes D_{q-t-k} \xrightarrow{d_0} D_p \otimes D_q$$

Where $|k| = \sum k_i$.

The author in [4] studied the resolution and exactness of the Weyl module in (8,7), Shaymaa [5] exhibited the resolution of Weyl module for the skew partition (8,6)/(2,t) ,where t=0,1. While auother [6] studied the resolution of Weyl module for the skew partition (9,7)/(S,0),where S=1,2. At work now , I specified the resolution and exactness of Weyl module in partition (6,4) and skew-partition (6,4)/(1,0).

2. The resolution of two rows Weyl module in the case of partition (6,4):

$M_0 = D6 \otimes D4$

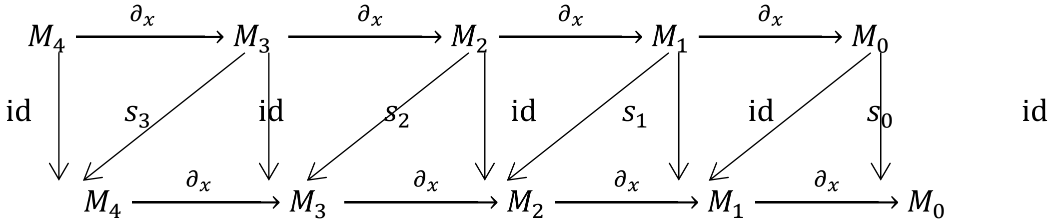
$M_1 = Z_{21} x D_7 \otimes D_3 \oplus Z_{21}^{(2)} x D_8 \otimes D_2 \oplus Z_{21}^{(3)} x D_9 \otimes D_1 \oplus Z_{21}^{(4)} x D_{10} \otimes D_0$

$M_2 = Z_{21} x Z_{21} x D_8 \otimes D_2 \oplus Z_{21}^{(2)} x Z_{21} x D_9 \otimes D_1 \oplus Z_{21} x Z_{21}^{(2)} x D_9 \otimes D_1$
$\oplus Z_{21}^{(3)} x Z_{21} x D_{10} \otimes D_0 \oplus Z_{21} x Z_{21}^{(3)} D_{10} \otimes D_0 \oplus Z_{21}^{(2)} x Z_{21}^{(2)} x D_{10} \otimes D_0$

$M_3 = Z_{21} x Z_{21} x Z_{21} x D_9 \otimes D_1 \oplus Z_{21}^{(2)} x Z_{21} x Z_{21} x D_{10} \otimes D_0 \oplus Z_{21} x Z_{21}^{(2)} x Z_{21} x D_{10} \otimes D_0$
$\oplus Z_{21} x Z_{21} x Z_{21}^{(2)} x D_{10} \otimes D_0$

$M_4 = Z_{21} x Z_{21} x Z_{21} x Z_{21} x D_{10} \otimes D_0$

Thus we got the following diagram:



$$S_0\left(\left(\begin{matrix} w \\ w' \end{matrix}\middle| \begin{matrix} 1^{(6)} 2^{(k)} \\ 2^{(4-k)} \end{matrix}\right)\right) = \begin{cases} Z_{21}^{(k)} x \left(\begin{matrix} w \\ w' \end{matrix}\middle| \begin{matrix} 1^{(6+k)} \\ 2^{(4-k)} \end{matrix}\right) & ; if\ k > 0 \\ 0 & ; if\ k \leq 0 \end{cases}$$

$S_1 : M_1 \longrightarrow M_2$

$$S_1\left(Z_{21}^{(k)} x \left(\begin{matrix} w \\ w' \end{matrix}\middle| \begin{matrix} 1^{(6+k)} 2^{(m)} \\ 2^{(4-k-m)} \end{matrix}\right)\right) = \begin{cases} Z_{21}^{(k)} x Z_{21}^{(m)} x \left(\begin{matrix} w \\ w' \end{matrix}\middle| \begin{matrix} 1^{(6+k+m)} \\ 2^{(4-k-m)} \end{matrix}\right) & ; if\ m = 1,2,3 \\ 0 & ; if\ m = 0 \end{cases}$$

$S_2 : M_2 \longrightarrow M_3$

$$S_2\left(Z_{21}^{(k_1)} x Z_{21}^{(k_2)} x \left(\begin{matrix} w \\ w' \end{matrix}\middle| \begin{matrix} 1^{(6+|k|)} 2^{(m)} \\ 2^{(4-|k|-m)} \end{matrix}\right)\right) =$$

$$\begin{cases} Z_{21}^{(k_1)} x Z_{21}^{(k_2)} x Z_{21}^{(m)} x \left(\begin{matrix} w \\ w' \end{matrix}\middle| \begin{matrix} 1^{(6+|k|+m)} \\ 2^{(4-|k|-m)} \end{matrix}\right) & ; if\ m = 1,2, \\ 0 & ; if\ m = 0 \end{cases} ;\ \text{where } |k| = k_1 + k_2$$

$S_3 : M_3 \longrightarrow M_4$

$$S_3\left(Z_{21}^{(k_1)}xZ_{21}^{(k_2)}xZ_{21}^{(k_3)}x\begin{pmatrix}w\\w'\end{vmatrix}\begin{matrix}1^{(6+|k|)}2^{(m)}\\2^{(4-|k|-m)}\end{pmatrix}\right)$$

$$=\begin{cases}Z_{21}^{(k_1)}xZ_{21}^{(k_2)}xZ_{21}^{(k_3)}xZ_{21}^{(m)}x\begin{pmatrix}w\\w'\end{vmatrix}\begin{matrix}1^{(6+|k|+m)}\\2^{(4-|k|-m)}\end{pmatrix} & ;if\ m=1\\ 0 & ;if\ m=0\end{cases};$$

where $|k|=k_1+k_2+k_3$

$$S_0\partial_{\varkappa}\left(Z_{21}^{(k)}x\begin{pmatrix}w\\w'\end{vmatrix}\begin{matrix}1^{(6+k)}2^{(m)}\\2^{(4-k-m)}\end{pmatrix}\right)=S_0\partial_{21}^{(k)}\begin{pmatrix}w\\w'\end{vmatrix}\begin{matrix}1^{(6+k)}2^{(m)}\\2^{(4-k-m)}\end{pmatrix}$$

$$=\binom{k+m}{m}Z_{21}^{(k+m)}x\begin{pmatrix}w\\w'\end{vmatrix}\begin{matrix}1^{(6+k+m)}\\2^{(4-k-m)}\end{pmatrix}$$

and

$$\partial_{\varkappa}S_1\left(Z_{21}^{(k)}x\begin{pmatrix}w\\w'\end{vmatrix}\begin{matrix}1^{(6+k)}2^{(m)}\\2^{(4-k-m)}\end{pmatrix}\right)=\partial_{\varkappa}\left(Z_{21}^{(k)}xZ_{21}^{(m)}x\begin{pmatrix}w\\w'\end{vmatrix}\begin{matrix}1^{(6+k+m)}\\2^{(4-k-m)}\end{pmatrix}\right)$$

$$=-\binom{k+m}{m}Z_{21}^{(k+m)}x\begin{pmatrix}w\\w'\end{vmatrix}\begin{matrix}1^{(6+k+m)}\\2^{(4-k-m)}\end{pmatrix}+Z_{21}^{(k)}x\begin{pmatrix}w\\w'\end{vmatrix}\begin{matrix}1^{(6+k)}2^{(m)}\\2^{(4-k-m)}\end{pmatrix}$$

$$=Z_{21}^{(k)}x\begin{pmatrix}w\\w'\end{vmatrix}\begin{matrix}1^{(6+k)}2^{(m)}\\2^{(4-k-m)}\end{pmatrix}$$

It is clear that $S_0\partial_{\varkappa}+\partial_{\varkappa}S_1=id_{M_1}$.

$$S_1\partial_{\varkappa}\left(Z_{21}^{(k_1)}xZ_{21}^{(k_2)}x\begin{pmatrix}w\\w'\end{vmatrix}\begin{matrix}1^{(6+|k|)}2^{(m)}\\2^{(4-|k|-m)}\end{pmatrix}\right)$$

$$=S_1\left(-\binom{|k|}{k_2}Z_{21}^{|k|}x\begin{pmatrix}w\\w'\end{vmatrix}\begin{matrix}1^{(6+|k|)}2^{(m)}\\2^{(4-|k|-m)}\end{pmatrix}+Z_{21}^{(k_1)}x\,\partial_{21}^{(k_2)}\begin{pmatrix}w\\w'\end{vmatrix}\begin{matrix}1^{(6+|k|)}2^{(m)}\\2^{(4-|k|-m)}\end{pmatrix}\right)$$

$$=-\binom{|k|}{k_2}Z_{21}^{|k|}xZ_{21}^{(m)}x\begin{pmatrix}w\\w'\end{vmatrix}\begin{matrix}1^{(6+|k|+m)}\\2^{(4-|k|-m)}\end{pmatrix}+$$
$$\binom{k_2+m}{m}Z_{21}^{(k_1)}xZ_{21}^{(k_2+m)}x\begin{pmatrix}w\\w'\end{vmatrix}\begin{matrix}1^{(6+|k|+m)}\\2^{(4-|k|-m)}\end{pmatrix},$$

and

$$\partial_{\varkappa}S_2\left(Z_{21}^{(k_1)}xZ_{21}^{(k_2)}x\begin{pmatrix}w\\w'\end{vmatrix}\begin{matrix}1^{(6+|k|)}2^{(m)}\\2^{(4-|k|-m)}\end{pmatrix}\right)=\partial_{\varkappa}\left(Z_{21}^{(k_1)}xZ_{21}^{(k_2)}xZ_{21}^{(m)}x\begin{pmatrix}w\\w'\end{vmatrix}\begin{matrix}1^{(6+|k|+m)}\\2^{(4-|k|-m)}\end{pmatrix}\right)$$

$$=\binom{|k|}{k_2}Z_{21}^{|k|}xZ_{21}^{(m)}x\begin{pmatrix}w\\w'\end{vmatrix}\begin{matrix}1^{(6+|k|+m)}\\2^{(4-|k|-m)}\end{pmatrix}-$$
$$\binom{k_2+m}{m}Z_{21}^{(k_1)}xZ_{21}^{(k_2+m)}x\begin{pmatrix}w\\w'\end{vmatrix}\begin{matrix}1^{(6+|k|+m)}\\2^{(4-|k|-m)}\end{pmatrix}+\ Z_{21}^{(k_1)}xZ_{21}^{(k_2)}x\begin{pmatrix}w\\w'\end{vmatrix}\begin{matrix}1^{(6+|k|)}2^{(m)}\\2^{(4-|k|-m)}\end{pmatrix},$$

where $|k/=k_1+k_2$.

It is clear that $S_1\partial_{\varkappa}+\partial_{\varkappa}S_2=id_{M_2}$.

$$S_2\partial_{\varkappa}\left(Z_{21}^{(k_1)}xZ_{21}^{(k_2)}xZ_{21}^{(k_3)}x\begin{pmatrix}w\\w'\end{vmatrix}\begin{matrix}1^{(6+|k|)}2^{(m)}\\2^{(4-|k|-m)}\end{pmatrix}\right)$$

$$=S_2\left(\binom{k_1+k_2}{k_2}Z_{21}^{(k_1+k_2)}xZ_{21}^{(k_3)}x\begin{pmatrix}w\\w'\end{vmatrix}\begin{matrix}1^{(6+|k|)}2^{(m)}\\2^{(4-|k|-m)}\end{pmatrix}-\binom{k_2+k_3}{k_3}Z_{21}^{(k_1)}xZ_{21}^{(k_2+k_3)}x\begin{pmatrix}w\\w'\end{vmatrix}\begin{matrix}1^{(6+|k|)}2^{(m)}\\2^{(4-|k|-m)}\end{pmatrix}+\right.$$
$$\left.Z_{21}^{(k_1)}xZ_{21}^{(k_3)}x\begin{pmatrix}w\\w'\end{vmatrix}\begin{matrix}1^{(6+|k|)}2^{(m)}\\2^{(4-|k|-m)}\end{pmatrix}\right)$$

$$=\binom{k_1+k_2}{k_2}Z_{21}^{(k_1+k_2)}xZ_{21}^{(k_3)}xZ_{21}^{(m)}x\begin{pmatrix}w\\w'\end{vmatrix}\begin{matrix}1^{(6+|k|+m)}\\2^{(4-|k|-m)}\end{pmatrix}-$$
$$\binom{k_2+k_3}{k_3}Z_{21}^{(k_1)}xZ_{21}^{(k_2+k_3)}xZ_{21}^{(m)}x\begin{pmatrix}w\\w'\end{vmatrix}\begin{matrix}1^{(6+|k|+m)}\\2^{(4-|k|-m)}\end{pmatrix}+$$
$$\binom{k_3+m}{m}Z_{21}^{(k_1)}xZ_{21}^{(k_2)}xZ_{21}^{(k_3+m)}x\begin{pmatrix}w\\w'\end{vmatrix}\begin{matrix}1^{(6+|k|+m)}\\2^{(4-|k|-m)}\end{pmatrix},$$

and

$$\partial_\varkappa S_3\left(Z_{21}^{(k_1)}xZ_{21}^{(k_2)}xZ_{21}^{(k_3)}x\begin{pmatrix} w \\ w' \end{vmatrix}\begin{matrix}1^{(6+|k|)}2^{(m)} \\ 2^{(4-|k|-m)}\end{pmatrix}\right) = \partial_\varkappa\left(Z_{21}^{(k_1)}xZ_{21}^{(k_2)}xZ_{21}^{(k_3)}xZ_{21}^{(m)}x\begin{pmatrix} w \\ w' \end{vmatrix}\begin{matrix}1^{(6+|k|+m)} \\ 2^{(4-|k|-m)}\end{pmatrix}\right)$$

$$= -\binom{k_1+k_2}{k_2}Z_{21}^{(k_1+k_2)}xZ_{21}^{(k_3)}\varkappa Z_{21}^{(m)}x\begin{pmatrix} w \\ w' \end{vmatrix}\begin{matrix}1^{(6+|k|+m)} \\ 2^{(4-|k|-m)}\end{pmatrix} +$$

$$\binom{k_2+k_3}{k_3}Z_{21}^{(k_1)}xZ_{21}^{(k_2+k_3)}xZ_{21}^{(m)}x\begin{pmatrix} w \\ w' \end{vmatrix}\begin{matrix}1^{(6+|k|+m)} \\ 2^{(4-|k|-m)}\end{pmatrix} -$$

$$\binom{k_3+m}{m}Z_{21}^{(k_1)}xZ_{21}^{(k_2)}xZ_{21}^{(k_3+m)}x\begin{pmatrix} w \\ w' \end{vmatrix}\begin{matrix}1^{(6+|k|+m)} \\ 2^{(4-|k|-m)}\end{pmatrix} +$$

$$Z_{21}^{(k_1)}xZ_{21}^{(k_2)}xZ_{21}^{(k_3)}x\partial_{21}^{(m)}\begin{pmatrix} w \\ w' \end{vmatrix}\begin{matrix}1^{(6+|k|+m)} \\ 2^{(4-|k|-m)}\end{pmatrix}$$

$$= -\binom{k_1+k_2}{k_2}Z_{21}^{(k_1+k_2)}xZ_{21}^{(k_3)}xZ_{21}^{(m)}x\begin{pmatrix} w \\ w' \end{vmatrix}\begin{matrix}1^{(6+|k|+m)} \\ 2^{(4-|k|-m)}\end{pmatrix} +$$

$$\binom{k_2+k_3}{k_3}Z_{21}^{(k_1)}xZ_{21}^{(k_2+k_3)}xZ_{21}^{(m)}x\begin{pmatrix} w \\ w' \end{vmatrix}\begin{matrix}1^{(6+|k|+m)} \\ 2^{(4-|k|-m)}\end{pmatrix} -$$

$$\binom{k_3+m}{m}Z_{21}^{(k_1)}xZ_{21}^{(k_2)}xZ_{21}^{(k_3+m)}x\begin{pmatrix} w \\ w' \end{vmatrix}\begin{matrix}1^{(6+|k|+m)} \\ 2^{(4-|k|-m)}\end{pmatrix} +$$

$$Z_{21}^{(k_1)}xZ_{21}^{(k_2)}xZ_{21}^{(k_3)}x\begin{pmatrix} w \\ w' \end{vmatrix}\begin{matrix}1^{(6+|k|)}2^{(m)} \\ 2^{(4-|k|-m)}\end{pmatrix},$$

where $|k| = k_1+k_2+k_3$.

It is clear that $S_2\partial_\varkappa + \partial_\varkappa S_3 = id_{M_3}$

From the foregoing we concluded $\{S_0, S_1, S_2, S_3,\}$ is a contracting homotopy [8] this proves on that the Complex is exact.

3. The resolution of two rows Weyl module in the skew – partition $(6,4)/(1,0)$

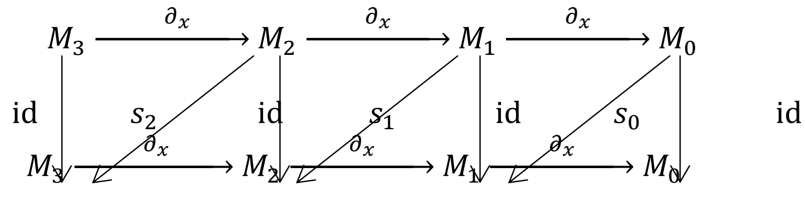
$M_0 = D_5 \otimes D_4$

$M_1 = Z_{21}^{(2)}xD_7 \otimes D_2 \oplus Z_{21}^{(3)}xD_8 \otimes D_1 \oplus Z_{21}^{(4)}xD_9 \otimes D_0$

$M_2 = Z_{21}^{(2)}xZ_{21}xD_8 \otimes D_1 \oplus Z_{21}^{(3)}xZ_{21}xD_9 \otimes D_0 \oplus Z_{21}^{(2)}xZ_{21}^{(2)}xD_9 \otimes D_0$

$M_3 = Z_{21}^{(2)}xZ_{21}xZ_{21}xD_9 \otimes D_0$

Thus we got the following diagram:



$S_0: D_5 \otimes D_4 \longrightarrow \sum_{k>0} Z_{21}^{(k+1)}\, x\, D_{5+k} \otimes D_{4-k}$

$$S_0\left(\begin{pmatrix} w \\ w' \end{vmatrix}\begin{matrix}1^{(5)}2^{(k)} \\ 2^{(4-k)}\end{pmatrix}\right) = \begin{cases} Z_{21}^{(k+1)}x\begin{pmatrix} w \\ w' \end{vmatrix}\begin{matrix}1^{(5+k)} \\ 2^{4-k)}\end{pmatrix} & ; if\ k = 2,3,4 \\ 0 & ; if\ k \le 1 \end{cases}$$

$S_1: \sum_{k>0} Z_{21}^{(k+1)}\, x\, \mathrm{D}_{6+k} \otimes \mathrm{D}_{3-k} \longrightarrow Z_{21}^{(k_1+1)}xZ_{21}^{(k_2)}xD_{6+k} \otimes D_{3-k}$ such that:

$$S_1\left(Z_{21}^{(k+1)}x\begin{pmatrix} w \\ w' \end{vmatrix}\begin{matrix}1^{(6+k)}2^{(m)} \\ 2^{(3-k-m)}\end{pmatrix}\right) = \begin{cases} Z_{21}^{(k+1)}xZ_{21}^{(m)}x\begin{pmatrix} w \\ w' \end{vmatrix}\begin{matrix}1^{(6+k+m)} \\ 2^{(3-k-m)}\end{pmatrix} & ; if\ m = 1,2 \\ 0 & ; if\ m = 0 \end{cases}$$

where $|k| = k_1 + k_2$

$S_2: \sum_{k_i>0} Z_{21}^{(k_1+1)}\, x\, Z_{21}^{(k_2)}x\mathrm{D}_{6+|k|} \otimes \mathrm{D}_{3-|k|} \longrightarrow Z_{21}^{(k_1+1)}xZ_{21}^{(k_2)}xZ_{21}^{(k_3)}x\mathrm{D}_{6+|k|} \otimes \mathrm{D}_{3-|k|}$

such that:

$$S_2\left(Z_{21}^{(k_1+1)}xZ_{21}^{(k_2)}x\begin{pmatrix}w\\w'\end{pmatrix}\left|\begin{matrix}1^{(6+|k|)}2^{(m)}\\2^{(3-|k|-m)}\end{matrix}\right)\right) =$$

$$\begin{cases}Z_{21}^{(k_1+1)}xZ_{21}^{(k_2)}xZ_{21}^{(m)}x\begin{pmatrix}w\\w'\end{pmatrix}\left|\begin{matrix}1^{(6+|k|+m)}\\2^{(3-|k|-m)}\end{matrix}\right) & ;if\ m=1\\0 & ;if\ m=0\end{cases};$$

where $|k| = k_1+k_2$

$$S_0\partial_\varkappa\left(Z_{21}^{(k+1)}x\begin{pmatrix}w\\w'\end{pmatrix}\left|\begin{matrix}1^{(6+k)}2^{(m)}\\2^{(3-k-m)}\end{matrix}\right)\right) = S_0\partial_{21}^{(k+1)}\begin{pmatrix}w\\w'\end{pmatrix}\left|\begin{matrix}1^{(6)}2^{(k+m)}\\2^{(3-k-m)}\end{matrix}\right)$$

$$= \binom{k+1+m}{m}Z_{21}^{(k+1+m)}x\begin{pmatrix}w\\w'\end{pmatrix}\left|\begin{matrix}1^{(6+k+m)}\\2^{(3-k-m)}\end{matrix}\right),$$

and

$$\partial_\varkappa S_1\left(Z_{21}^{(k+1)}x\begin{pmatrix}w\\w'\end{pmatrix}\left|\begin{matrix}1^{(6+k)}2^{(m)}\\2^{(3-k-m)}\end{matrix}\right)\right) = \partial_\varkappa\left(Z_{21}^{(k+1)}xZ_{21}^{(m)}x\begin{pmatrix}w\\w'\end{pmatrix}\left|\begin{matrix}1^{(6+k+m)}\\2^{(3-k-m)}\end{matrix}\right)\right)$$

$$= -\binom{k+1+m}{m}Z_{21}^{(k+1+m)}x\begin{pmatrix}w\\w'\end{pmatrix}\left|\begin{matrix}1^{(6+k+m)}\\2^{(3-k-m)}\end{matrix}\right) + Z_{21}^{(k+1)}x\begin{pmatrix}w\\w'\end{pmatrix}\left|\begin{matrix}1^{(6+k)}2^{(m)}\\2^{(3-k-m)}\end{matrix}\right),$$

It is clear that $S_0\partial_\varkappa + \partial_\varkappa S_1 = id_{M_1}$.

$$S_1\partial_\varkappa\left(Z_{21}^{(k_1+1)}xZ_{21}^{(k_2)}x\begin{pmatrix}w\\w'\end{pmatrix}\left|\begin{matrix}1^{(6+|k|)}2^{(m)}\\2^{(3-|k|-m)}\end{matrix}\right)\right)$$

$$= S_1\left(-\binom{|k|+1}{k_2}Z_{21}^{|k|+1}x\begin{pmatrix}w\\w'\end{pmatrix}\left|\begin{matrix}1^{(6+|k|)}2^{(m)}\\2^{(3-|k|-m)}\end{matrix}\right) + Z_{21}^{(k_1+1)}x\partial_{21}^{(k_2)}\begin{pmatrix}w\\w'\end{pmatrix}\left|\begin{matrix}1^{(6+|k|)}2^{(m)}\\2^{(3-|k|-m)}\end{matrix}\right)\right)$$

$$= -\binom{|k|+1}{k_2}Z_{21}^{|k|+1}xZ_{21}^{(m)}x\begin{pmatrix}w\\w'\end{pmatrix}\left|\begin{matrix}1^{(6+|k|+m)}\\2^{(3-|k|-m)}\end{matrix}\right) +$$

$$\binom{k_2+m}{m}Z_{21}^{(k_1+1)}xZ_{21}^{(k_2+m)}x\begin{pmatrix}w\\w'\end{pmatrix}\left|\begin{matrix}1^{(6+|k|+m)}\\2^{(3-|k|-m)}\end{matrix}\right),$$

and

$$\partial_\varkappa S_2\left(Z_{21}^{(k_1+1)}xZ_{21}^{(k_2)}x\begin{pmatrix}w\\w'\end{pmatrix}\left|\begin{matrix}1^{(6+|k|)}2^{(m)}\\2^{(3-|k|-m)}\end{matrix}\right)\right) = \partial_\varkappa\left(Z_{21}^{(k_1+1)}xZ_{21}^{(k_2)}xZ_{21}^{(m)}x\begin{pmatrix}w\\w'\end{pmatrix}\left|\begin{matrix}1^{(6+k+m)}\\2^{(3-k-m)}\end{matrix}\right)\right)$$

$$= \binom{|k|+1}{k_2}Z_{21}^{(|k|+1)}xZ_{21}^{(m)}x\begin{pmatrix}w\\w'\end{pmatrix}\left|\begin{matrix}1^{(6+|k|+m)}\\2^{(3-|k|-m)}\end{matrix}\right) -$$

$$\binom{k_2+m}{m}Z_{21}^{(k_1+1)}xZ_{21}^{(k_2+m)}x\begin{pmatrix}w\\w'\end{pmatrix}\left|\begin{matrix}1^{(6+|k|+m)}\\2^{(3-|k|-m)}\end{matrix}\right) + Z_{21}^{(k_1+1)}xZ_{21}^{(k_2)}x\begin{pmatrix}w\\w'\end{pmatrix}\left|\begin{matrix}1^{(6+|k|)}2^{(m)}\\2^{(3-|k|-m)}\end{matrix}\right),$$

where $|k/= k_1+k_2$.

It is clear that $S_1\partial_\varkappa + \partial_\varkappa S_2 = id_{M_2}$.

From the foregoing we concluded $\{S_0, S_1, S_2\}$ is a contracting homotopy [8] this proves on that the complex is exact.

4. Conclusions:

We concluded from this work the sequence in the cases of (6,4) and (6,4) / (1,0) are exact.

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