Measuring Curvature Method for the Exact Value of the Ellipse Perimeter

Laith H. M. Al-Ossmi[1]

[1] Civil Engineering Department /College of Engineering/ University of Thi-Qar, Thi-Qar, Iraq.
E-mail: laith-h@utq.edu.iq , hardmanquanny@gmail.com
ORCID iD: 0000-0002-6145-9478

Nesreen Kurdy Al-Obaidy

[2] Civil Engineering Department /College of Engineering/ University of Thi-Qar, Thi-Qar, Iraq.
E-mail: nesreen.kurdy@utq.edu.iq
ORCID iD: 0000-0002-1441-1682

ABSTRACT

This article introduces a novel geometric approach known as the Measuring Curvature Method MCM for precisely determining the perimeter of an ellipse. The MCM involves the construction of three centered circles: the Unit Circle, which has a radius equal to the major axis of the ellipse b; the Base Circle, with a radius of 2b/π; and the Measuring Circle. The radius of the Measuring Circle intersects with a newly defined curvature termed the Measuring Curvature. By utilizing this method, the circumference of the Measuring Circle provides the exact value of the ellipse perimeter.
To establish a practical procedure for the MCM, data is generated, refined, and organized to create a chart and establish an empirical relationship between the radius of the Measuring Circle and the lengths of the ellipse's semi-axes. As a result, the perimeter can be readily determined by multiplying the radius obtained from the developed equation by the constant 2π. Additionally, a mathematical analysis is conducted to derive an equation that connects the radius of the Measuring Circle with four parameters: the major and minor axes of the ellipse, and the horizontal and vertical coordinates of the intersection point on the Measuring Curvature between the ellipse and its measuring circle. This equation enables the perimeter to be conventionally and simply calculated. To further validate the accuracy of the MCM theorem, numerical comparisons are performed using Ramanujan's method with the PRI test. The outcomes demonstrate that employing the MCM theorem yields highly reliable and precise calculations for determining the perimeter of an ellipse.

Keywords: Base Circle, Ellipse Perimeter, Geometric Method, Measuring Circle, Measuring Curvature.

Introduction

An ellipse is a curve formed by a plane, that intersects a cone at an angle with respect to the base. The perimeter of an ellipse is the total distance run by its outer boundary. Each of the two fixed points is called a focus. Unfortunately, there is no easy technique to determine the perimeter of the ellipse by using elementary functions of a and b, instead it requires complicated functions beyond trigonometric, exponential, and logarithmic functions [1]. Geometrically, an ellipse is symmetry and closed-form of conic curves that are built adopting geometric methods to generate points and connect them to determine the curve and shape it. Mathematically, there are many formulas that typically dealt with the perimeter of the ellipse which all still help to determine its approximation value [2]. The most recent work on the subject has been carried out by Barnard et al., 2001[3], Abbott, 2009 [4], Adlaj, 2012 [5], Ahmedi, 2018 [6], and recently by works of Qureshiet al., 2020 [7], and recently in innovative work of Rohman, 2022 [8], and Tiehonget al., 2022 [9].

Promptly, all these methods also prove the major conclusion in fundamental point; the formulae for calculating the perimeter of an ellipse indicated that there are not exact but approximate methods and formulas. The problem is erroneously resumed that the ellipse does not have a compact analytical solution and is confirmed by hyper-geometric functions [10,11] which is not satisfactory, accordingly there is still a need to bridge the geometric gap for closed-form methods to deal with such field, which is later infrequently studied [12-15].

The article's motivation is linked to the problem of finding the perimeter of an ellipse, which generally is summarized by the fact that the ellipse perimeter is not defined accurately. As a result, researchers have already attempted to find an appropriate solution for this issue. In general, there is a perfect formula using an integral, which is exact formulae, but they need an "infinite series" of calculations to be exact, in fact, the terms continue on infinitely, and unfortunately, they must calculate a lot of terms to get a reasonably close answer, so in practice, it still only get an approximation. Nevertheless, the formulae of these geometric methods are also well known for their simplicity and are eloquently expressible by two-dimensional geometric mean. With this paper, the quest for a concise formula giving rise to an exact swiftly convergent method permitting the calculation of the perimeter value of an ellipse is over. In this article, a new approach of the Measuring Curvature Method MCM is proposed to define the perimeter of any form of ellipse given a and b. The article also addresses the challenge of accurately determining the perimeter of an ellipse, which has traditionally relied on complex mathematical functions beyond elementary trigonometric, exponential, and logarithmic functions.

Materials and Methods

This method aims to provide a concise formula that enables swift convergence and exact calculation of the ellipse's perimeter. In this article, the Measuring Curvature Method MCM is extensively simulated and applied to various numerical scenarios to establish a new general approach for accurately determining the perimeter of an ellipse. To ensure the reliability of the results, all outcomes are thoroughly verified and statistically analyzed to obtain the numerical values of the ellipse's perimeter for different geometries.

To enhance accuracy, the efficiency of the MCM calculations, obtained through empirical analysis, is evaluated by comparing them with Ramanujan's method. The results are validated and tabulated using PRI's test, providing further confidence in the accuracy of the proposed method. Additionally, detailed figures and measurements of the MCM are generated and illustrated using AutoCAD, a computer drawing application (Version 2020). This serves to ensure consistency between the results obtained from both methods and presented in this paper. Furthermore, a mathematical relationship is derived to calculate the circumference of an ellipse based on the radius of the corresponding Measuring Circle. This relationship involves the axes of the ellipse and the intersection point on the Measuring Curvature. As a result, given the values of a and b for an ellipse and specifying the intersection point on the Measuring Curvature, the perimeter of the associated ellipse can be determined accurately using the proposed method.

The construction of Measuring Curvature Method (MCM)

The MCM's geometric approach, as its name says, is related to studying the geometry of two-dimensional mean by using the values of the ellipse major and minor axes, (a) and (b) to determine the perimeter of an ellipse. Where the ellipse center (C) is at the origin (0,0) and the foci are on the x-axis, thus the minor axis (a) on the y-axis, then let an ellipse, at which (a) and

(b) are the length of semi-major and minor axes respectively, as shown in Fig.1.

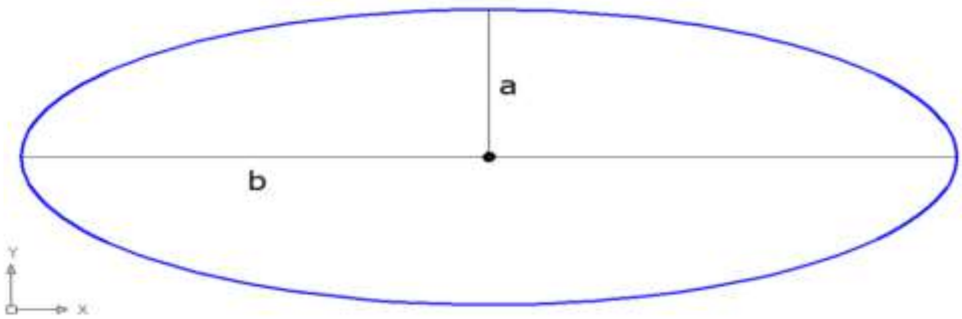
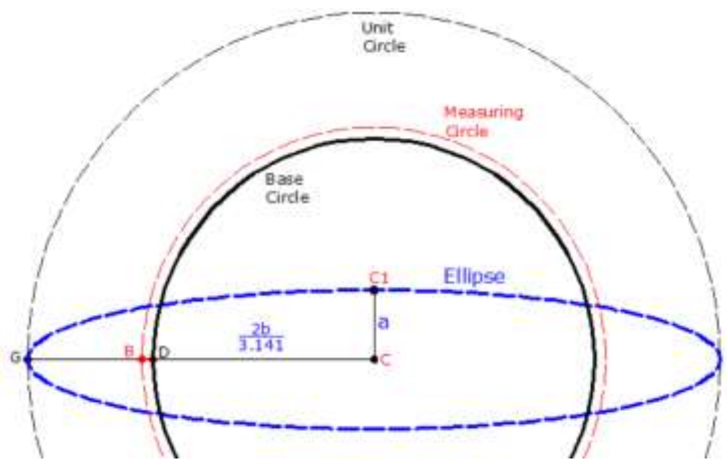


Figure 1. Plot an ellipse, where (a) and (b) are the length of semi-major and minor axes respectively.

To plot the new method produced in this paper, MCM, the following steps illustrated in Fig.2 are required:

- Let an ellipse, where (a) and (b) are the length of semi-major and minor axes respectively.
- From the ellipse center point (C), draw the Base Circle with radius of ($\frac{2b}{\pi}$),
- From the ellipse center point (C), draw the Unit Circle with radius of (b),
- From the ellipse center point (C), draw a ray to intersect the Unit Circle at (C1),
- From the ellipse center point (C), draw a horizontal line to intersect the Base Circle at (D1),
- Let the Measuring Curvature lies from points; (D1) and (C1) respectively,
- The intersection point between the ellipse and the Measuring curvature is (B),
- Construct a line from point (B) and (C),
- The line segment (BC) is the radius of the Measuring Circle,

The circumference of the Measuring Circle is the exact value of the ellipse perimeter.



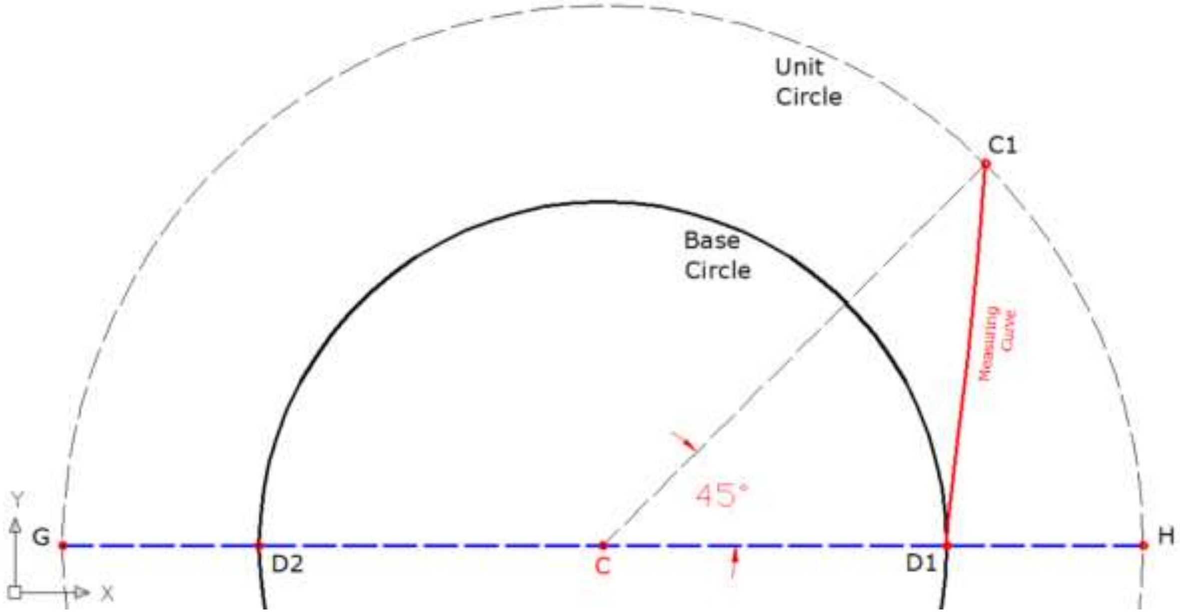


Figure2. the Measuring Curvature Method (MCM) construction, where the Base Circle with radius of ($\frac{2b}{\pi}$), and the Unit Circle with radius of (b)

It is noticeable here from figure 2, that the Base Circle's radius (CD1) determines the minimize value of the Measuring Circle's radius (CB=r), hence the circumference of Base Circle presents the smallest value of the Measuring Circle's radius which is accrued when the length of semi- minor axes of the ellipse (a = 0) then, the Measuring Circle and the Base Circle share the same value of the radius, CD1= ($\frac{2b}{\pi}$), thus, the obtained curve is not an ellipse instead it is a line (GH) with length of (2b). Also, the radius value of the Base Circle is a constant, as:

$$CD1 = \left(\frac{2b}{\pi}\right) = 0.63661977236758134307553505349006\ b,$$

while D1H = $b - \left(\frac{2b}{\pi}\right) = \ 0.36338022763241865692446494650994\ b,$

Then:

$$\left(\frac{b(\pi - 2)}{\pi}\right) = \ 0.36338022763241865692446494650994\ b \qquad 1$$

From Figure 2 and these steps, the Base Circle with radius of (CD1= r) shares the same center point of the ellipse, hence the circumference of the Measuring Circle is the exact value of the ellipse perimeter then:

$$Ellipse\ Primeter = 2\pi r \qquad 2$$

$$Base\ Circle\ (area) = \pi \left(\frac{2b}{\pi}\right)^2 \qquad 3$$

$$Base\ Circle\ (area) = \left(\frac{4b^2}{\pi}\right) \qquad 4$$

$$Measuring\ Circle\ (area) = \pi b - \frac{4b^2}{\pi} \qquad 5$$

$$Measuring\ Circle\ (area) = \left(\frac{\pi^2 b - 4b^2}{\pi}\right) \qquad 6$$

$$Measuring\ Circle\ (area) = \left(\frac{b(\pi^2 - 4b)}{\pi}\right) \quad 7$$

According to the ellipse proportions, there are three types of curvatures, one of them is a straight line. The MCM deals with all possible cases of the curvature; an ellipse, a circle, and a line, hence the values of (a) varied from (0) to (b), the results vary from a line with length of $(4b)$, and to an ellipse and when $(0 > a \leq b)$, since (a) value comes less but close to (b), and finally if (a) value became equal to (b) it is a circle. In general, these 3 cases can be listed as following:

1) When $(a = 0)$, then it is a Line with length of $(4b)$.
2) When $(a = b)$, then it is a Circle with radius of (b) and a circumference of $\left(\frac{\text{EP}}{b}\right)$, where (EP) is the Ellipse Perimeter.
3) When $(0 > a \leq b)$, then it is an Ellipse with circumference of $(r\ 2b\pi)$.

The Measuring Curvature's properties

The Measuring Curvature Method (MCM) is a geometric approach that enables the exact calculation of the perimeter of an ellipse. This method introduces a unique curvature known as the Measuring Curvature, which is generated within the MCM framework. When the ellipse intersects with this curvature at point (B), the radius of the Measuring Circle (CB) is determined by this intersection point.

One key characteristic of the MCM is that the length of the Measuring Curvature remains constant throughout the method. You can refer to Figure 3 and Table 1 in the article for visual representation and further details regarding this constant length.

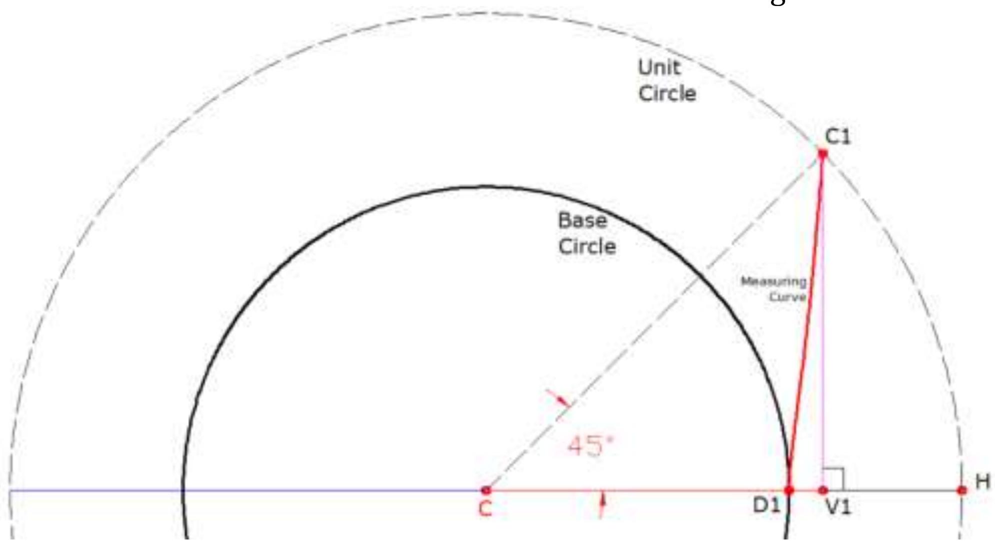


Figure 3. plot the Measuring Curvature's geometrical properties according to MCM.

In MCM, the Measuring Curvature (C1D1) is a constant which has three point of twisting and a specific length of (0.7102 b), (see Table 1). The angle from the center point (C) and the top point of the curvature (C1) is a constant, $(\frac{\pi}{4})$, and then from the right-angled triangle (CC1V1), it easily to found;

$$CC1 = b$$
$$CV1 = b\cos 45$$
$$C1V1 = b\,sin\,45$$
$$CD1 = \left(\frac{2b}{\pi}\right)$$
$$D1V1 = b\cos 45 - \left(\frac{2b}{\pi}\right) \quad 8$$

According to MCM's properties, to determine the Measuring Curvature, draw a ray from the center point (C) with an angle of $(\frac{\pi}{4})$, then the intersection point (C1) is a top point of the Measuring curvature, the curve is extending from point (C1) till point (D1) and through it will pass the curve which creates two peaks and tumbling, (D1), (L1), (L2) and (C1), hence:

1) -The area enclosed by the Measuring Curve and both segments; (C1V1) and (D1V1), is a constant, its value equals to (0.02390 b).

2) -The area enclosed by the Measuring Curve and both segments; (CC1) and the Base Circle is a constant, its value equals to (0.06710 b).

As they are listed in Table 1.

Table 1. The Measuring Curvature's properties in MCM

Code	Value	Type	Description
B2H	b 0.7065	Line	(B2H=CV1) if angle (B2CH) =90
CC1	b	Line	b
C1H	b 0.7645	Line	Constant
C1V1	b 0.7065	Line	$b \sin 45$
D1V1	b 0.0711	Line	$b \cos 45 - \left(\frac{2b}{\pi}\right)$
CD1	b 0.6366	Line	Constant = $\left(\frac{2b}{\pi}\right)$
C1D1	b 0.7102	Curve	Measuring Curvature (Constant)
CH	b	Line	b
CV1	b 0.7077	Line	Constant $= b \cos 45$
V1H	b 0.2923	Line	Constant
C1CH	45	Angle	Constant
CB	$CD1 \geq CB \leq b$	Line	$\left(\frac{2b}{\pi}\right) \geq r \leq b$

Where (r) is the Measuring Circle's radius, BC.

From the above properties, Table1, this curvature (the Measuring Curve) is neither a parabola's segment and also nor a circle's segment, it is a special form of curvature enclosed by two points, (C1) and (D1). And it has two loops (L1) and (L2), hence the intersection point of the ellipse and the Measuring Circle is (B), which lies at the Measuring Curve (D1C1), as it is shown in Fig.4, where;

$$CD2 = CD1 = \left(\frac{2b}{\pi}\right),$$

As it is shown in Fig.4, whenever the intersection point (B) lies at (C1), then it is a Circle where; (a = b), and then (CB = C1C), therefore; (CB = a). whereas if the intersection point (B) lies at (D1), then it is a line and; (a =0), (CB = D1C). While if the intersection point (B) lies between (C1) and (D1), then it is an ellipse and formerly;

$$(0 > a < b), \left(\frac{2b}{\pi} > CB < b\right) \qquad 9$$

Point (B) is the intersection of the ellipse with the Measuring Curvature, (C1D1), then, (CB) is the radius of the Measuring Circle, the circumference of the Measuring Circle = the Ellipse perimeter, and consequently;

$$Ellipse\ primeter = 2\pi(CB) \tag{10}$$

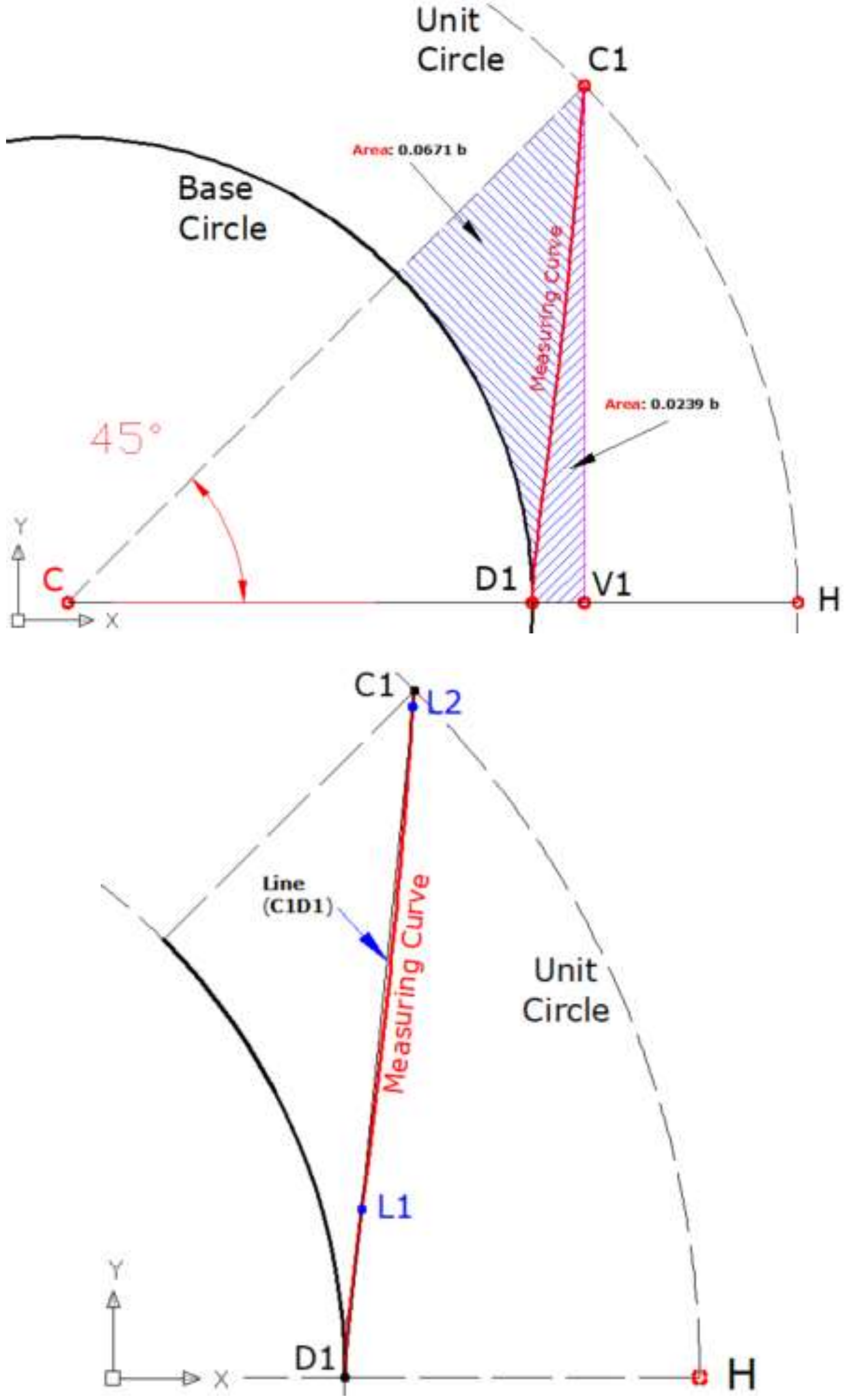


Figure 4. plot the properties of Measuring Curvature according to MCM.

Table 2. Measuring Curvature's key properties

Code	Measuring Curve	Line

(C1L2)	0.01697449 b	0.01697420 b
ratio	1.00001708475215326790	
Code	Measuring Curve	Line
(L1L2)	0.5194 b	0.5193 b
ratio	1.00019256691700300000	
Code	Measuring Curve	Line
(D1L1)	0.1739 b	0.1738 b
ratio	1.00057537399309551208	
Code	Measuring Curve	Line
(D1L1L2C1)	0.71023974 b	0.71005633 b
ratio	1.00025830345037555497820	

In this paper, the focus is on the geometric aspect of the Measuring Curvature Method (MCM), specifically in determining the radius of the Measuring Circle (BC). This radius plays a crucial role in Equation (10), which is used to calculate the exact value of the ellipse's perimeter.

However, in an upcoming paper, a new and innovative formula will be developed to determine the exact value of the ellipse's perimeter. This formula will likely build upon the foundations of the MCM and incorporate additional mathematical considerations. It is important to note that the reliability of the MCM has been extensively validated through numerical comparisons, confirming its accuracy and effectiveness in determining the perimeter of an ellipse. By combining the insights from this paper and the forthcoming one, the field of ellipse perimeter calculation is expected to benefit from improved techniques and formulas that offer precise and reliable results.

Determination the exact value of the ellipse perimeter by MCM

Consider an ellipse with a minor axis (a) and a major axis (b). Let's also introduce the Base Circle, which shares the same center point (C) as the ellipse. The radius of the Base Circle is denoted as (CD1). Now, draw an auxiliary ellipse centered at point (C), where the length of its semi-major axis is (b) and the length of its semi-minor axis is (a).

In this auxiliary ellipse, the Measuring Curvature is positioned between points (D1) and (C1). The point of intersection between the ellipse and the Measuring Curvature is labeled as (B). The line segment connecting points (B) and (C) represents the radius of the Measuring Circle and is denoted as (BC).

It is important to note that the circumference of the Measuring Circle, which is equal to 2π times the length of (BC), represents the exact value of the ellipse's perimeter. Refer to Figure 5 for a visual representation of these geometric relationships.

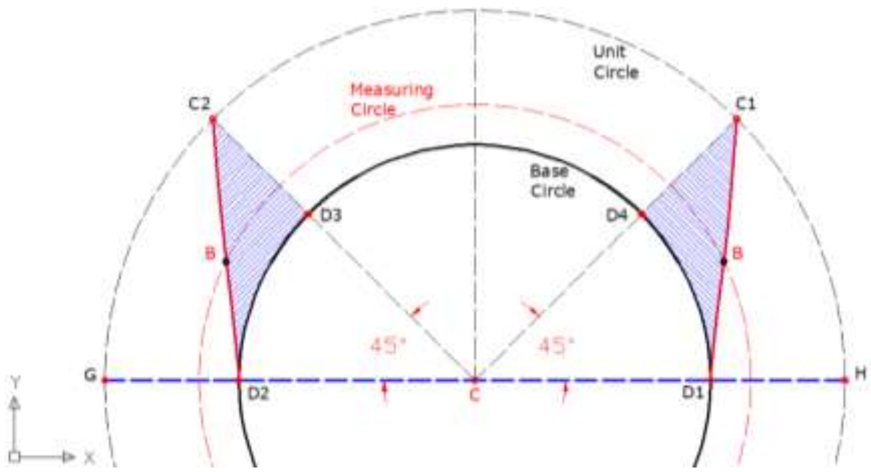


Figure 5. the MCM, the intersection point (B) of the ellipse with the Measuring Curvature determines the radius of the Measuring Circle which its circumference value is exactly the ellipse perimeter.

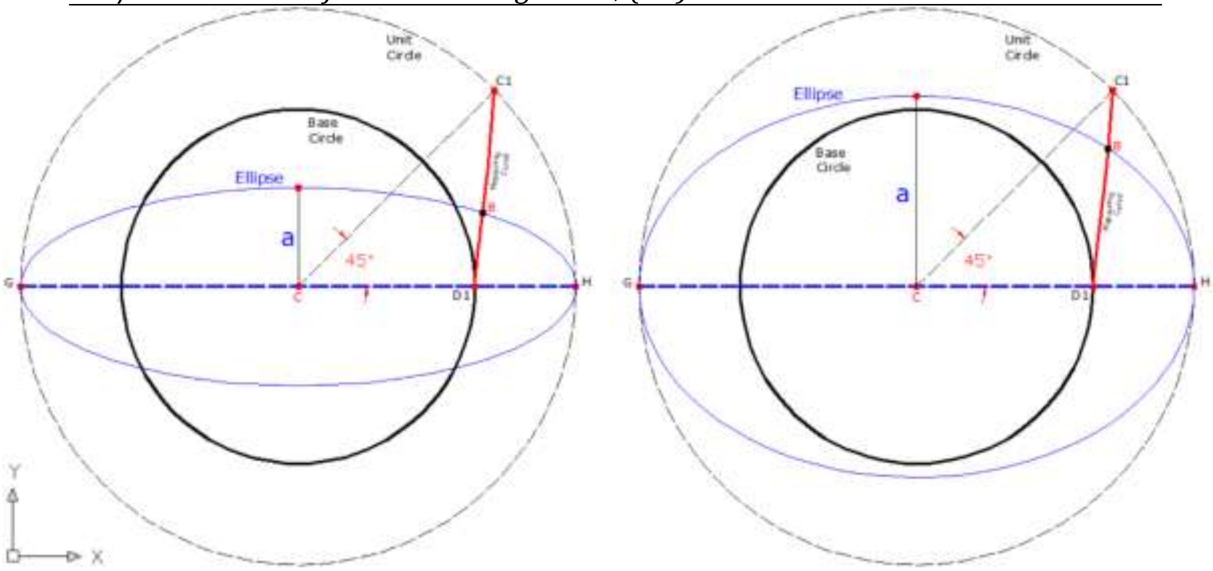
The MCM helps to determine geometrically the real length of the Measuring Circle's radius by determine the position of point (B) which is the intersection of ellipse and the Measuring Curvature, (Fig.5). Note that the point (B) is parametrically moving through a path of the Measuring curvature leading by value of (a), since when it comes closely to x-axis, then the value of (a) is declined closely to (0), whereas if it ,(B), raised forward to come close to point (C1), then the value of (a) comes gradually to value of (b), consequently the obtained curve (ellipse form) is roughly turned into an inflated ellipse and finally to a circle with radius of (b), (when; a = b). Although the Measuring Curvature has a limited length and geometrical properties, the MCM can deal with any form of ellipse, these flatted and inflated ellipses, which is a marvelous advantage point to this novel method. In addition, the coordinates of point (B) can easily define at both x-axis and y-axis, hence in MCM position of (B) is limited with a range of angle (C1CB), (a) and (b), (Table 2).

The coordinators of (B) along x-axis is limited between points (D1) and (V1), which is a constant with value of (0.07109540 b), while the vertical rang (along the y-axis) is another constant with value of (0.70648192 b), In general, there are 4 parameters control the radius length of Measuring Circle (CB) which are; (a), and these two constants; (C1V1) and (D1V1) respectively, in addition to the Measuring Curvature. Also, the MCM gave correct results of the ellipse perimeter at all cases in which the method was checked, when (a = 0), (a = b), and when $(0 > a \leq b)$, (as it is shown in Table 3 and Fig. 6).

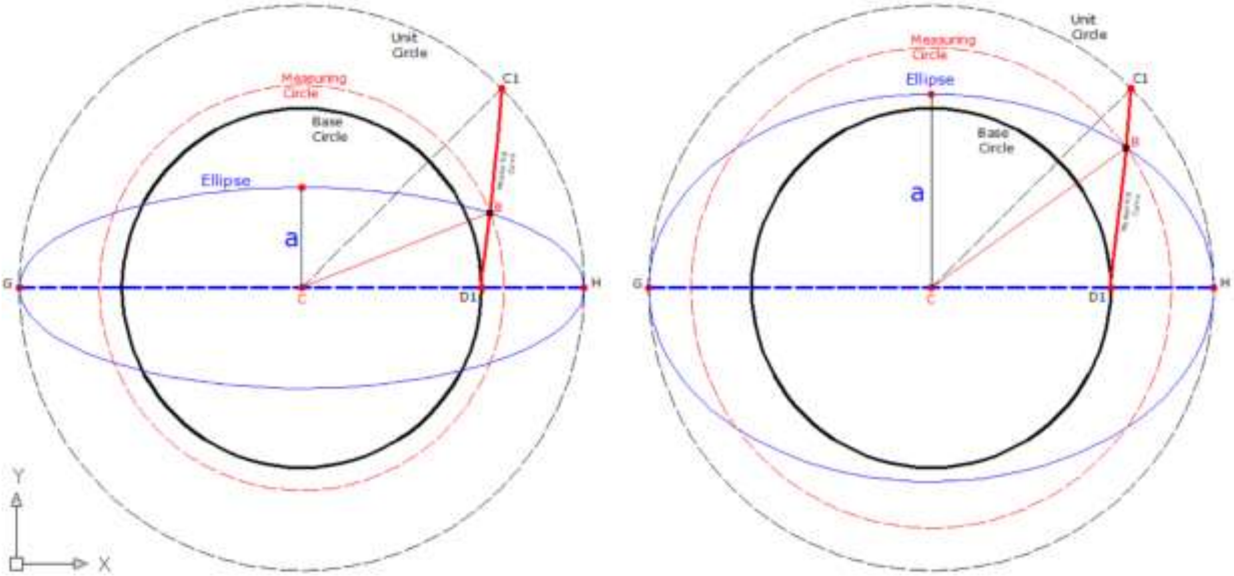
Table 3. key properties of the MCM

a	$\frac{EP}{2b\pi}$	EP
0	$\left(\frac{2b}{\pi}\right)$	$4b$
$0.0000000001\ b$	$0.6366197723675813\ b$	$4.00000000000b$
$0.0001\ b$	$0.6366213639250\ b$	$4.00001b$
$0.002\ b$	$0.6366356878\ b$	$4.0001b$
$0.010\ b$	$0.6367948428\ b$	$4.0011b$
$0.10\ b$	$0.6468\ b$	$4.0640b$
$0.20\ b$	$0.6688\ b$	$4.2020b$
$0.30\ b$	$0.69804\ b$	$4.3859b$
$0.40\ b$	$0.73252\ b$	$4.6026b$
$0.50\ b$	$0.770978\ b$	$4.8442b$
$0.60\ b$	$0.81254964\ b$	$5.1054b$
$\left(\frac{2b}{\pi}\right)$	$0.828433309782\ b$	$5.2052b$
$0.70\ b$	$0.8566355657\ b$	$5.3824b$
$0.80\ b$	$0.9027745837\ b$	$5.6723b$
$0.90\ b$	$0.95066430610\ b$	$5.9732b$
b	b	$2b\pi$

Where; (a) and (b) are the minor and major axes of the ellipse.
EP abbreviation of Ellipse Perimeter.
EP/2bπ = radius of the Measuring Circle, (BC).



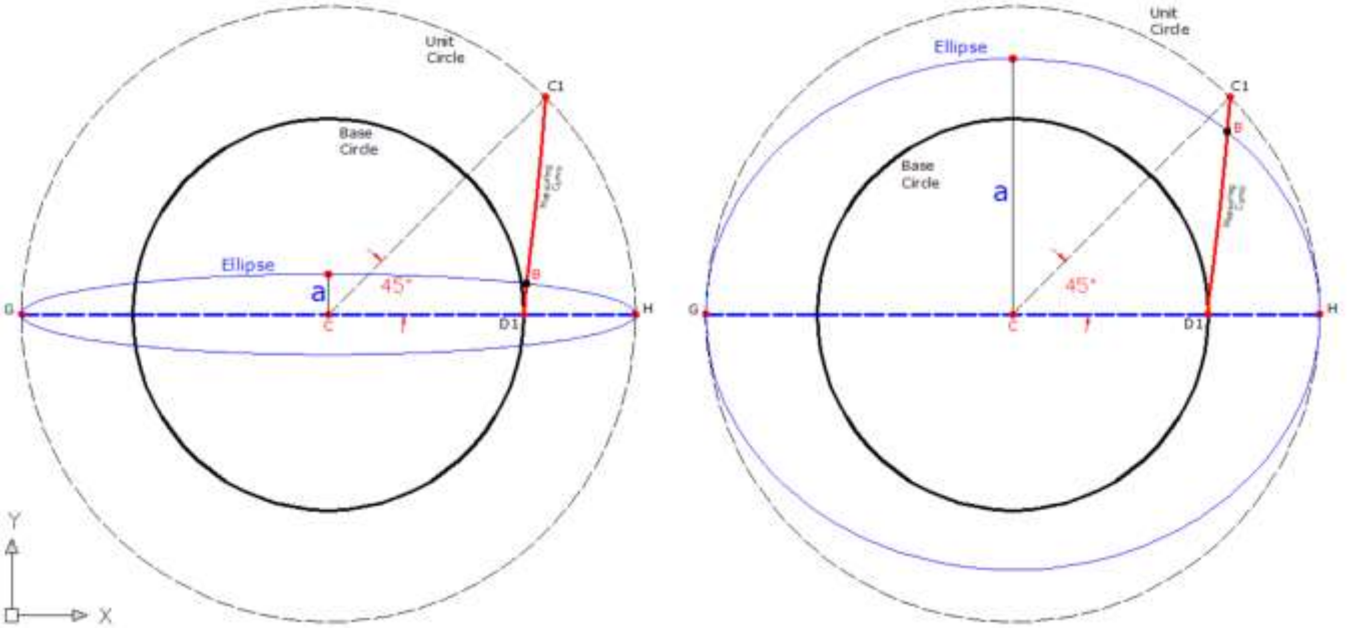
MCM, when; (a = 0.68570692b), and (a = 0.35552655 b).



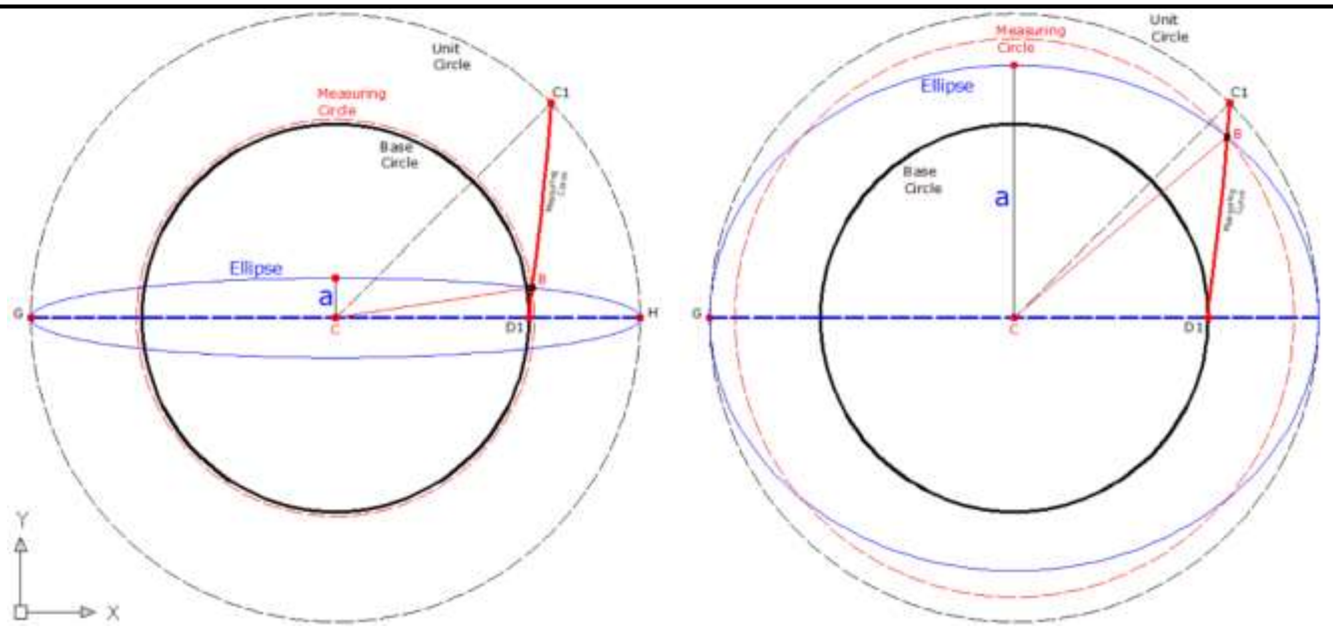
MCM, the Measuring Circle’s radius; (CB = 0.85020428 b), and (CB = 0.71663728 b).

Figure 6. plot two nominated examples of ellipses using MCM, with two values of (a), since the perimeter of the ellipses are defined accurately by value of (CB).

Table 3 presents these varied values of the Measuring Circle's radius (CB) produced by using MCM, which are gotten by dividing the ellipse perimeter (given here) on the circumference value of ($2\pi b$), hence the data in the table gathered by a set of range of ellipses from (a = 0) till ($a = b$). It is perceptible that the radius value of the Base Circle is a constant, ($2b/\pi$), which is accurately measured as; ($0.63663569b$), this constant can be consider as MCM's key factor to determination the exact value of the ellipse perimeter by MCM, (as it is shown in Fig. 7-8).

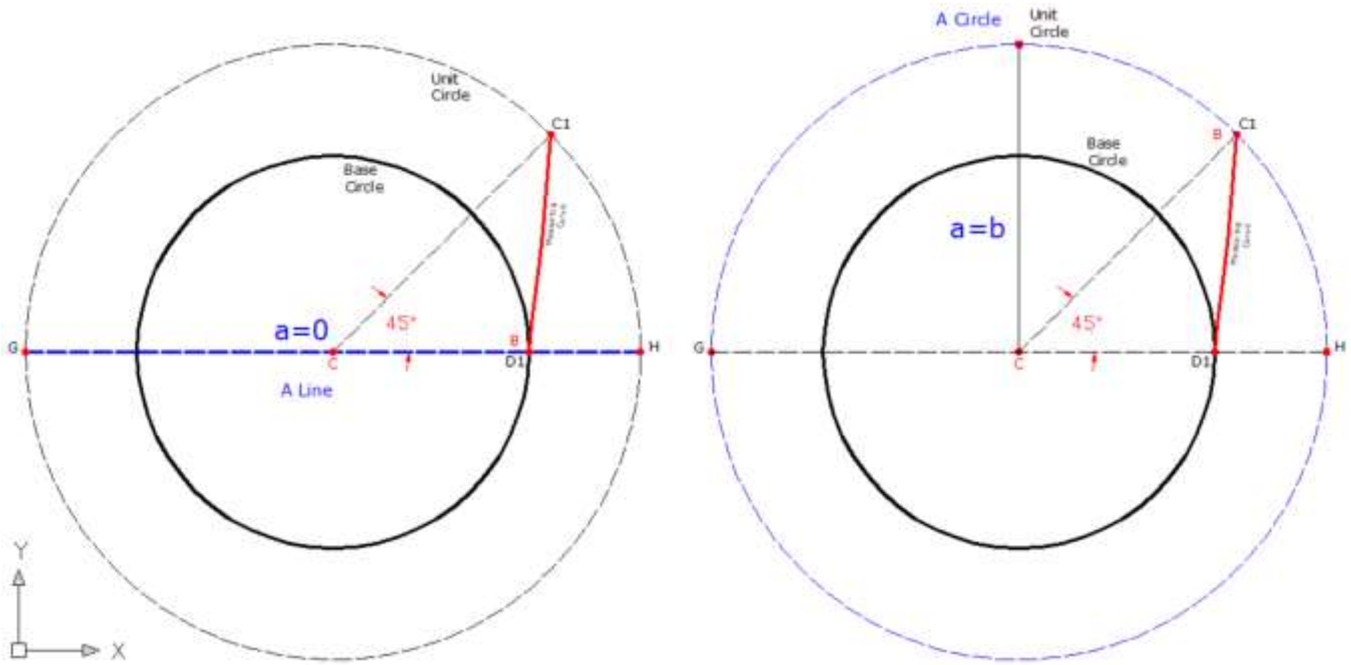


MCM, when; (a = 0.13080759b), and (a = 0.83127405b).

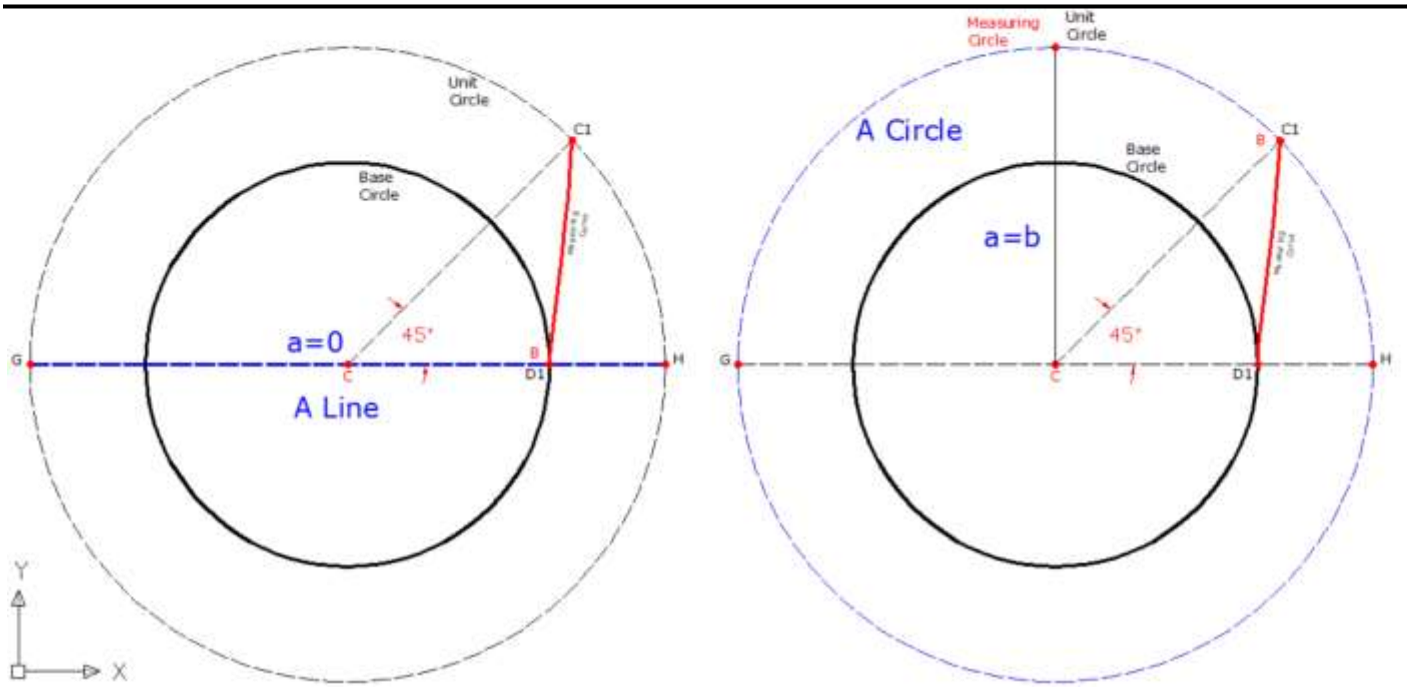


MCM, the Measuring Circle radius; (CB = 0.91759019b), and (CB = 0.65266674b).

Figure 7. plot two nominated samples of ellipses using MCM, with two values of (a), since the perimeter of the ellipses are defined accurately by value of (CB).



MCM, when; (a = 0), and (a = b).



MCM, the Measuring Circle radius; (CB = $\frac{2b}{\pi}$), and (CB = b).

Figure 8. plot two nominated samples of ellipses using MCM, with two values of (a), since the perimeter of the ellipses are defined accurately by value of (CB).

Relationship between Ellipse Perimeter and its Semi Axes Based on MCM Data

It is worth mentioned here that the MCM data corresponding to first and second column presented in Table3, can be visualized as illustrated in Fig.9 to form a chart of determination the radius of Measuring circle CB based on the values of the semi axes (a) and (b). For instance, if (a) and (b) values of an ellipse are given and the ellipse perimeter is required. First, the ratio of both axes (a/b) is to be determined and established on y-axis of the chart, then by plotting a horizontal line which will intersect the data curve.

Therefore, its project on the x-axis represents the value CB/b from which and by multiplying by b-value, the magnitude of CB is predicted. Finally, the ellipse perimeter can be simply estimated by applying eq.10.

On other hand, the radius CB can be estimated and related to the values of semi axes (a) and (b) simply by applying the following expression from the best fitting to the same data trend using the option of second degree polynomial trendline:

$$CB = 0.2014(a^2/b) + 0.1709a + 0.335b$$

Then, by substituting eq,11 into eq, 10, it yields:

$$Ellipse\ primeter\ (EP) = 2\pi[0.2014(a^2/b) + 0.1709a + 0.335b]$$

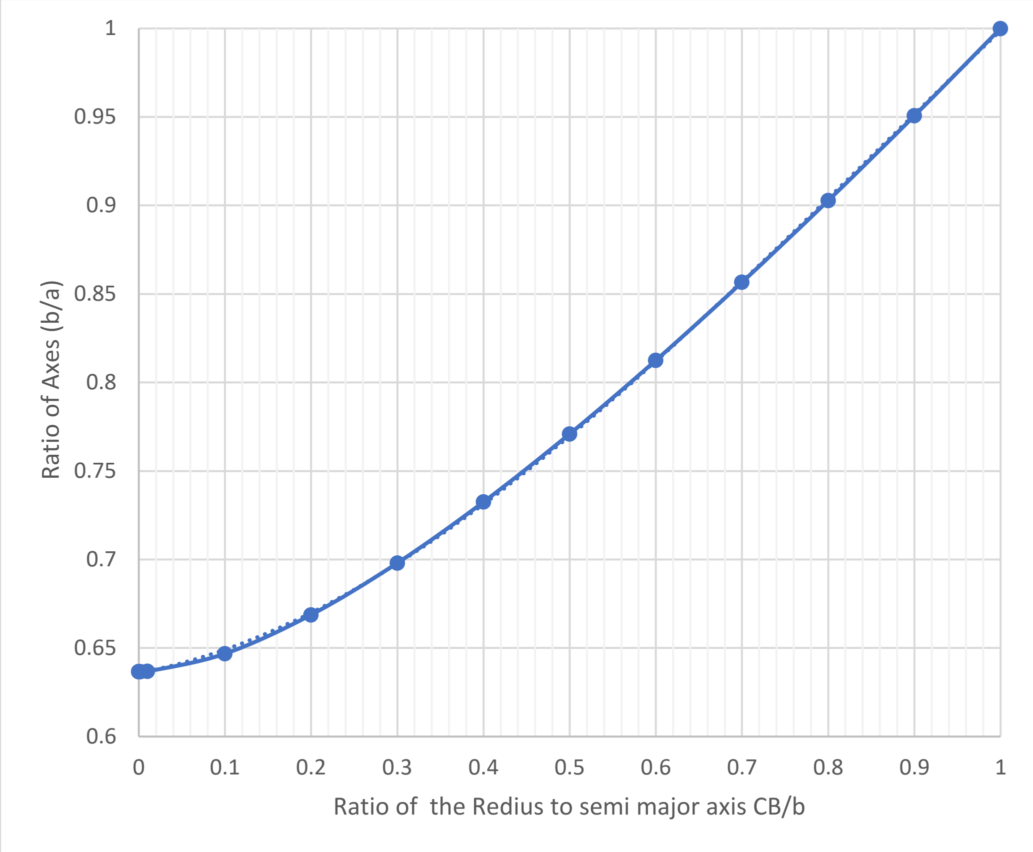


Figure 9. Chart of determination the radius CB from ellipse semi axes based on the MCM method data

Comparison the MCM with Ramanujan's Method

For supplementary accuracy, by fixing the value of (b), a set of 12 series of (a) value are used to check MCM's efficiency which have correspondingly confirmed by these obtained by Ramanujan's, subsequently the perimeter of the ellipses are defined accurately by value of (CB). Table 4 and Fig.10 demonstrate this comparison. Additionally, the efficient calculations of results which are empirically obtained by the MCM are evaluated to these gained by Ramanujan and all were comprised and tabulated using PRI test. Likewise, where point (C) is the center point and both (a) and (b) are the length of semi-major and minor axes respectively, the test of PRI showed a full fitting over the 5 digits, since the obtained ratio of accuracy which is compared with Ramanujan's, given a value of (2.4015e-07), this fitting is shown in Fig. 11.

Table 4. Comparison MCM's efficiency by Ramanujan's method.

a	MCM's	Ramanujan's	Ratio
0	4b	4b	1.0000000000000000
0.01 b	4.00102262 b	4.00109833 b	1.0000189226623267
0.1 b	4.06396426 b	4.06397418 b	1.0000024409663484
0.2 b	4.20213150 b	4.20200891 b	0.9999708267102064
0.3 b	4.38589970 b	4.38591007 b	1.0000023643951547
0.4 b	4.60255900 b	4.60262252 b	1.0000138010180858
0.5 b	4.84419997 b	4.84422411 b	1.0000049832790036
0.6 b	5.10539996 b	5.10539977 b	0.9999999627845023
0.7 b	5.38240009 b	5.38236898 b	0.9999942200506317
0.8 b	5.67229971 b	5.67233358 b	1.0000059711231302

0.9 b	5.97320000 b	5.97316043 b	0.9999933754101654
b	6.28318531 b	6.28318531 b	1.0000000000000000

where (a) and (b) are the length of semi-major and minor axes respectively.

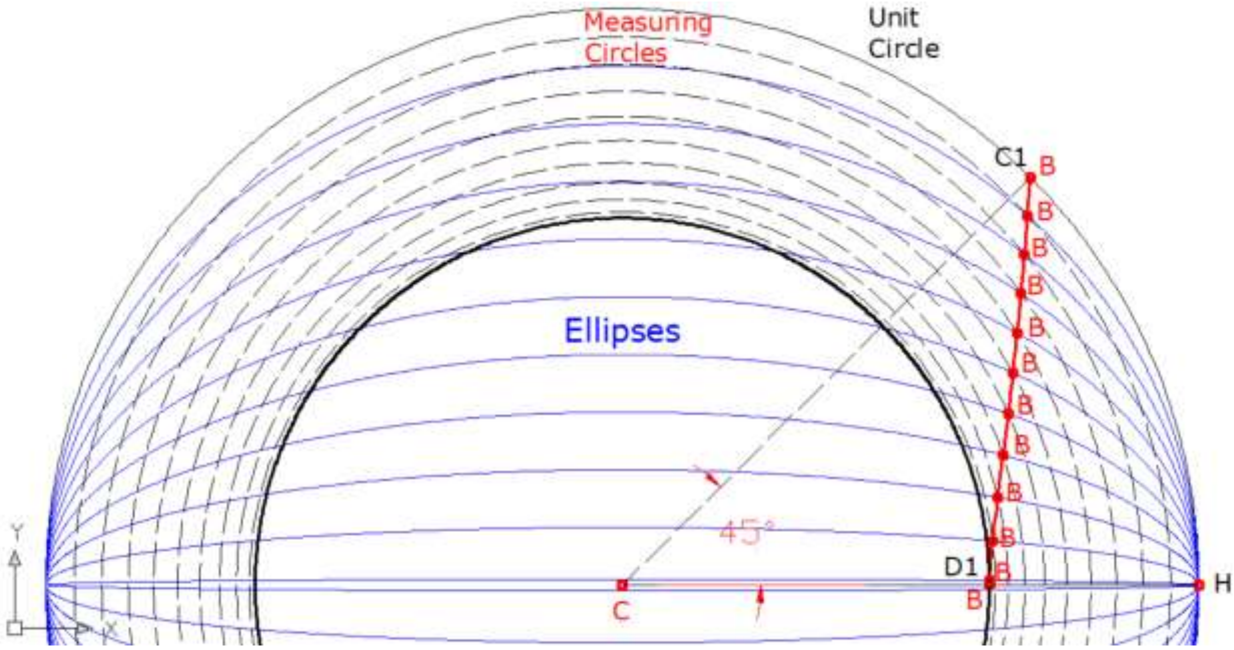


Figure 10. plot comparison MCM's efficiency by Ramanujan's method using 10 samples where the perimeter of the ellipses is defined accurately by value of (CB), the Measuring Circle's radius.

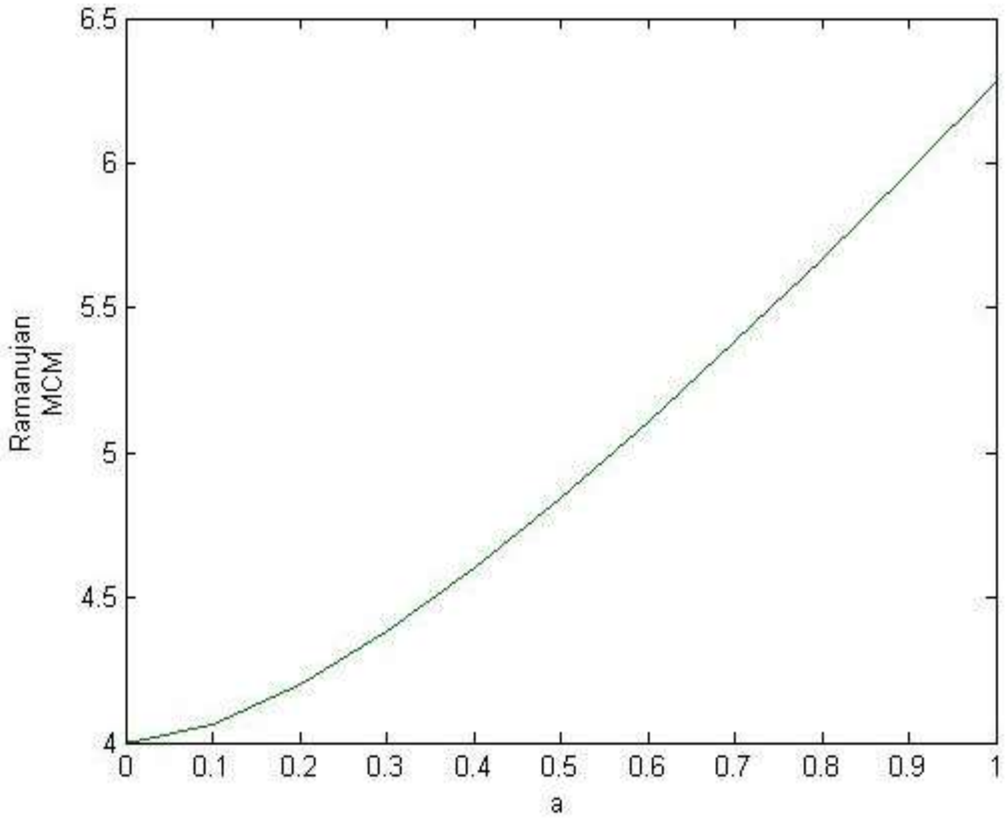


Figure 11. plot comparison of MCM's efficiency by Ramanujan's method using PRI test.

The Mathematical Analysis of MCM

Once semi axes of an ellipse (a) and (b) are given and the intersection point B(x,y) on the Measuring curvature is geometrically specified, the associated ellipse perimeter can be determined according to an equation deduced from the following steps:

Since the intersection point B(x,y) ∈ the ellipse and the measuring circle, so it satisfies both ellipse and circle standard equations as follows:

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \qquad 13$$

and $x^2+y^2=r^2$ *which can be written as:*

$$x^2+y^2=CB^2 \qquad 14$$

From eq,14, two expressions can be obtained, they are:

$$x^2=CB^2-y^2 \qquad 15$$

$$and \quad y^2=CB^2-x^2 \qquad 16$$

By substituting eq. 15 and eq, 16 into eq.13, it yields:

$$\frac{CB^2-y^2}{a^2}+\frac{CB^2-x^2}{b^2}=1 \qquad 17$$

Which can be written as:

$$\frac{b^2(CB^2-y^2)+a^2(CB^2-x^2)}{a^2b^2}=1 \quad or \quad b^2(CB^2-y^2)+a^2(CB^2-x^2)=a^2b^2$$

The last equation leads to:

$$b^2CB^2-b^2y^2+a^2CB^2-a^2x^2)=a^2b^2$$

$$which\ equivalant\ to \qquad (b^2+a^2)CB^2=a^2b^2+b^2y^2+a^2x^2$$

By simplification:

$$CB=\sqrt{\frac{b^2y^2+a^2(b^2+x^2)}{(a^2+b^2)}} \qquad 18$$

By substitution eq.18 into eq.10, it yields:

$$Ellipse\ primeter\ (EP)=2\pi\sqrt{\frac{b^2y^2+a^2(b^2+x^2)}{(a^2+b^2)}} \qquad 19$$

Conclusion

This paper deals with ellipse perimeter. The ellipse perimeter is measured by a geometric method called MCM with accuracy of (2.4015e-07). The MCM is intended by mathematics and geometry which is a drawn according to 3 circles shared the same center point, with two constant radiuses, (b) and $\left(\frac{2b}{\pi}\right)$, where value ($a$) is the minor axis of ellipse laying on the y-axis, hence the ellipse major axis (b) is passed through the x-axis. This article gives a new geometric method for key constructing features of this form of the conic curve; the measurements demonstrated by this method help to produce exactly value of the ellipse perimeter. Conceptually obtained results were examined through results obtained by Ramanujan(b). Also, it has been proved with appropriate PRI's test and rigor drawings of AutoCAD program, having tremendous sides of accuracy. In addition A mathematical analysis on the MCM has been performed by which an equation is provided such that Once semi axes of an ellipse (a) and (b) are given and the intersection point B(x,y) on the Measuring curvature is geometrically specified, the associated ellipse perimeter can be determined.

In this paper, four constants are presented and a new form of special curvature is produced. by fixing these constants, the MCM can be used to calculate the exact value of the ellipse circumference which is appropriate for all forms of flatted and inflated ellipses.

Furthermore, the coordinators of (B) along x-axis is limited between points (D1) and (V1), which is a constant with a value of (0.07109540 b), while the vertical rang (along the y-axis) is another constant with a value of (0.70648192 b).

In general, there are 4 parameters control the radius length of Measuring Circle (CB) which are; (a), and these two constants; (C1V1) and (D1V1) respectively, in addition to the Measuring Curvature (C1L2L1D1) with length value of (0.71019548 b). Correspondingly, the Measuring curvature covers under its vertical and horizontal projections an area zone which is a constant; (0.0239 b^2).

The research proposed in the present paper could be extended by finding new modules of (algebraic) tactic, probably by investigating projective planes of conics, where other rich arrangements arise. Also, moving to higher geometric and algebraic properties of MCM would reveal possible importance to application in mathematics and physic. Hence this geometrical method has higher dimensions of accuracy. With its original simplicity, accuracy, and geometric form, the future predictions are expected for the MCM to be usefully engaged in important practical applications.

Acknowledgment

It gives immense pleasure to thank all those who extended their coordination, moral support, guidance and encouraged to reach the publication of the article. My special thanks to my family; Angham S.M., Noor, Nada, Fatima and Omneya, for their full support and co-ordination.

References

1. Chandrupatla, T. R., & Osler, T. J.The perimeter of an ellipse. Mathematical Scientist, 2010;35(2). http://web.tecnico.ulisboa.pt/~mcasquilho/compute/com/,ellips/PerimeterOfEllipse.pdf
2. Hanifat A., Suryadi D, and Sumiaty E. Reflective inquiry: Why area is never negative? J Phys Conf Ser. 2019;1280(4). DOI 10.1088/1742-6596/1280/4/042048.
3. Barnard, R. W., Pearce, K., & Schovanec, L. Inequalities for the perimeter of an ellipse. Journal of Mathematical Analysis and Application.2001; 260(1), 295–306. https://doi.org/10.1006/jmaa.2000.7128
4. Abbott, P. On the perimeter of an ellipse. The Mathematical Journal.2009;11(2), 172-185. https://content.wolfram.com/uploads/sites/19/2009/11/Abbott.pdf
5. Adlaj, S. An eloquent formula for the perimeter of an ellipse. Notice of The AMS.2012; 59(8), 1094–1099. https://doi.org/http://dx.doi.org/10.1090/noti879
6. Saeb Amir Ahmedi, C. An exact formula for the perimeter of an ellipse. Independent Researcher, Rasht, Guillan, Iran. 2018;415(56)-4678.
7. Qureshi I, Akhtar N., Ahamad D. Analytical expressions for curved surface area of revolution and arc-length of an ellipse: A hypergeometric mechanism. Trans Natl Acad Sci Azerb Ser Phys-Tech Math Sci Mathematics. 2020; 40(1): 152–160
8. Harun A. Rohman. Deriving the exact formula for perimeter of an ellipse using coordinate transformation. Journal Pendidikan dan Pembelajaran Matematika, June. 2022; (4), 1, DOI:10.35316/alifmatika.2022.v4i1.1-16
9. Tiehong Z., Miaokun W. & Yuming C. On the bounds of the perimeter of an ellipse. Acta Mathematica Scientia.2022; 23(42): 491–501
10. Al-ossmi, Laith H. M. “Nada’s curve towards a new curvature produced by the tangent of a circle and an ellipse: the Nada’s curve”, IJCSM. 2022;4 (1), DOI: 10.52866/ijcsm.2022.01.01.001
11. Al-ossmi, Laith H. M. Noor's curve, a new geometric form of agnesi witch, a construction method is produced, Baghdad Sci.J [Internet].2021 June 2 (Suppl.June), E-ISSN: 2411-7986, DOI:

http://dx.doi.org/10.21123/bsj.2021.18.2(Suppl.).1113

12. Asaad SH, Ahmed IS, Ebrahim HH. On finitely null-additive and finitely weakly null-additive relative to the σ–ring. Baghdad Sci.J [Internet]. 2022 Oct. 1;19(5):1148.
13. Wang M K, Chu H, Chu Y M. On the approximation of some special functions in Ramanujan's generalized modular equation with signature 3. Ramanujan J, 2021; 56(1): 1–22.
14. Zhao, T., Wang, M., & Chu, Y. On the bounds of the perimeter of an ellipse. Acta Mathematica Scientia, 2022, 42(2), 491-501. https://link.springer.com/article/10.1007/s10473-022-0204-y
15. Koshy, K. Idicula. "A new ellipse perimeter approximation formula that reduces the relative error to the order of 10^{-8}. Bulletin of Mathematics and Statistics Research, 2023,11(1):25-31. http://www.bomsr.com/11.1.23/25-31%20K.%20Idicula%20Koshy.pdf