



Optimal Solution Algorithm for Transport Problem With practical application

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ABSTRACT

Transportation problem is an important mathematical method in the decision-making process in transferring the quantity of goods from the sources of manufacturing to the centers requested at the lowest possible cost.

In this research, a model was presented for the transport problem, and this model was solved by using the exponential method to find the optimal solution for the transport problem. This method is considered as a modern and important method for economic and service decision makers for the production facilities. For the purpose of comparing the results using WIN.QSB, to solve the problem of transferring the black oil product from the warehouses to the governorates requesting the product of the intermediate refineries, good results were achieved in reducing the cost of transportation. After the implementation of the exponential method came at a total cost of (113225) ID. The results of the method of solving the linear programming amounted to (111075) ID

Keywords:

The optimal Solution, The exponential method, Transportation Problem

Introduction:

Companies that produce oil derivatives are among the important establishments, since their products are among the main pillars of many vital activities and other industries. There are also many methods of operations research which are suitable for solving problems that researchers face in the fields of production and decision-making. As a result of the development of the oil refining industry worldwide along with the increased demand for petroleum products with all its derivatives, this industry is witnessing a significant change as a result of the decrease and rise in crude oil prices at the local level on the one hand and at the global level on the other hand as a result of the lack and weakness of the available capabilities for production.. The exponential method is an important tool that contributes to making the optimal decision represented by maximizing profits or minimizing costs, while determining the optimal quantities transferred

from sources to demand destinations to solve the transportation problem (5).

The research consists of:

The theoretical aspect: which includes the theoretical concept of the transportation problem and model, as well as an explanation of the exponential method used in the research. The applied aspect: explains the application of the exponential method in the Midland Refineries Company on the black oil product, building a model for the transportation problem and comparing it to the simplified method with the lowest costs.

It includes the most important conclusions and recommendations that were reached through the research, which would benefit researchers and specialists in this subject

Research target:

The aim of the research is to build a mathematical model to obtain the lowest transportation cost for the black oil product

(fuel oil) using the exponential method in solving the transportation model to find the optimal solution with determining the optimal quantities for transporting the product from the main warehouses to the requesting governorates for the material, and comparing the results with the linear programming solution method.

Research importance:

Finding the lowest possible cost for the problem of fuel oil transporting, while determining the quantities transferred for the product.

Search problem

Black oil is considered an oil derivative and one of the most economically vital products for the country. The high cost of transporting the product from storage depots to governorates using tanker trucks is costing a lot of money economically.

Theoretical aspect

Transport Problem is a special case of linear programming models which can solve in one way or another the problems of transport and distribution of goods and find the optimal solution for distribution with finding optimal quantities transferred. Transport models are known to be one of the methods in the science of production management and operations research, because it is one of the ways to show how to best store the produced materials, and to transport the products at the lowest possible cost. This has helped achieve the economic goals of the establishments, which represent the main pillar for the advancement of the national economy. It is necessary to pay attention to transport methods, as they are an important

step on the road to the economic progress of the country. (1).

Transport model assignments

Transportation must have a number of basic assumptions in the transportation model to solve a problem (6):

- 1- Distribution locations (stores, warehouses) each with a specific capacity (supply quantity).
- 2- Demand locations (shopping center, customers) for each specific quantity order.
- 3- A transport cost which is predetermined for product transporting.
- 4- The supply quantities are equal to the demand quantities.

Transportation problem mathematical model:

The main objective of the transport model solution is to find the optimal solution to the target function of least possible transport cost for the transport of the product with the aim of meeting the production or consumption centers. The mode of transport is also determined in the process of allocation of human or financial resources and best if these resources are pre-determined (7). The model can also be mathematically represented" on the assumption that there are (m) sources, (n) locations where the transport problem can be:

S_i : Represents units displayed at source (i)

D_j :

Represents the units requested in the location (j)

C_{ij} : Cost of unit transport at route (i,j) between source (i) and location (j)

X_{ij} :

Represents units transferred from the source (i) to location (j)

The main objective is to determine the optimal transfer quantities from the source (i) to the location (j); to make the total cost of the transport is as low as possible, as shown in the table below:

Table (1-2) public transport model

From	D ₁	D ₂	D _n	supply
S ₁	X ₁₁ C ₁₁	X ₁₂ C ₁₂		X _{1n} C _{1n}	a ₁
S ₂	X ₂₁ C ₂₁	X ₂₂ C ₂₂		X _{2n} C _{2n}	a ₂
....				
S _m	X _{m1} C _{m1}	X _{m2} C _{m2}		X _{mn} C _{mn}	a _m
Demand	b ₁	b ₂		b _n	$\sum a_i = \sum b_j$

A linear programming model for the transport problem, assuming transport costs are linear, would be as follows:

$$Min z = \sum_{i=1}^m \sum_{j=1}^n c_{ij} x_{ij}$$

Subject to :

$$\sum_{i=1}^m x_{ij} \leq a_i$$

$$\sum_{j=1}^n x_{ij} \leq b_j$$

$$x_{ij} \geq 0$$

Because of the balance of the transport model (the sum of the quantities processed equals the quantities required) which consists of (m) source and (n) demand center with a number of structural constraints equal to (m+n)

The first step to the problem of transport is to find the primary solution. The second step is to find the best solution to the problem, i.e. a plan for transport from the sources to the demand centers, in order to take the appropriate decision to solve the problem (4).

Below is an explanatory chart showing the main stages of solving the transport model (2).

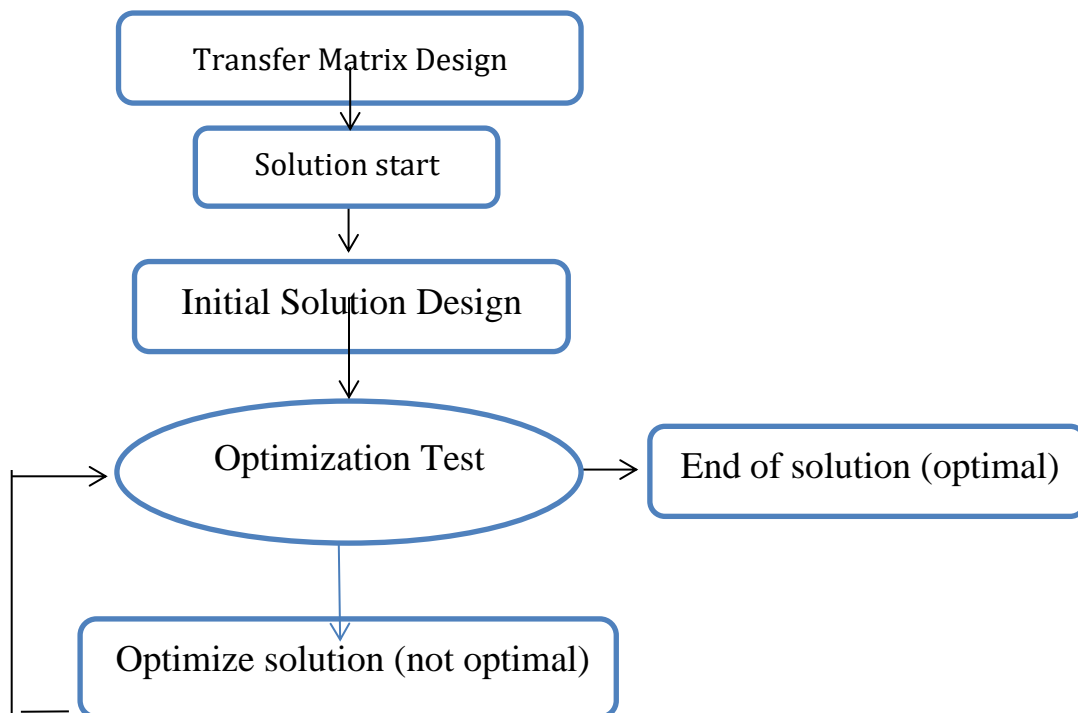


Figure (1.1) the main stages of the transport model

Methods of finding transportation costs:

The main goal of the methods of solving the problem is to find the lowest final cost of transporting goods from their places of production (rows) to the consumer (columns), provided that the total supply equals the total demand (3).

There are four ways to solve the transportation problem:

1. North West – Corner Method
2. Minimum –Cost Method
3. Vogel’s Approximation method
4. Minimum-Cost Sum- Modified method

The results of the solution by the above methods are the lowest value for the target function (the lowest total cost of transport) (7).

Suggested algorithm (exponential method):

A modern method of obtaining the most direct optimal solution was proposed in 2013 by the researchers (Rekha& Vannan), which came to be named as such because cell allocation in the transport matrix is based on the choice of cells that have the lowest baseline value for allocation, a method that has proven to be efficient in terms of results and is easy to apply and understand. This method also has the advantage of determining the optimal solution because it is suitable for solving economic problems and therefore can be defined as a method that helps determine the optimal plan for the transportation problem, a method and technique for optimizing the optimal mathematical solution. The algorithm (simplex) is known as a set of rules known for solving the problem of linear programming (8).

For the purpose of applying the proposed algorithm, we follow the following:

Step 1 - Select the lowest cost per row of the transport table and then subtract this cost from all row elements and from the process to all rows.

Step 2 - Select the lowest cost per table column and subtract it for the cost from the column elements, with the operation for all columns and in turn.

Step 3 - After the previous step, make sure that there is one zero in each row and column after which we calculate the base values for zeros (i.e., select the first zero in the transport table, then calculate the total number of zeros in the row and the column corresponding to the first zero (except the specified zero) in the transport matrix, after which the exponential values are determined above each zero in the transport table).

Step four - From the previous step we choose the zero that has the lowest exponential value, and then the allocation process for the given cell is done within the bounds of supply and demand if the exponential values of the two table cells are equal to the exponential values of the two table cells that are processed, we take the rate of demand and supply for the two equivalent cells and choose the cell that has the lowest supply and demand.

Step five - After getting a reduced transfer schedule we make sure there is at least one zero in each row and column and vice versa the steps repeat from the first to the fourth step.

Step 6 - After the cell allocation process, delete the row or column and repeat the previous steps until all of the table's supply and demand quantities have been exhausted.

Step 7 - Calculate final transfer cost of transfer schedule

The applied aspect:

The concern with the transport problem is important because it has contributed to the advancement of the economic development of the country.

This has led many researchers and experts to carry out studies and research that would develop the methods of solving this problem, using computers for the purpose of studying and comparing modern methods of transport. Therefore, in this aspect, we will present what was discussed previously, and will

apply the basic algorithm to find the best solution to transfer the black oil product from the main warehouse of the Central

Refineries Company to the provinces requesting the product via tankers.

The Central Refineries Company is one of the Ministry of Oil's formations and includes four main depots.

The headquarters of the company is the oldest and largest refinery, the Dora refinery, located in AlDora area in Baghdad, which is considered the beginning of Iraq's first oil start; (AlDiwaniya refinery, AlSamawa refinery depot, the Najaf refinery depot). Black oil is an important product because it is used for brick factories and power plants in Iraq. The high cost of transporting it has a negative impact on the country's economy, which is why attention has been given to developing the methods used to transport this product.

The data shall include the black oil deposits of the Central Refineries

Company, as well as the capacity of each refinery and the cost of transporting the product from the warehouses to the provinces supplied with the material for the year 2017, as shown in table (1.3), (2.3), and (3.3).

Table (3.1) Warehouse capacity

No.	Warehouse	Capacity
1	Al-Dora refinery	4500 ton
2	Al-Diwaniya refinery	1500 ton
3	Al-Samawa refinery	2500 ton
4	Najaf refinery	2500 ton

Table (3.2) the daily demand for black oil in provinces for a year 2018

No.	Province	Daily demand (ton)
1	Baghdad	3095
2	Babel	800
3	Al-Samawa	600
4	Al-Diwaniya	950
5	Najaf	500
6	Karbala	1575
7	Kut	725

The cost of transporting black oil from main warehouses to governorates is calculated after the data is documented

Based on the formula used by the Central Refineries Company :

Transport cost = load (tons) * distance (km) * Transport price (dinars per ton)

Table (3.3) Transport cost of black oil (fuel oil)

المستودع	Baghdad	Babel	Al-Samawa	Al-Diwaniya	Najaf	Karbala	Kout
Al-Dora	10	20	40	28	32	20	33
Al-Diwaniya	33	18	30	15	20	28	20
Al-Samawa	40	30	10	20	32	38	34
Najaf	28	16	33	20	10	16	28

*The cost of transporting a unit of a product is measured in thousand dinars per ton from the main depots to the provinces requesting the product.

Applying the exponential algorithm to the problem of black oil transport:

The method is applied to the transport schedule to solve the problem of transporting black oil from the main warehouses of the mid-refineries to their requisitioners. The results of this method are then compared with the results of the linear programming method to find the optimal solution, as shown in the table below:

table (4.3) Black oil (fuel oil) transportation problem

المستودع	Baghdad	Babel	Al-Samawa	Al-Diwaniya	Najaf	Karbala	Kout	Supply
Al-Dora	10	20	40	28	32	20	33	4500
Al-Diwaniya	33	18	30	15	20	28	20	1500
Al-Samawa	40	30	10	20	32	38	34	2500
Najaf	28	16	33	20	10	16	28	2500
Demand	3095	800	600	950	500	1575	725	

Note that the sum of supply (11000) \neq the total demand (8245) means that the transport matrix is unbalanced so an imaginary column is added to the transport table that costs cells of this column zero, the amount of demand is (2755) as shown in the table below:

Table (5.3) Budget of the transport table with the dummy column

المستودع	Baghdad	Babel	Al-Samawa	Al-Diwaniya	Najaf	Karbala	Kout	Dummy column	Supply
Al-Dora	10	20	40	28	32	20	33	0	4500
Al-Diwaniya	33	18	30	15	20	28	20	0	1500
Al-Samawa	40	30	10	20	32	38	34	0	2500
Najaf	28	16	33	20	10	16	28	0	2500
Demand	3095	800	600	950	500	1575	725	2755	

The row subtraction is then followed by the column subtraction of the transportation table. The resulting table is called the reduced transfer table, after we have ascertained that there is at least one zero in each row or column. We then calculate the base values for the zeros in the table below:

Table (5.3) Subtract rows and columns and specify the key values for zeros in the transport table

المستودع	Baghdad	Babel	Al-Samawa	Al-Diwaniya	Najaf	Karbala	Kout	Dummy column	Supply
Al-Dora	0 ⁽¹⁾	4	30	13	22	4	13	0 ⁽⁴⁾	4500
Al-Diwaniya	23	2	20	0 ⁽²⁾	10	12	0 ⁽²⁾	0 ⁽⁵⁾	1500
Al-Samawa	30	14	0 ⁽¹⁾	5	22	22	14	0 ⁽²⁾	2500
Najaf	18	0 ⁽³⁾	23	5	0 ⁽³⁾	0 ⁽³⁾	8	0 ⁽⁶⁾	2500
Demand	3095	800	600	950	500	1575	725	2755	

Then the process of assigning the cells with the lowest exponential value takes place. The procedures for the exponential method described above and in the table below continue. We explain the cell allocation process until all rows and columns in the transport table have been deleted, that is, all the requirements for supply and demand have been met, and as follows:

Table (6.3) Allocation of optimal quantities for black oil transport according to the exponential method

المستودع	Baghdad	Babel	Al-Samawa	Al-Diwaniya	Najaf	Karbala	Kout	Dummy column	Supply
Al-Dora	0	4	30	13	22	4	13	0	4500
	3095			175	375			855	
Al-Diwaniya	23	2	20	0	10	12	0	0	1500
				775			725		
Al-Samawa	30	14	0	5	22	22	14	0	2500
			600					1900	
Najaf	18	0	23	5	0	0	8	0	2500
		800			125	1575			
Demand	3095	800	600	950	500	1575	725	2755	

According to the quantities allocated in the reduced table, we calculate the total final cost of transport as follows:

$$\text{Min. } Z = (3095 \times 10) + (16 \times 800) + (10 \times 600) + (28 \times 175) + (15 \times 775) + (375 \times 32) + (10 \times 125) + (16 \times 1575) + (20 \times 725) = 113225$$

Apply the (Win Q.S.B.) linear programming method to solve the transport problem:

After the linear model of the transport problem was applied in WINQSP, the value of the target function was:

$$\text{Min } (Z) = 111075$$

The results are presented in Annex I.

Conclusions:

- 1- The final cost of transporting black oil from the main depots to the provinces requesting the product is 113,225 dinars.
- 2- The exponential method demonstrated its efficiency through the extracted results, being close to optimal, and comparing the researcher's results with the method of solving linear programming using the (Win Q.S.B.), the total cost of the transfer was 111,075 dinars.

Recommendations:

- 1- Development of modern methods based on the exponential method of finding the optimal solution to transport problems.
- 2- Attention to real-life transport problems and ways of developing them in order to assist economic decision-makers in making appropriate decisions in productive enterprises in order to reduce their transport costs, and to establish a database containing all the data on the issues of transfer, distribution and cost in order to be used in the future by researchers.

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Appendix I Results of WINQSB Program to Solve Black Oil Transport Problem Linear Model Using Linear Programming Method

	01:52:30		Wednesday	December	20	2017		
	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	x11	3,095.0000	10.0000	30,950.0000	0	basic	-M	32.0000
2	x12	375.0000	20.0000	7,500.0000	0	basic	16.0000	20.0000
3	x13	0	40.0000	0	30.0000	at bound	10.0000	M
4	x14	0	28.0000	0	8.0000	at bound	20.0000	M
5	15	0	32.0000	0	18.0000	at bound	14.0000	M
6	x16	0	20.0000	0	0	at bound	20.0000	M
7	17	0	33.0000	0	8.0000	at bound	25.0000	M
8	x21	0	33.0000	0	28.0000	at bound	5.0000	M
9	x22	0	18.0000	0	3.0000	at bound	15.0000	M
10	x23	0	30.0000	0	25.0000	at bound	5.0000	M
11	24	775.0000	15.0000	11,625.0000	0	basic	8.0000	18.0000
12	x25	0	20.0000	0	11.0000	at bound	9.0000	M
13	26	0	28.0000	0	13.0000	at bound	15.0000	M
14	x27	725.0000	20.0000	14,500.0000	0	basic	-M	27.0000
15	31	0	40.0000	0	30.0000	at bound	10.0000	M
16	x32	0	30.0000	0	10.0000	at bound	20.0000	M
17	33	600.0000	10.0000	6,000.0000	0	basic	-M	35.0000
18	x34	175.0000	20.0000	3,500.0000	0	basic	17.0000	24.0000
19	35	0	32.0000	0	18.0000	at bound	14.0000	M
20	x36	0	38.0000	0	18.0000	at bound	20.0000	M
21	37	0	34.0000	0	9.0000	at bound	25.0000	M
22	x41	0	28.0000	0	22.0000	at bound	6.0000	M
23	42	425.0000	16.0000	6,800.0000	0	basic	16.0000	20.0000
24	x43	0	33.0000	0	27.0000	at bound	6.0000	M
25	44	0	20.0000	0	4.0000	at bound	16.0000	M
26	x45	500.0000	10.0000	5,000.0000	0	basic	-M	21.0000
27	x46	1,575.0000	16.0000	25,200.0000	0	basic	-M	16.0000
28	x47	0	28.0000	0	7.0000	at bound	21.0000	M
	Objective	Function	(Min.) =	111,075.0000				
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS

