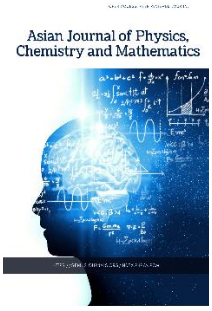

Primary-Fc Modules

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ABSTRACT

The main purpose of this paper is to present another kind of fully cancellation module namely primary –Fully Cancellation and we highlight the main features of this new concept and its relationship with fully cancellation module. We inquire about some results of such modules.

Keywords: Fully Cancellation module, Max-Fully Cancellation module, Artinian Ring ,Boolean Ring and Primary-FC Module

Interoduction

In this paper we denote to submodule: subm., submodules: subms., module: md., modules: mds, cancellation module: cance. md., cancellation ideal: cance. Ideal, fully cancelation module : FC md., maximal fully cancellation module: MFC-md, primary –fully cancellation module: primary – FC md., such that: s.t.

Throughout this paper all rings are commutative with identity and all mds. are unitary. Let X be a T-md. . Then X is said to be FC md. , if for each ideal I of T and for each subms. D_1,D_2 of X s.t. ID_1=ID_2 implies D_1=D_2 [1] .In this case ,if for every maximal ideal I of T, $I \neq 0$ and for every subms. $D_1$and D_2 of X s.t. ID_1=ID_2 ,then D_1=D_2 and we call it MFC -md. [2] . Now in this paper ,we define the concept of primary –FC md., we give some equivalent conditions for a primary- FC md..

Also ,we will find some relations between MFC-md. and Primary- FC md..

2.Main Results

Definition (2.1):- Let X be a T-md. . X is called Primary-FC md. if for every primary ideal I of T, $I\neq 0$ and for every subms. D_1,D_2 of X s.t. ID_1=ID_2 ,then D_1=D_2.

Remarks and Examples(2.2)

(1) Z as Z-md. is primary-FC md..

(2) Z_6 as a Z_6-md. is not primary-FC md. .Since $(\bar{3})$ is primary ideal of Z_6 and $(\bar{3})$,Z_6 are subms. of Z_6 such that $(\bar{3})(\bar{3}) = (\bar{3})Z_6$ but $((\bar{3})\neq Z_6$.

(3) Every FC md. is primary-FC md. ,but the convers is not true in general for example:-
Let T =Z_{36} and X=$(\bar{6})$ as a T-md.. We have $(\overline{12})$ and $(\overline{24})$ are subms. of $(\bar{6})$ such that $(\bar{3})(\overline{12}) = (\bar{3})(\overline{24}) = (\bar{0})$ where $(\bar{3})$ is primary ideal of Z_{36} and hence $(\overline{12}) = (\overline{24})$. Therefore X=$(\bar{6})$ is primary-FC md. but it is not FC md.. To show this, take $(\bar{4})$ is a nonzero ideal of Z_{36} and $(\bar{6})$, $(\bar{0})$ are subms. of X such that $(\bar{4})(\bar{6}) = (\bar{4})(\bar{0}) = (\bar{0})$ which implies $(\bar{6}) \neq (\bar{0})$. Which gives what we wanted.

(4) Every subm. of a primary-FC md. is primary-FC md..

(5) Let $X_1$and X_2 be a T-mds. such that $X_1 \subset X_2$ .Then X_1 is Primary-FC md. if and only if X_2 is Primary-FC md..

The following theorem is illustration of primary-FC md. :-

Theorem(2.3):-

Let X be a T-md. , D_1 ,D_2 be two subms. of X , and let I be a primary ideal of T, I$\neq$ 0. Then the coming are equivalent:-

(1) X is Primary-FC md. .

(2) if $ID_1 \subseteq ID_2$,then $D_1 \subseteq D_2$.

(3) if I$< a >\subseteq ID_2$, then $a \in D_2$ where $a \in$ X.

(4) $(ID_1 :_T ID_2) = (D_1 :_T D_2)$.

Proof :-

(1) ⇒ (2) If $ID_1 \subseteq ID_2$ then $ID_2 = ID_1 + ID_2$ Which Implies $ID_2 = I(D_1 + D_2)$,

But X is primary-FC md., then $D_2 = (D_1 + D_2)$ and hence $D_1 \subseteq D_2$

(2) ⇒ (3) If $I\langle a\rangle \subseteq ID_2$ then $\langle a\rangle \subseteq D_2$ by (2) Which implies ,$a \in D_2$.

(3) ⇒ (4) If $ID_1 = ID_2$, to prove that $D_1 = D_2$. Let $a \in D_1$ then $I\langle a\rangle \subseteq ID_1 \subseteq ID_2$ And hence $a \in D_2$ by (3) similarly , we can show $D_2 \subseteq D_1$. Thus $D_1 = D_2$.

(1) ⇒ (4) Let $r \in (ID_1 :_T ID_2)$ .Then $r\, ID_2 \subseteq ID_1$ so , $IrD_2 \subseteq ID_1$ and since (1) implies (2) , we have $D_2 \subseteq D_1$.

Thus $r \in (D_1 :_T D_2)$and hence $(ID_1 :_T ID_2) \subseteq (D_1 :_T D_2)$

Let $r \in (D_1 :_T D_2)$. Then $rD_2 \subseteq D_1$ which implies $IrD_2 \subseteq ID_1$ and hence $rID_2 \subseteq ID_1$.

Therefore $r \in (ID_1 :_T ID_2)$ and hence $(D_1 :_T D_2) \subseteq (ID_1 :_T ID_2)$. Then we get $(D_1 :_T D_2) = (ID_1 :_T ID_2)$

(4)⇒ (1)

Let $ID_1 = ID_2$ Then by (4) $(ID_1 :_T ID_2) = (D_1 :_T D_2)$. But $(ID_1 :_T ID_2) = T$

(since $ID_1 = ID_2$). Then $(D_1 :_T D_2) = T$ so $D_2 \subseteq D_1$..

Similarly $(ID_2 :_T ID_1) = (D_2 :_T D_1)$

Thus $(D_2 :_T D_1) = T$ Which implies $D_1 \subseteq D_2$. Therefore $D_1 = D_2$. .

Before we give our proposition ,the following concepts are needed.

A ring T is called a Boolean ring in case each of its elements is an idempotent. And a commutative ring T with unity is called an Artinian ring if and only if for any descending chain of ideals $I_1 \supseteq I_2 \supseteq I_3 \supseteq \cdots \ldots..$ of T , $\exists n \in Z^+$ such that $I_n = I_{n+1} = \cdots$ [3]

Now ,the following proposition gives the relationship between MFC-md. and Primary-FC md..

Proposition(2.4):-

Every Primary-FC md. is MFC-md..

Proof:- It is easy

The reverse of proposition (2.4) is true under the condition that the ring T is PID or regular or Artinian or Boolean ring .

Proposition(2.5):-

Let T be a PID (regular or Artinian or Boolean) and X be a T-md. .Then X is Primary-FC md. if and only if M is MFC-md..

Proof :- It is obvious

Proposition(2.6):-

Let X be a Primary-FC md. over a ring T . If X is a cance. md. ,then every primary ideal I of T, I$\neq$ 0 is cance. ideal .

Proof:-

Let I be a primary ideal of T , I$\neq$ 0 s.t. CI=DI ,where C ,D are two ideal of T . CIX=DIX ,then ICX=IDX . But X is Primary-FC md.

, therefore CX=DX. As X is cance. md. , then C=D by [4] .

Proposition(2.7):-

Let X_1 ,X_2 be two R-mds. .If $X_1 \cong X_2$,then X_1 is Primary-FC md. if and only if X_2 is Primary-FC md..

Proof:-

Let $\theta: X_1 \longrightarrow X_2$ be an isomorphism . Suppose X_1 is a Primary-FC md..

To prove X_2 is a Primary-FC md..

For every primary ideal I of T, I$\neq$ 0 and every subms. D_1,D_2 of X_2 .Let $\overline{ID_1} = \overline{ID_2}$

Now , there exists two subms. D_1 ,D_2 of X_1 such that $\theta(D_1) = \overline{D}_1$, $\theta(D_2) = \overline{D}_2$

Then I $\theta(D_1)$ = I $\theta(D_2)$, Which implies $\theta(I\, D_1) = \theta(I\, D_2)$. Therefore $ID_1 = ID_2$

since θ is (1-1))But X_1 is Primary-FC md. .Then $D_1 = D_2$ and hence

$\theta(D_1) = \theta(D_2)$ Therefore $\overline{D}_1 = \overline{D}_2$ That is X_2 is Primary-FC md..

Conversely:-

Suppose that X_2 is Primary-FC md. over the ring. Let $ID_1 = ID_2$ for every prime ideal I of T, I$\neq$ 0 and every subms. D_1 , D_2 of X_1 . Now , $\theta(I\, D_1) = \theta(I\, D_2)$. Which implies I $\theta(D_1)$= I $\theta(D_2)$,where $\theta(D_1)$, $\theta(D_2)$ are two subms. of X_2

Also X_2 is Primary-FC md.. Then $\theta(D_1) = \theta(D_2)$ Which implies $D_1 = D_2$

since θ is (1-1)) Which completes the proof.

Conclusions

In this paper we study the concept of Primary-FC md.. The results: Z as Z-md. is primary-FC md., every FC md. is primary-FC md. but the convers is not true and we gave an example explain that. Also Every subm. of a primary-FC md. is primary-FC md.. If $X_1$and X_2 are a T-mds. such that $X_1 \subset X_2$,then X_1 is Primary-FC md. if and only if X_2 is Primary-FC md.. Every Primary-FC md. is MFC-md.. . We proved that if X is a Primary-FC md. over a ring T and X is a cance. md. ,then every non zero primary ideal of T is cance. ideal . finally if X_1 ,X_2 are two R-mds. and $X_1 \cong X_2$,then X_1 is Primary-FC md. if and only if X_2 is Primary-FC md..

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