Dynamic Macsherry Model with Heredity Included

Alimov Kh.T.	National University of Uzbekistan named after Mirzo Ulugbek, Tashkent, Uzbekistan
Parovik R.I	Vitus Bering Kamchatka State University, Petropavlovsk-Kamchatsky, Russia

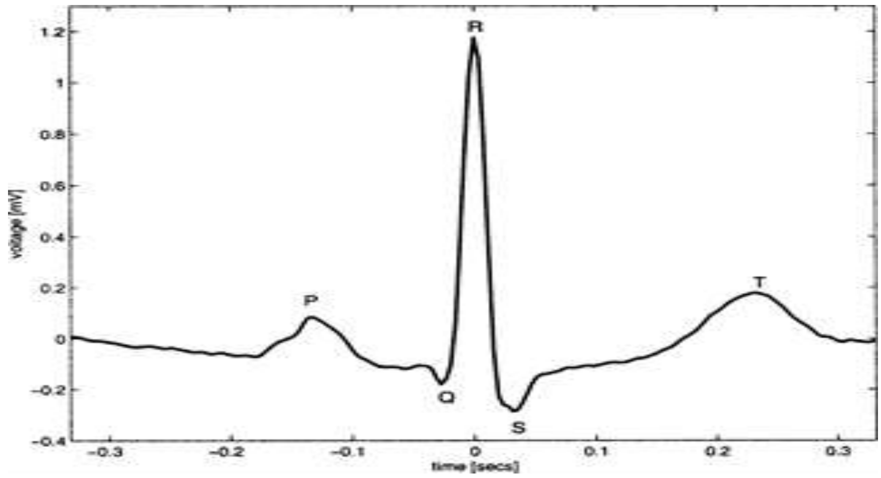
ABSTRACT

Mathematical modeling in biophysical systems is essential. For example, a study of the work of the heart based on the contraction of its muscles. This article proposes a new mathematical model for constructing an artificial electrocardiogram of a healthy person, and develops a numerical solution algorithm based on finite difference schemes.

Keywords: ECG morphology, Heart rate variability, Dynamic model.

Introduction

An electrocardiogram (ECG) is a time-varying signal that reflects the flow of an ionic current that causes contraction and subsequent relaxation of the heart fibers. Surface ECG is obtained by recording the potential difference between two electrodes placed on the surface of the skin. One normal ECG cycle is a series of atrial depolarization/repolarization and ventricular depolarization/repolarization that occur with each heart beat. They can be roughly related to the peaks and valleys of the ECG trace labeled P, Q, R, S, and T [11] as shown in Fig. 1



Rice. Fig. 1. Morphology of the average PQRST-complex of an ECG recorded in a healthy person.

Reliable signal processing methods are required to extract useful clinical information from a real (noisy) ECG [12]. These include R peak detection [13], [14], QT interval determination [15], and ECG heart rate and respiration rate detection [16], [17]. The RR interval is the time between successive R peaks, the reciprocal of this time interval gives the instantaneous heart rate. The series of RR intervals is known as the RR tachogram, and the variability of these RR intervals reveals important information about the physiological state of the subject [18]. Currently, new biomedical signal processing algorithms are usually evaluated by applying them to ECG in a large database such as the Physionet database [18]. While this gives the operator an idea of the accuracy of this algorithm in relation to real data, it is difficult to conclude how performance will vary in different clinical settings with different noise levels and sample rates. Access to realistic artificial ECG signals may facilitate this assessment. This article presents a model for generating a synthetic ECG signal with realistic PQRST morphology and given heart rate dynamics.

The purpose of this model is to provide a standard, realistic ECG signal with known characteristics that can be generated using specific statistics such as heart rate mean and standard deviation, and frequency domain heart rate variability (HRV) characteristics.), such as the low/high frequency (LF/HF) ratio, defined as the power ratio between 0.015–0.15 Hz and 0.15–0.4 Hz on the RR tachogram [18]. Generating a signal that is representative of a typical human ECG facilitates comparison of different signal processing techniques. A synthetic ECG can be generated with different sampling rates and different noise levels to establish the effectiveness of this method. This performance can be represented, for example, as the number of true positives, false positives, true negatives, and false negatives for each test. This performance rating could be used as a "standard" and would allow clinicians to determine which biomedical signal processing techniques are best suited for a given application. signal processing methods are best suited for a given application.

In this article, we will give the main provisions of the mathematical apparatus of fractional calculus, which we will often refer to in this dissertation. The definition of fractional integration and differentiation is related to the Abel integral equation [14].

Definition 1. *Let* $x(t) \in L(0,T)$*. Integral*

$$I_{0t}^{\alpha} x(\tau) = \frac{1}{\Gamma(\alpha)} \int_0^t \frac{x(\tau)\, d\tau}{(x-\tau)^{1-\alpha}}, \qquad (3)$$

Where $\alpha > 0$ *and* $\Gamma(z)$ *– Euler's gamma function is called the fractional order integral* α*.*

Using formula (3), we naturally introduce the definition of fractional differentiation as the inverse operation of integration.

Definition 2. *For function* $x(t) \in L(0,T)$ *the ratio*

$$D_{0t}^{\alpha} x(\tau) = \frac{1}{\Gamma(1-\alpha)} \frac{d}{dt} \int_0^t \frac{x(\tau)\, d\tau}{(x-\tau)^{\alpha}}, \qquad (4)$$

which is called the fractional derivative $0 < \alpha < 1$*.*

Formulas (3) and (4) are called Riemann-Liouville integral and derivative [12].

Formula (4) can be generalized to the case when $[\alpha] < \alpha < [\alpha] + 1$, где $[\alpha]$ – the integer part of the number α. In this case, the generalization looks like:

$$D_{0t}^{\alpha} x(\tau) = \frac{1}{\Gamma(1+[\alpha]-\alpha)} \frac{d^{[\alpha]+1}}{dt^{[\alpha]+1}} \int_0^t \frac{x(\tau)\, d\tau}{(x-\tau)^{\alpha-[\alpha]}}, \qquad (5)$$

If we consider $[\alpha] = n - 1$, then we get another representation of the fractional derivative of order $n < \alpha < n + 1$:

$$D_{0t}^{\alpha}x(\tau)=\frac{1}{\Gamma(n-\alpha)}\frac{d^{n}}{dt^{n}}\int\limits_{0}^{t}\frac{x(\tau)\,d\tau}{(x-\tau)^{\alpha-n+1}}, \qquad (6)$$

Let us introduce another definition of a fractional order derivative.

Definition 3. Let $x(t) \in C^{m-1}[0,T]$, $m \geq 1$, and $x^{(n)}(t) \in L[0,T]$. Then the operator of the following form

$$\partial_{0t}^{\alpha}x(\tau)=\frac{1}{\Gamma(n-\alpha)}\int\limits_{0}^{t}\frac{x^{(n)}(\tau)\,d\tau}{(x-\tau)^{\alpha-n+1}}, \qquad (7)$$

Where $x^{(n)}(\tau)=\dfrac{d^{n}x(\tau)}{d\tau^{n}}$,

is called the regularized Riemann-Liouville fractional derivative or the Gerasimov-Caputo fractional derivative of order n < α < n + 1.

Remark 1. It should be noted that in the foreign literature operator (7) is called the Caputo fractional derivative and is denoted as . This mathematical construction was introduced by the Italian mathematician M. Caputo in 1967 in [5] and was widely used in his monograph [15]. However, the Soviet mechanic A.N. Gerasimov in 1948, in his work [13], devoted to problems of plasticity, introduced a partial fractional derivative of the order $0 < \alpha < 1$:

$$D_{-\infty,t}^{\alpha}u(x,\tau)=\frac{1}{\Gamma(1-\alpha)}\int\limits_{-\infty}^{t}\frac{u_{\tau}(x,\tau)\,d\tau}{(x-\tau)^{\alpha}}.$$

Therefore, in the future, in the dissertation work, we will call the operator (7) the Gerasimov-Caputo operator.

Remark 2. The Gerasimov-Caputo operator of order $n < \alpha < n + 1$ is related to the Riemann-Liouville operator by the relation:

$$\partial_{0t}^{\alpha}x(\tau)=D_{0t}^{\alpha}x(\tau)-\sum_{k=0}^{n-1}\frac{x^{(n)}(0)\,t^{k-\alpha}}{\Gamma(k-\alpha+1)}. \qquad (8)$$

According to relation (8), the Gerasimov-Caputo operator coincides with the Riemann-Liouville operator if the relation $x^{(n)}(0) = 0$ is satisfied. We introduce several important definitions.

Definition 4. For arbitrary $\alpha \in R$ and $\beta \leq 0$, the composition law

$$D_{0t}^{\alpha}D_{0t}^{\beta}x(\tau)=D_{0t}^{\alpha+\beta}x(\tau). \qquad (9)$$

Definition 5. For $n - 1 < \beta \leq n$, $n \in N$, the generalized Newton-Leibniz formula is true

$$D_{0t}^{\alpha}D_{0t}^{\beta}x(\tau)=D_{0t}^{\alpha+\beta}x(\tau)-\sum_{k=1}^{n}\frac{t^{-\alpha-k}}{\Gamma(1-\alpha-k)}\lim_{t\to 0}D_{0t}^{\beta-k}x(\tau). \qquad (10)$$

Definition 6. An integration-by-parts formula is valid for $\alpha \leq 0$

$$\int\limits_{0}^{t}x(\tau)\,D_{0\tau}y(\tau_{1})\,d\tau=\int\limits_{0}^{t}h(\tau)\,D_{t\tau}y(\tau_{1})\,d\tau. \qquad (11)$$

Definition 7. The fractional differentiation of the product of two functions can be found by the generalized Leibniz rule

$$D_{0t}^{\alpha}\left[x\left(\tau\right)y\left(\tau\right)\right]=\sum_{k=0}^{\infty}\begin{pmatrix}\alpha\\ k\end{pmatrix}D_{0t}^{\alpha-k}x\left(\tau\right)y^{(n)}\left(t\right), \qquad (12)$$

where coefficient

$$\begin{pmatrix}\alpha\\ k\end{pmatrix}=\frac{\Gamma\left(\alpha+1\right)}{\Gamma\left(k+1\right)\Gamma\left(\alpha-k+1\right)}.$$

Consider fractional operators on some elementary functions. The fractional Riemann-Liouville derivative of unity is different from zero:

$$D_{0t}^{\alpha}1=\frac{t^{-\alpha}}{\Gamma\left(1-\alpha\right)},\alpha>0,t>0 \qquad (13)$$

However, the fractional Gerasimov-Caputo derivative of unity is zero.

$$\partial_{0t}^{\alpha}1=0,\alpha>0,t>0. \qquad (14)$$

According to relations (13) and (14), the following relations are valid:

$$D_{0t}^{\alpha}C=\frac{Ct^{-\alpha}}{\Gamma\left(1-\alpha\right)},\partial_{0t}^{\alpha}C=0,C-const.$$

The fractional Riemann-Liouville derivative of a power function has the form

$$D_{0t}^{\alpha}t^{\beta}=\frac{1}{\Gamma\left(n-\alpha\right)}\frac{d^{n}}{dt^{n}}\int\limits_{0}^{t}\tau^{\beta}\left(t-\tau\right)^{n-\alpha-1}d\tau= \qquad (15)$$

$$=\frac{1}{\Gamma\left(n-\alpha\right)}\frac{d^{n}t^{n-\alpha+\beta}}{dt^{n}}\int\limits_{0}^{t}\left(\frac{\tau}{t}\right)^{\beta}\left(1-\frac{\tau}{t}\right)^{n-\alpha-1}d\left(\frac{\tau}{t}\right)=$$

$$=\frac{1}{\Gamma\left(n-\alpha\right)}\frac{d^{n}t^{n-\alpha+\beta}}{dt^{n}}\int\limits_{0}^{1}\xi^{\beta}\left(1-\xi\right)^{n-\alpha-1}d\xi=$$

$$=\frac{\Gamma\left(\beta+1\right)}{\Gamma\left(\beta+1-\alpha\right)}t^{\beta-\alpha}.$$

Here we have used the definition of the beta function. Similarly, it can be shown that the fractional Gerasimov-Caputo derivative of a power function coincides with (15).

$$\partial_{0t}^{\alpha}t^{\beta}=\frac{\Gamma\left(\beta+1\right)}{\Gamma\left(\beta+1-\alpha\right)}t^{\beta-\alpha}. \qquad (16)$$

The solution of linear differential equations of fractional orders can be found using the Fourier and Laplace integral transforms. For the Riemann-Liouville operator, the Laplace transform has the form:

$$L\left[D_{0t}^{\alpha}x\left(\tau\right)\right]\left(p\right)=\begin{cases}p^{\alpha}x\left(p\right)-\sum\limits_{j=1}^{n}p^{j-1}\lim\limits_{t\to0}D_{0t}^{\alpha-j}x\left(\tau\right),\alpha\geq1\\ p^{\alpha}x\left(p\right),\alpha<1.\end{cases} \qquad (17)$$

For the Gerasimov-Caputo operator, the following Laplace transform formula is valid:

$$L\left[\partial_{0t}^{\alpha}x(\tau)\right](p)=\begin{cases}p^{\alpha}x(p)-\sum_{j=0}^{n-1}p^{\alpha-j-1}x^{(n)}(0),\alpha\geq 1,\\ p^{\alpha}x(p),\alpha<1.\end{cases} \quad (18)$$

The Fourier transform of the Riemann-Liouville fractional derivative has the form:

$$F\left[D_{0t}^{\alpha}x(\tau)\right](p)=x(p)(-ik)^{\alpha},\alpha<1. \quad (19)$$

Remark 3. Formula (19) does not apply when α ≥ 1, because its right-hand side may not exist in the usual sense [15]. Therefore, sometimes another definition of the fractional derivative is introduced.

Definition 8. The fractional Riesz derivative is the following operator

$$\partial_{0t}^{\alpha}x(\tau)=\frac{1}{2\Gamma(n-\alpha)\cos(\pi(n-\alpha)/2)}\frac{d^n}{dt^n}\int_{-\infty}^{\infty}\frac{x(\tau)\,d\tau}{|x-\tau|^{\alpha-n+1}},n-1<\alpha<n. \quad (20)$$

Fourier transform of the Riesz derivative:

$$F\left[\partial_{0t}^{\alpha}x(\tau)\right]=|k|^{\alpha}x(k). \quad (21)$$

Consider the difference analogue of the Riemann-Liouville operator.

Definition 9. The fractional derivative of Grunwald-Letnikov is the following operator

$$D_{0t}^{\alpha}x(\tau)=\lim_{\tau\to 0}\frac{1}{\tau^{\alpha}}\sum_{k=0}^{[t/\tau]}(-1)^k\frac{\Gamma(\alpha+1)}{k!\Gamma(\alpha-k+1)}x(t-k\tau) \quad (22)$$

where $\tau=\frac{t}{N}$ *– sampling step, N – amount of points.*

Using formula (22), one can approximate the Riemann-Liouville operator as follows:

$$D_{0t}^{\alpha}x(\tau)\approx\frac{1}{\tau^{\alpha}}\sum_{k=0}^{n}(-1)^k\frac{\Gamma(\alpha+1)}{k!\Gamma(\alpha-k+1)}x(t_{n-k}) \quad (23)$$

where $t_{n-k}=t_n-k\tau$.

The Gerasimov-Caputo operator is approximated differently [4]:

$$\partial_{0t}^{\alpha}x(\tau)=\frac{1}{\Gamma(1-\alpha)}\int_0^t\frac{x'(\tau)\,d\tau}{(t-\tau)^{\alpha}}\approx\frac{1}{\Gamma(1-\alpha)}\sum_{j=0}^{k}\int_{j\tau}^{(j+1)\tau}\frac{x'(\tau)\,d\tau}{(t_{k+1}-\tau)^{\alpha}}= \quad (24)$$

$$= \frac{1}{\Gamma(1-\alpha)} \sum_{j=0}^{k} \frac{x(t_{j+1}) - x(t_j)}{\tau} \int\limits_{j\tau}^{(j+1)\tau} \frac{d\tau}{(t_{k+1}-\tau)^{\alpha}} =$$

$$= \frac{1}{\Gamma(1-\alpha)} \sum_{j=0}^{k} \frac{x(t_{j+1}) - x(t_j)}{\tau} \int\limits_{(k-j)\tau}^{(k-j+1)\tau} \frac{d\eta}{\eta^{\alpha}} =$$

$$= \frac{1}{\Gamma(1-\alpha)} \sum_{j=0}^{k} \frac{x(t_{k-j+1}) - x(t_{k-j})}{\tau} \int\limits_{j\tau}^{(j+1)\tau} \frac{d\eta}{\eta^{\alpha}} =$$

$$= \frac{\tau^{-\alpha}}{\Gamma(2-\alpha)} \sum_{j=0}^{k} \left[x(t_{k-j+1}) - x(t_{k-j})\right] \left[(j+1)^{1-\alpha} - j^{1-\alpha}\right], 0 < \alpha < 1 .$$

Here $(k + 1)\tau = t$. In the case $1 < \alpha < 2$, the approximation has the form:

$$\partial_{0t}^{\alpha} x(\tau) \approx \frac{\tau^{-\alpha}}{\Gamma(3-\alpha)} \sum_{j=0}^{k} a_j \left[x(t_{k-j+1}) - 2x(t_{k-j}) + x(t_{k-j-1})\right], \quad (25)$$

where $a_j = (j + 1)^{2-\alpha} - j^{2-\alpha}$.

In the general case, when $n - 1 < \alpha < n$, formulas (24) and (25) are generalized:

$$\partial_{0t}^{\alpha} x(\tau) \approx \frac{\tau^{-\alpha}}{\Gamma(n+1-\alpha)} \sum_{j=0}^{k} \sum_{i=0}^{n} \binom{n}{i} (-1)^{i} a_j x(t_{k-j-i+1}), \quad (26)$$

where $a_j = (j + 1)^{n-\alpha} - j^{n-\alpha}$.

Remark 4. Note that all the above formulas are valid for functions of several variables. In this case, we will work with fractional partial derivatives.

Remark 5. There are other definitions of fractional derivatives and integrals. Some are modifications of the Riemann-Liouville or Gerasimov-Caputo operators. Others are defined differently. The question of choosing one or another fractional operator for solving an applied problem is open. Usually, fractional operators are chosen for reasons of simplicity in mathematical transformations and interpretation of simulation results. For example, for an equation with the Gerasimov-Caputo operator, traditional initial and boundary conditions are set, which is important for physical applications. The Riesz operator is convenient for the integral Fourier transform.

The fractional calculus is considered in more detail in the books [4]-[7], [3], [2]. It should be noted that there are different directions in the theory of fractional calculus, for example, fractional analysis based on the d-operator [2] or fractional stable distributions (stochastic approach) [4].

Definition 10. Dynamic systems or models in which derivatives of fractional orders are present will be called fractional dynamic systems or fractional dynamic models.

Based on the definitions discussed above, we can write a fractional mathematical model for constructing an artificial ECG of a healthy person in the form of the following Cauchy problem:

$$\begin{cases} \partial_{0t}^{\alpha} x(t) = \lambda x(t) - \omega y(t), \\ \partial_{0t}^{\beta} y(t) = \lambda y(t) + \omega x(t), \\ \partial_{0t}^{\gamma} z(t) = \sum\limits_{i \in \{P,Q,R,S,T\}} a_i \exp\left(-\dfrac{\Delta\theta_i^2}{2b_i^2} \right) - \left(z(t) - z_0(t) \right), \\ x(0) = x_0, y(0) = y_0, z(0) = z_0. \end{cases} \quad (27)$$

Here derivatives of fractional orders are understood in the sense

Definitions 7, where 0 < α,β,γ < 1.

Remark 6. Note that in the particular case α = β = γ = 1, the fractional McSherry mathematical model goes over into the ordinary McSherry mathematical model (1).

To solve problem (27), we use the theory of finite difference schemes. To do this, we assume that the functions x(t), y(t), z(t) have the necessary smoothness conditions. Consider a uniform grid along the time coordinate. Let's divide the segment [0,T] into N equal parts with the discretization step τ = T/N. Then the solution functions x(t),y(t),z(t) transform into grid functions x(tk),y(tk),z(tk), where tk = kτ, k = 1,..,N. Approximations of derivative fractional orders according to Definition 26 for n=1 are given by the following formulas:

$$\begin{aligned} \partial_{0t}^{\alpha} x(t) &= K_{\alpha} \sum_{i=0}^{k-1} w_i^{\alpha} \left(x_{k-i+1} - x_{k-i} \right) + O(\tau), \\ \partial_{0t}^{\beta} x(t) &= K_{\beta} \sum_{i=0}^{k-1} w_i^{\beta} \left(x_{k-i+1} - x_{k-i} \right) + O(\tau), \\ \partial_{0t}^{\gamma} x(t) &= K_{\gamma} \sum_{i=0}^{k-1} w_i^{\gamma} \left(x_{k-i+1} - x_{k-i} \right) + O(\tau), \end{aligned} \quad (28)$$

where $K_{\alpha} = \dfrac{\tau^{-\alpha}}{\Gamma(2-\alpha)}, K_{\beta} = \dfrac{\tau^{-\beta}}{\Gamma(2-\beta)}, K_{\gamma} = \dfrac{\tau^{-\gamma}}{\Gamma(2-\gamma)}$, $w_i^{\alpha} = (i+1)^{1-\alpha} - i^{1-\alpha}$, $w_i^{\beta} = (i+1)^{1-\beta} - i^{1-\beta}$ $w_i^{\gamma} = (i+1)^{1-\gamma} - i^{1-\gamma}$.

Taking into account the approximations given above, we can write down the discrete analogue of problem (28).

$$\begin{cases} x_{k+1} = \dfrac{1}{K_{\alpha}} \left(\left(1 - \sqrt{x_k^2 + y_k^2} \right) x_k - \omega y_k - K_{\alpha} \sum\limits_{i=1}^{k-1} w_i^{\alpha} \left(x_{k-i+1} - x_{k-i} \right) + K_{\alpha} x_k \right) \\ y_{k+1} = \dfrac{1}{K_{\beta}} \left(\left(1 - \sqrt{x_k^2 + y_k^2} \right) x_k + \omega x_k - K_{\beta} \sum\limits_{i=1}^{k-1} w_i^{\beta} \left(y_{k-i+1} - y_{k-i} \right) + K_{\beta} y_k \right) \\ z_{k+1} = \dfrac{1}{K_{\gamma}} \left(-\sum\limits_{i=1}^{5} a_i \Delta\theta_{k,i} \exp\left(-\dfrac{\Delta\theta_{k,i}^2}{2b_i^2} \right) + z_k^0 - K_{\gamma} \sum\limits_{i=1}^{k-1} w_i^{\gamma} \left(z_{k-i+1} - z_{k-i} \right) + \left(K_{\gamma} - 1 \right) z_k \right) \\ z_k^0 = A \sin\left(2\pi f k \tau \right) \end{cases} \quad (29)$$

Conclusions

The article gives the concept of heredity (hereditary) and explains the transition to fractional calculus. Some aspects of fractional calculus are given, definitions, properties, remarks are given. The statement of the problem is given, as well as the method of its solution. The solution technique is based on the approximation of the Gerasimov-Caputo fractional order derivatives and the solution of the discrete problem (29). Discrete problem (29) is a nonlocal explicit finite difference scheme. Such schemes have the first order of accuracy. It should also be noted that the issues of stability and convergence were not considered in the dissertation work. The correctness of the calculations was confirmed by the well-known artificial ECG graphs obtained earlier in [1] and [3].

References

1. McSharry P. E., Clifford G. D., Tarassenko L., Smith L. A. A dynamical model for generating synthetic electrocardiogram signals, IEEE transactions on biomedical engineering., 2003. vol. 50, no. 3, pp. 289-294.
2. Алимов Х. Т. Дробная математическая модель Макшерри / Х. Т. Алимов, Ф. Х. Дзамихова, Р. И. Паровик // Вестник КРАУНЦ. Физико-математические науки. – 2023. – Т. 42, № 1. – С. 164-179. – DOI 10.26117/2079-6641-2023-42-1-164-179. – EDN BSUNVQ.
3. Марценюк В. П., Сарабун Р. О. Исследование нелинейной динамики в модели МакШерри на основе экспонент Ляпунова // Вестник Воронежского государственного университета. Серия: Системный анализ и информационные технологии. – 2014. – №. 2. – С. 57-61.
4. Oldham K., Spanier J. The Fractional Calculus. Theory and Applications of Differentiation and Integration to Arbitrary Order. London: Academic Press, 1974. 240 pp.
5. Miller K., Ross B. An Introduction to the Fractional Calculus and Fractional Differntial Equations. New York: A Wiley-Interscience Publication, 1993. 384 pp.
6. Нахушев А. М. Дробное исчисление и его применение. Москва: Физматлит, 2003. 272 с.
7. Kilbas A. A., Srivastava H. M., Trujillo J. J. Theory and Applications of Fractional Differential Equations, vol. 204: Amsterdam, 2006. 523 pp.
8. Petras I. Fractional Order Nonlinear Systems. Modeling, Analysis and Simulation.—Beijing, Springer-Verlag Berlin Heidelberg : Springer, 2011.
9. Parovik R. I. Mathematical Models of Oscillators with Memory // Oscillators–RecentDevelopments. - IntechOpen, 2018. DOI:10.5772/intechopen.81858. - URL: https://www.intechopen.com/online-first/mathematicalmodels-of-oscillators-with-memory.
10. Паровик Р. И. Математическое моделирование линейных эредитарных осцилляторов. — Петропавловск-Камчатский: КамГУ им. Витуса Беринга, 2015.
11. Einthoven W. Le télécardiogramme. — Paris: Arch Int Physiol, 1906. — No 4. — P. 132-164.
12. Goldberger A. L., Goldberger E. Clinical Electrocardiography, Mosby, St. – 1977.
13. Pan J., Tompkins W. J. A real-time QRS detection algorithm //IEEE transactions on biomedical engineering. – 1985. – no. 3. – P. 230-236.
14. Kaplan D. T. Simultaneous QRS detection and feature extraction using simple matched filter basis functions // Proceedings Computers in Cardiology. – IEEE, 1990. – С. 503-506.
15. Davey P. A new physiological method for heart rate correction of the QT interval, Heart, 1999. vol. 82, no. 2, pp. 183-186.
16. Moody G. B. et al. Derivation of respiratory signals from multi-lead ECGs //Computers in cardiology. – 1985. – Т. 12. – №. 1985. – С. 113-116.
17. Moody G. B. et al. Clinical validation of the ECG-derived respiration (EDR) technique //Computers in cardiology. – 1986. – Т. 13. – №. 1. – С. 507-510.
18. Malik M., Camm A. J. Heart rate variability //Clinical cardiology. – 1990. – Т. 13. – №. 8. – С. 570-576.