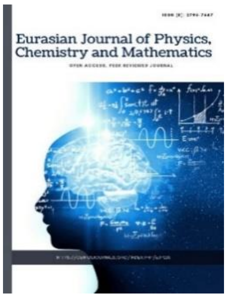

Complex solutions of trigonometric equations and The roots of the equation sin(x) = a

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ABSTRACT

This article deals with complex solutions of trigonometric equations in an undefined interval. That is, we were taught that trigonometric functions, especially equations such as Sin(x), do not have a solution if they are not in the interval [-1;1]. But in this article, the range of values of Sin(x) is [-1; 1] accepts non-interval states and we will see that they have a complex form

Keywords: Complex number, trigonometric equation, system of equations, area of values, exponential representation of a complex number, vertical axis, horizontal axis, complex plane, natural logarithm, abstract and real parts of a complex number, Re(z), Im(z).

As for the topics we will encounter in starting this topic, we first need to have a general understanding of complex numbers. In addition, the range of values of the trigonometric equation Sin(x) is discussed.
The equation we will consider is Sin(x) = a in an undefined interval
a ∉ [-1; 1] is to obtain a solution in the interval. So, the first concept we will consider is complex numbers.
Let us be given a complex number in the form z = a+ib. Let's make it trigonometric.
z = cosφ + i sinφ
From here, taking φ= x, we can see z = sosx + i sinx. Now if we use the index representation of a complex number:
z = e^{ix};
z = cosx + i sinx;
In general, we have expressions in this case. From the equal strength of these expressions, by equating the left side of the equation and making the variable x negative, we get the following system of equations:

$$\begin{cases} e^{ix} = Cos(x) + iSin(x) \\ e^{-ix} = Cos(-x) + iSin(-x) \end{cases}$$

By dividing this system of equations, we get the expression
2isin (x) =e^{ix}-e^{-ix} . That is, it allows us to form two equalities such as sin(x) =$\frac{e^{ix}-e^{-ix}}{2i}$ and sin(x) = a.

By equating these two equations, we get the following exponent and complex number equation:

$\frac{e^{ix}-e^{-ix}}{2i}$ = a

We will try to find the unknown x from this equation. For this, we will perform the following steps:

e^{ix}- e^{-ix}= 2ia
By equality in the form of e^{ix}= t, we get this simpler representation:
t - $\frac{1}{t}$ = 2ia
t^2-2iat-1 = 0

We find the unknowns from this quadratic equation through the discriminant.

D = 4 - $4a^2$

$t_{1;2}=\frac{2ia \pm 2\sqrt{1-a^2}}{2}$;

That is, the expression e^{ix} = ia $\pm\sqrt{1-a^2}$ is formed. In order to simplify this solution, we introduce the natural logarithm on both sides

$\ln e^{ix} = \ln(ia \pm \sqrt{1-a^2})$

$ix = \ln(ia \pm \sqrt{1-a^2})$

$ix = \ln(ia \pm \sqrt{(-1)(a^2-1)})$

We know that the square of complex number i is equal to (-1).

$ix = \ln(ia \pm i\sqrt{(a^2-1)})$

$ix = \ln i\left(a \pm \sqrt{a^2-1}\right)$

$ix = \ln i + \ln\left(a \pm \sqrt{a^2-1}\right)$

$x = \frac{\ln i + \ln\left(a \pm \sqrt{a^2-1}\right)}{i}$;

Let's calculate the natural logarithm of the unknown complex number involved in the solution of this quadratic equation.

We have i as a complex number. If we insert the natural logarithm ln on both sides of z = i, then $\ln i = \ln z$ is expressed in the general form $\ln z = \ln r + i\varphi$.

The proof is:

If it is known that $z = e^{i\varphi} \times r$ representation is the exponential representation of a complex number, if we introduce the natural logarithm to both sides of it,

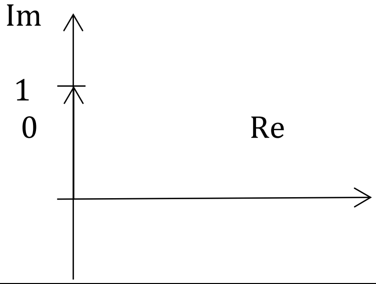
$z = e^{i\varphi} \times r$

$$\ln z = \ln r + \ln e^{i\varphi}$$

$\ln e = 1$, it follows that:

$\ln z = \ln r + i\varphi$;

Now, to find the modulus r of the unknown complex number and the angle z = i, if we describe the complex number on the coordinate axis, here the real part, i.e. Re(z) part is 0 and the abstract part is The part of Im(z) is equal to 1:



As a reference, we can say that the set of points of complex numbers forms a plane.

In the above view, the real part of the complex number is marked on the vertical axis, and the abstract part is marked on the horizontal axis.

Re(z)=0

Im(z)=1

Since $r = |z| = \sqrt{(Re(z))^2 + (Im(z))^2}$, it is known that r = 1 and $\varphi = \frac{\pi}{2}$ is the angle between, then the initial

$\ln z = \ln r + i\varphi$

to return to equality:

$\ln i = \ln 1 + \frac{\pi}{2} i$; $\ln 1 = 0$

from this it can be seen that we will have the value $\ln i = \frac{\pi}{2} i$.

Returning to the original solution,

$x = \frac{\ln i + \ln\left(a \pm \sqrt{a^2-1}\right)}{i} = \frac{\pi}{2} + \frac{1}{i}\ln\left(a \pm \sqrt{a^2-1}\right)$

$x = \frac{\pi}{2} \pm i\ln\left(a \pm \sqrt{a^2-1}\right)$;

we will have equality.

The main case that we need to see is that trigonometric equations in an undefined interval can take the value of x in the trigonometric equation Sin(x) = a that we considered, can also take values in the complex form derived above. From the above it can be concluded that it is possible to find the solution of the equation even in undefined intervals. It is no exaggeration to say that the solutions of the equations that we consider as non-existent actually exist and belong to the family of complex solutions.

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