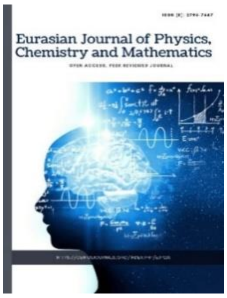

Use Of Information Technologies And Computer Mathematics Systems In The Process Of Teaching The Subject Of Differential Equations

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ABSTRACT

The article provides information about information technology and systems of computer mathematics. The advantages of Mathcad and Maple are described in detail, as well as how to use these programs to solve differential equations. In addition, the article provides a number of suggestions and recommendations for mathematics teachers on the use of Mathcad and Maple in the educational process.

Keywords: information technology, computer mathematics systems, Mathcad, Maple, differential equation, learning process, picture, diagram, Runge-Kutta method, Euler's method.

It is known that the use of information technologies is one of the new and effective forms of organizing the educational process. This is mainly the implementation of a specific curriculum aimed at the independent work of students. The transition to the information society opens up new opportunities for modernization of educational content and teaching methods. The computer is becoming a powerful tool for organizing and systematizing mathematical knowledge and skills, shaping the worldview and developing the student's mind. In the systematic use of computers in teaching mathematics, the following main points should be taken into account:

In order to obtain the expected result, it is necessary to use the computer continuously during the educational process;

It is necessary for the teacher to be familiar with the computer, to use a flexible methodology to use the educational material in various educational activities aimed at activating students [1].

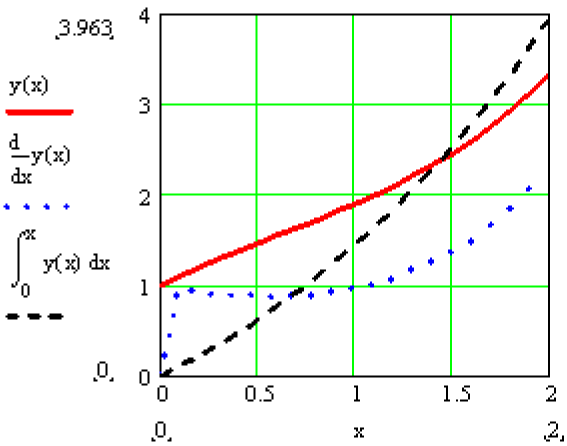
Recently, a new fundamental-scientific direction "computer mathematics" appeared in the area of mathematics and computer science and is widely used in scientific calculations and educational processes. Currently, the rapid development of computer mathematics, computer industry and programming technologies is recognized as the basis of automation of educational, scientific-methodical and scientific research work. At the same time, as a result of the application of the achievements in the field of modern information technologies, there are many software tools aimed at automating the solution of scientific-research, scientific-methodical, scientific-technical, engineering, financial and economic, chemical, biological issues. For example: universal software environments such as Mathematica, Maple, Matlab, Mathcad, Derive, Scientific, Workplace, Femlab, FeexPDE are among them. Two of these are widely used by professional mathematicians and researchers.

Mathcad was developed as a tool for engineering calculations, and is currently used to perform calculations of sufficient complexity, perform various numerical algorithms and analytical substitutions in scientific research. The use of Mathcad and Maple software environments, considered among the most advanced achievements in the field of information technology in the teaching of mathematics, is one of the main criteria for making the lesson interesting and effective. Mathcad is a computing software for professors, interns, researchers, graduate students, students, technical engineers, physicists, and more. With this program, various professions can solve problems related to their fields and get the necessary graphs and diagrams. Mathcad can be called, in other words, a programming language. For example, when solving differential equations in Mathcad, the Given-Odesolve calculation blog can be used [2]:

Given $\quad y'''(x) - 2^{-x}\cdot y(x) = x\cdot\sin(x)$

$y(0) = 1 \qquad y'(0) = 1 \qquad y''(0) = -0.5$

$y := \text{Odesolve}(x, 2) \qquad x := 0, 0.1 .. 2$



Here is an algorithm for solving a third-order variable coefficient differential equation.

It is known that in the educational process we are taught to solve a narrow class of differential equations, but in the course of education, we have the opportunity to solve a wide class of differential equations as a result of the use of computer mathematics systems, which makes the lesson more effective and interesting. Using the Mathcad Given-Odesolve calculation blog, the solution can be derived from a function and integrated. This can be seen in the graph above. It should also be noted that Mathcad does not have the ability to find general and analytical solutions of differential equations. This problem can be solved in Maple. The **dsolve(equation, variable, option)** command is used to solve differential equations in the Maple program, where the **equation** is a differential equation, the **variable** is the solution of a differential equation, and the **option** is an optional parameter, given in the form **keyword=value** [3]. The general and analytical solutions of a large number of differential equations can be found using the Maple **dsolve** command. If the **option type=exact** is given, an analytical solution will be attempted. If the **option type=series** is given, then the solution will be searched in series form. If the **option type=numeric** is given, a numeric solution is sought. Now we will try to analytically solve the differential equation:

$$\frac{\partial}{\partial x} y(x) = \sqrt{x^2 - y(x)} + 2x$$

> **eq:=diff(y(x),x)=sqrt(x^2-y(x))+2*x;**

$$eq := \frac{\partial}{\partial x} y(x) = \sqrt{x^2 - y(x)} + 2x$$

> **dsolve(eq,y(x));**

$$8 \frac{y(x)\sqrt{x^2-y(x)}}{2\sqrt{x^2-y(x)}-x} + \frac{4\,y(x)\,x}{2\sqrt{x^2-y(x)}-x} - \frac{6x^2\sqrt{x^2-y(x)}}{2\sqrt{x^2-y(x)}-x} - \frac{3x^3}{2\sqrt{x^2-y(x)}-x} - \_C1 = 0$$

The solution was found in an obscure form.

We make the solution analytical with the help of the command > **isolate(%,y(x));**

$$y(x) = \frac{5}{4}x^2 + \frac{1}{2}(-x + \sqrt{-\_C1})\,x + \frac{1}{4}\_C1$$

If we want to find a particular solution that satisfies the initial condition

$y(1) = 0$

we find the value of the constant **C1** using Maple's **solve** command:

> **x:=1;y:=0;solve(y = 5/4*x^2+1/2*(-x+sqrt(-C1))*x+1/4*C1,C1);**

$x := 1$

$y := 0$

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In that case, the private solution would be as follows:

$$y(x) = \frac{5}{4}x^2 + \frac{1}{2}(-x+3)\,x - \frac{9}{4}.$$

Here are some more examples of how to solve differential equations using Maple:

> **diff(y(x),x$2)-y(x)=sin(x)*x;**

$$\left(\frac{\partial^2}{\partial x^2} y(x)\right) - y(x) = \sin(x)\,x$$

> **dsolve(diff(y(x),x$2)-y(x)=sin(x)*x,y(x));**

$$y(x) = -\frac{1}{2}\cos(x) - \frac{1}{2}\sin(x)\,x + \_C1\,e^x + \_C2\,e^{(-x)}$$

Here, a general solution was found. *C1* and *C2* in the solution are optional constants.

Initial conditions in differential equations are given by commas and combined with the equation:

> **restart;dsolve({diff(v(t),t)+2*t=0,v(1)=5},v(t));**

$$v(t) = -t^2 + 6$$

The derivatives are written in the form of the operator in the initial conditions: D(D(y))(0) or D(@@2)(y)(0):

> **de1:=diff(y(t),t$2)+5*diff(y(t),t)+6*y(t)=0;**

$$de1 := \left(\frac{\partial^2}{\partial t^2} y(t)\right) + 5\left(\frac{\partial}{\partial t} y(t)\right) + 6\,y(t) = 0$$

> **dsolve({de1,y(0)=0,D(y)(0)=1},y(t),method=laplace);**

$$y(t) = -e^{(-3t)} + e^{(-2t)}.$$

Now we solve the fourth order equation:

> **de2:=diff(y(x),x$4)+2*diff(y(x),x$2)-cos(x)=3;**

$$de2 := \left(\frac{\partial^4}{\partial x^4} y(x)\right) + 2\left(\frac{\partial^2}{\partial x^2} y(x)\right) - \cos(x) = 3$$

> **dsolve(de2,y(x)):combine(%);**

$$y(x) = -\cos(x) - \frac{1}{2}\_C1\cos(\sqrt{2}\,x) - \frac{1}{2}\_C2\sin(\sqrt{2}\,x) + \frac{3}{4}x^2 + \_C3\,x + \_C4.$$

The solution for the following equation is found using the method of substitution of variables:

```
> restart;q:=(2*sqrt(x*y(x))-x)*diff(y(x),x)+y(x);
```

$$q := (2\sqrt{x\,y(x)} - x)\left(\frac{\partial}{\partial x}y(x)\right) + y(x)$$

To change a variable, the **Dchangevar** command of the **DEtools** package is used:

```
> restart;q:=(2*sqrt(x*y(x))-x)*diff(y(x),x)+y(x);
```

$$q := (2\sqrt{x\,y(x)} - x)\left(\frac{\partial}{\partial x}y(x)\right) + y(x)$$

```
> with(DEtools):f:=Dchangevar({y(x)=v(x)*x},[q],x);
```

$$f := (2\sqrt{x^2\,v(x)} - x)\left(\frac{\partial}{\partial x}v(x)\,x\right) + v(x)\,x.$$

Since the 50s of the 20th century, the kinetics of processes that occur under chemical reactions that take place at a very slow and sufficiently high speed at the same time began to be studied. Many such practical problems are brought to the solution of the Cauchy problem for ordinary differential equations and particular types of systems of ordinary differential equations. Such equations can be called special differential equations or systems of special differential equations. When numerically solving this type of differential equations and their system using the Runge-Kutta method, which is considered to be the most reliable, it was observed that the obtained solution changes slowly in the near-zero part of the integration interval, and suddenly changes when moving to the next part, that is, in the transition phase. The observed phenomenon means that other methods known from the course of computational mathematics such as Runge-Kutta, Euler, etc., are not suitable for solving this type of equations. In practice, there are such differential equations of this class that it is necessary to integrate millions, billions, or even more points to numerically solve them by the methods mentioned above. The solution of special differential equations or their system consists of two parts. One of them is a function that changes slowly enough, and the other tends to zero with a large speed. There are certain practical difficulties in calculating the values of this second function. For example, consider the following Cauchy problem:

$$y'' + 101\cdot y' + 100\,y = 0, \quad (1)$$

$$y(0) = 1.01, \qquad y'(0) = -2. \quad (2)$$

Since the characteristic equation of this homogeneous differential equation with constant coefficients of the second order has solutions

$$k^2 + 101\cdot k + 100 = 0$$

$k_1 = -1, k_2 = -100$, the general solution of equation (1) is written in the form

$$y(x) = C_1\cdot e^{-x} + C_2\cdot e^{-100x} \quad (3).$$

A particular solution satisfying the given initial conditions will look like this:

$$y(x) = e^{-x} + 0.01\cdot e^{-100x} \quad (4).$$

The obtained analytical solution consists of the sum of two functions, the values of the first of which change relatively flat and slowly, and the values of the second function change rapidly and tend to zero with great speed. The table below shows the variation pattern of the approximate values of these two functions in the section [0;0.1]:

The regularity of change of the approximate values of the function $y(x) = e^{-x} + 0.01\cdot e^{-100x}$ *in the section [0;0.1]*

Table.

x	$Y_1 = e^{-x}$	$Y_2 = 0{,}01 \cdot e^{-100x}$
0	1	0,01
0,00001	0,99999	0,009999
0,0001	0,9999	0,0099
0,001	0,999	0,009
0,01	0,99	0,004
0,1	0,9	0,0000004

As it can be seen from the values in the table, [0;0.1] is outside the cross section, that is, in the transition phase, the second additive of the solution will have such small values that it cannot be taken into account. From this, it can be concluded that it is necessary to numerically find the solution of problem (1)-(2) with a sufficiently small step in the cross-section [0;0.1] and to increase the integration step in order to save computer time in the transition phase and reduce rounding errors. Practical calculations have shown that this conclusion is incorrect. Because in order to obtain a stable solution using the familiar methods presented above, a sufficiently small integration step is required, which is the same in the entire part of the integration interval, due to the first function.

Such problems are easily solved in the Maple system. The general solution of the given equation is easily found in Maple as:

```
> restart;
> eq:=diff(y(x),x$2)+101*diff(y(x),x)+100*y(x)=0;
```

$$eq := \left(\frac{d^2}{dx^2}\,y(x)\right) + 101\left(\frac{d}{dx}\,y(x)\right) + 100\,y(x) = 0$$

```
> dsolve(eq,y(x));
```

$$y(x) = \_C1\,e^{(-x)} + \_C2\,e^{(-100\,x)}$$

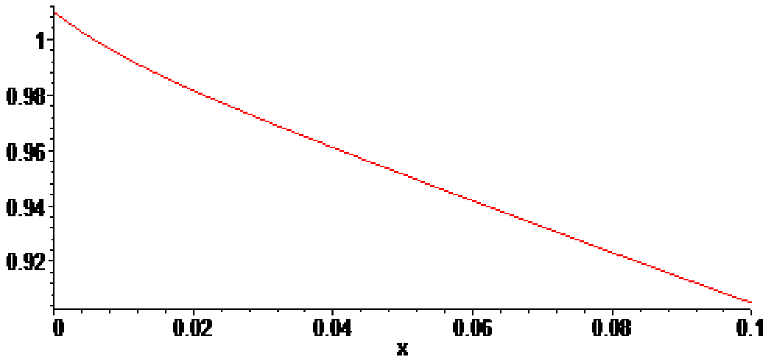
The particular solution of the equation satisfying the initial conditions is found in Maple as follows:

```
> cond:=y(0)=1.01,D(y)(0)=-2;de:=dsolve({eq,cond},y(x));
```

$$cond := y(0) = 1.01,\ D(y)(0) = -2$$

$$de := y(x) = e^{(-x)} + \frac{1}{100}\,e^{(-100\,x)}.$$

> **plot(exp(-x)+1/100*exp(-100*x),x=0..0.1);** command automatically draws the graph of the function that is the solution of the differential equation on the section x=[0;0.1] and helps the student to have a complete idea of the graph of the function that is the solution of the differential equation.



The graph of the function $y(x) = e^{-x} + 0.01 \cdot e^{-100x}$ *on the section x=[0;0.1].*

The use of the Maple program in the educational process allows you to see the graph of the solution function in practice, and students will fully understand how well the solution was found based on the graph of the function. The use of software environments such as Mathematika, Maple, and Mathlab, which are among the most advanced achievements in the field of information technology in the teaching of mathematics, allow students to be fully engaged in the lesson and increase the enthusiasm of students to learn mathematics and its latest achievements. Students will consciously understand the need for in-depth study of mathematics and engage in independent learning of the latest advances in mathematics [4]. In conclusion, it should be noted that the use of Mathcad and Maple in the educational process and scientific research works greatly helps in solving many problems.

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