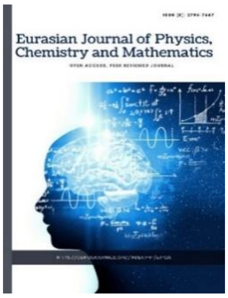

Derivation of temperature dependent Cauchy dispersion relation for Germanium

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ABSTRACT

In this study, Cauchy dispersion relation for Germanium in a specific temperature were derived, and that dependent on temperature. The temperatures taken are ranging from 30 to 300 K and wavelengths from 1.8 to 5.5 μm. The results of the computed refractive index were compared with a previously published data, and showed a good agreement.

Keywords: Cauchy dispersion equation, refractive index, phase velocity, cryogenic temperature.

1. Introduction:

The existence of the Germanium element (Z=32) with properties intermediate between the metal Zn and the non-metal Si had been predicted by D.I. Mendeleev, the Russian chemist, in 1871 due to the regular nature of the Periodic Table elements. However, it was the German chemist Clemens Winkler who first detected it in 1886 as a component of the mineral argyrodite (Ag8GeS6) in silver ores. Typically, germanium is recovered as a by-product from zinc and copper ores and coal[1].

Germanium (Ge) is one of the elements of modern technologies that are used in high-technology applications such as infrared systems, polymer catalysis, fiber optics, electronics, and solar cells. Its demand is expected to increase due to the deficiency of proper substitutes, increasing demand for solar cells and 5G networks, and the continuous increasing trend of Ge demand for the past few decades[ ].

This material, Germanium, is also essential in the field of infrared lens design. The optical instruments that designed for the wavelength range, where this material is transmissive, attain the best performance when cooled to cryogenic temperatures to improve the signal from the scene over instrument background radiation. To enable high-quality lens designs using germanium at cryogenic temperatures, B. J Frey et. al. measured the absolute refractive index of multiple prisms of this material using the Cryogenic, High-Accuracy Refraction Measuring System (CHARMS) at NASA's Goddard Space Flight Center, as a function of both wavelength and temperature[3].

In this research, Cauchy's dispersion relation was used to compute the refractive index of Germanium, which is an empirical relationship, named by the mathematician Augustin-Louis Cauchy who defined it in 1837, describe the dependence of the refractive index on the wavelength of light for a particular material. In 2002 D. Y. Smith express a novel process of describing optically transparent substances using dispersion theory, the index of refraction is found by a generalized equation of Cauchy dispersion with coefficients that are moments of the ultraviolet and infrared absorptions. Abbe´ number, Mean dispersion, and partial dispersion are combinations of these moments[4].

In 2013 E. Stoumbou et al. presented results on refractive index dispersion modeling in 61 different crystals, and made a comparative study on the use of the extended-Cauchy dispersion equation for fitting refractive index data in crystals[5].

2. Results:

In this research, Cauchy's dispersion relation was used to compute the refractive index of Germanium at cryogenic temperatures. Cryogenics, In physics, is the behavior and production of materials at very low temperatures),

To find refractive index values, Cauchy formula of eq, 1 was used as follows[6]:

$$n(\lambda) = A(T) + \frac{B(T)}{\lambda^2} + \frac{C(T)}{\lambda^4} + \frac{D(T)}{\lambda^6} \text{..........(1)}$$

For solving this equation that contains four parameters A, B, C, and D, four simultaneous equations must be solved by taking four practical values for the refractive index with the wavelength, at each temperatures, and here three temperatures will be taken 30 K, 90 K, and 200 K. These equations have been solved using Matlab code and extracting the values of the four constants as in Table (1)

Table (1): parameters values of Eq. (1) for three different temperatures

T(K)	A	B(μm²)	C(μm⁴)	D(μm⁶)
30	3.92708	0.33183	0.12102	0.11762
90	3.934982	0.341603	0.058697	0.28919
200	3.96624	0.36927	-0.00073	0.50545

To assure the correctness of these parameters, a comparison of the values evaluated from Eq.(1) was made with the practical values of reference [3], at the IR region wavelengths, as in Table (2) and Fig.(1), and it is obvious that the values are nearly the same to 3 digits.

Table (2): Values of refractive index evaluated from Eq.(1), n, with the empirical values from ref.[3],np.

λ	30 K		90 K		200 K	
(μm)	n	np	n	np	n	np
1.8	4.04448	4.0445	4.0545	4.0545	4.09501	4.0950
2.0	4.01944	4.0192	4.0285	4.0285	4.0664	4.0666
2.5	3.98375	3.9837	3.9923	3.9924	4.02948	4.0276
3.0	3.96560	3.9656	3.9740	3.9741	4.00796	4.0080
3.5	3.95504	3.9551	3.9634	3.9633	3.99666	3.9967
4.0	3.94832	3.9483	3.9566	3.9565	3.98745	3.9894
4.5	3.94377	3.9438	3.9520	3.9520	3.98454	3.9845
5.0	3.94055	3.9406	3.9487	3.9488	3.98105	3.9811
5.5	3.93819	3.9382	3.9463	3.9463	3.97847	3.9784

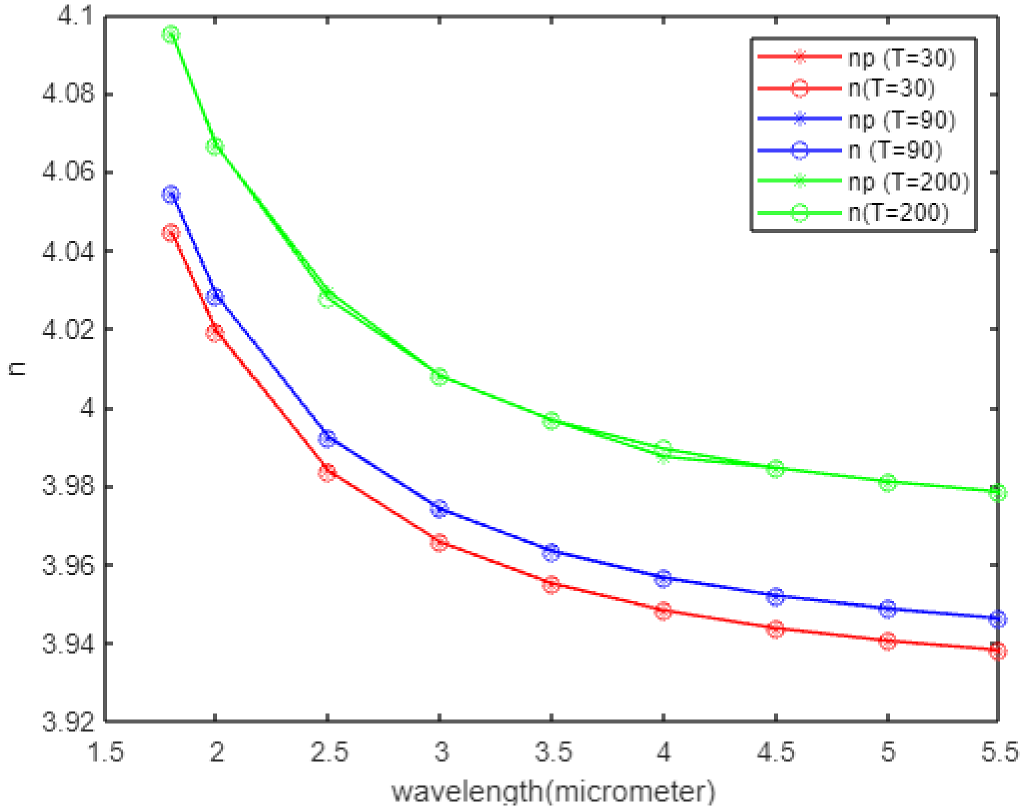

Fig. (1): Values of refractive index evaluated from Eq.(1), n, with the empirical values of ref.[3],np.

3- Cauchy dispersion equation as a function of temperature

To write Cauchy equation with four coefficients that depend on wavelength in terms of temperature, these coefficients must be written in terms of temperature by defining them in a quadratic equation with three coefficients, as follows:

$A=a_1T^2+ a_2T+ a_3$
$B=b_1T^2+ b_2T+ b_3$
$C=c_1T^2+ c_2T+ c_3$
$D=d_1T^2+ d_2T+ d_3$ (2)

Thus, the temperature-dependent Cauchy dispersion relation is of the form:

$$n(\lambda,T) = a_1T^2 + a_2T + a_3 + \frac{b_1T^2+b_2T+b_3}{\lambda^2} + \frac{c_1T^2+c_2T+c_3}{\lambda^4} + \frac{d_1T^2+d_2T+d)}{\lambda^6} \quad (3)$$

These twelve parameters of Eq. (3) are extracted by solving 3 equations for each one of the four parameters (A, B, C,and D) at the three temperatures (30, 90, and 200 K) as in Table (3). K.

Table (3): parameters values of Eq. (3).

A		**B**		**C**		**D**	
a1	$8.97353*10^{-7}$ K^{-2}	**b1**	$5.224356 *10^{-7}$ K^{-2}	**c1**	$2.9324 *10^{-6}$ K^{-2}	**d1**	$-5.23348*10^{-6}$ K^{-2}
a2	$2.39699*10^{-5}$K	**b2**	.0 0001 K	**c2**	-0.00139 K	**d2**	0.00349 K
a3	3.92556	**b3**	0.32837	**c3**	0.16011	**d3**	0.01765

Eq.(3) were used to calculate refractive indecies for wavelength range from 1.8 μm to 5.5μm for temperature range 30K-300K as in tables (4-6) , and Fig.s (2-4), and they were compared with empirical values of ref. [3]. The tables and figures shows that this dispersion equation, (eq.3), that depending on wavelength and temperature is more suitable for the law temperatures (T< 0C°), while for temperatures (T> 0C°), there were shift in the results as it is obvious in Fig.(4).

Table (4): Values of refractive index evaluated from Eq.(3), n, with the empirical values from ref.[3],np. For temperatures (30, 40, 50, 60)K.

T(K)	30		40		50		60	
□□□	n	np	n	np	n	np	n	np
1.8	4.04	4.044	4.045	4.045	4.0469	4.0465	4.04448	4.048
2.0	4.01	4.019	4.020	4.020	4.0216	4.0211	4.01944	4.022
2.5	3.98	3.983	3.984	3.984	3.9858	3.9855	3.98375	3.986
3.0	3.96	3.965	3.966	3.966	3.9676	3.9673	3.96560	3.968
3.5	3.95	3.955	3.956	3.955	3.9571	3.9568	3.95504	3.958
4.0	3.94	3.948	3.949	3.949	3.9503	3.9500	3.94832	3.951
4.5	3.94	3.943	3.944	3.944	3.9458	3.9455	3.94377	3.946
5.0	3.94	3.940	3.941	3.941	3.9426	3.9422	3.94055	3.943
5.5	3.93	3.938	3.939	3.938	3.9402	3.9399	3.93819	3.941

Table (5): Values of refractive index evaluated from Eq.(3), n, with the empirical values from ref.[3],np. For temperatures (70, 80, 90, 100)K.

T (K)	70		80		90		100	
□□□m□	n	np	n	□	n	np	n	np
1.8	4.0502	4.0499	4.0522	4.0519	4.0545	4.0545	4.0570	4.0575
2.0	4.0246	4.0243	4.0265	4.0260	4.0286	4.0285	4.0309	4.0314
2.5	3.9886	3.9885	3.9904	3.9901	3.9923	3.9924	3.9945	3.9952
3.0	3.9705	3.9702	3.9722	3.9719	3.9741	3.9741	3.9762	3.9767
3.5	3.9599	3.9596	3.9615	3.9611	3.9634	3.9633	3.9655	3.9659
4.0	3.9531	3.9528	3.9548	3.9543	3.9566	3.9565	3.9587	3.9591
4.5	3.9485	3.9483	3.9502	3.9499	3.9520	3.9520	3.9541	3.9546
5.0	3.9453	3.9450	3.9469	3.9466	3.9488	3.9488	3.9508	3.9513
5.5	3.9429	3.9426	3.9445	3.9442	3.9463	3.9463	3.9484	3.9489

Table (6): Values of refractive index evaluated from Eq.(3), n, with the empirical values from ref.[3],np. For temperatures (150, 200, 250, 295)K.

T(K)	150		200		250		295	
□□□m)	n	np	n	np	n	np	n	np
1.8	4.0731	4.0744	4.0950	4.0950	4.1229	4.1193	4.1530	4.1443
2.0	4.0458	4.0477	4.0664	4.0666	4.0927	4.0883	4.1211	4.1101
2.5	4.0083	4.0102	4.0274	4.0276	4.0516	4.0475	4.0778	4.0674
3.0	3.9896	3.9912	4.0080	4.0080	4.0312	4.0271	4.0564	4.0463
3.5	3.9787	3.9802	3.9967	3.9967	4.0194	4.0153	4.0440	4.0340
4.0	3.9717	3.9732	3.9894	3.9894	4.0119	4.0077	4.0361	4.0260
4.5	3.9670	3.9685	3.9845	3.9845	4.0068	4.0026	4.0307	4.0206
5.0	3.9636	3.9652	3.9810	3.9811	4.0031	3.9990	4.0269	4.0169
5.5	3.9611	3.9626	3.9785	3.9785	4.0004	3.9964	4.0241	4.0142

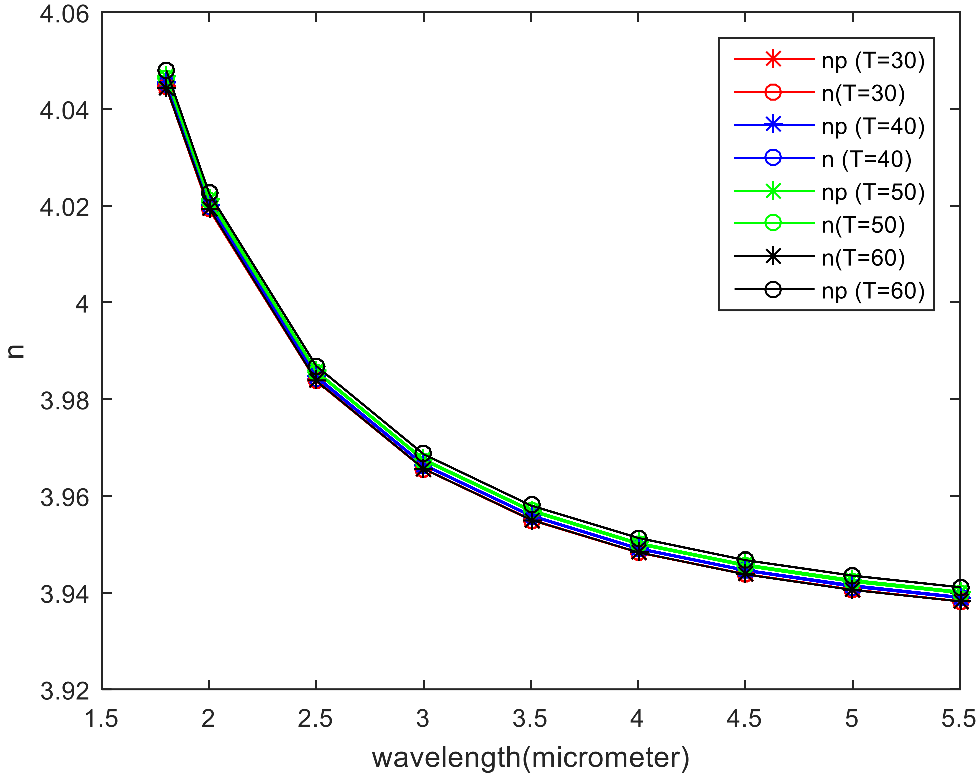

Fig. (2): Values of refractive index evaluated from Eq.(3), n, with the empirical values from ref.[3],np. For temperatures (30, 40, 50, 60)K.

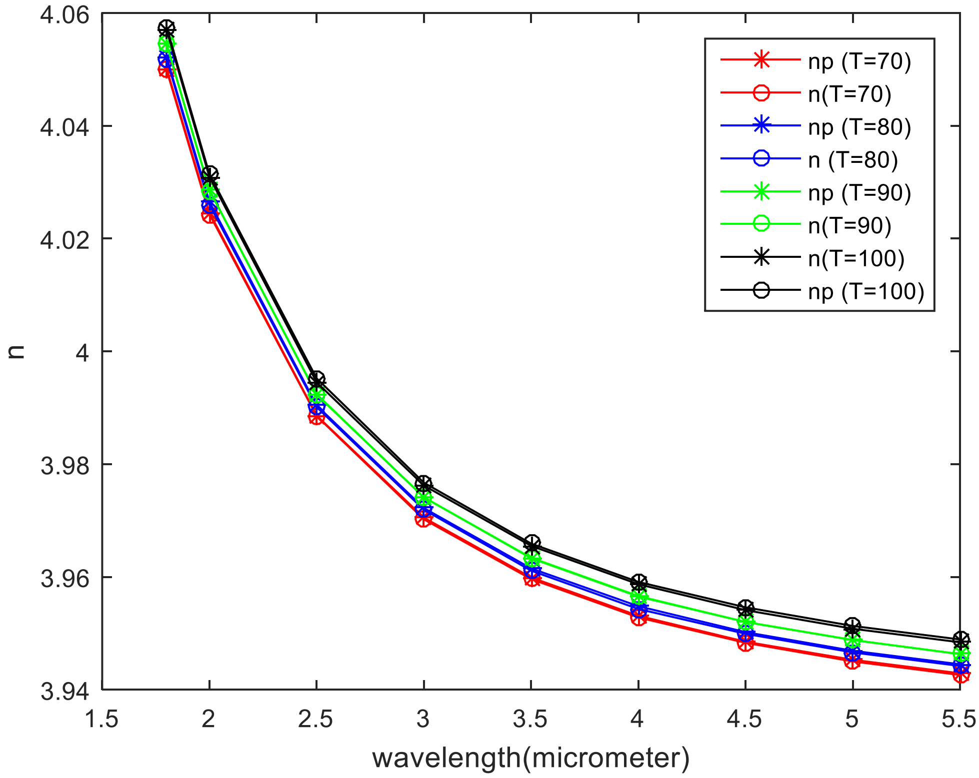

Fig. (3): Values of refractive index evaluated from Eq.(3), n, with the empirical values from ref.[3],np. For temperatures (70, 80, 90, 100)K.

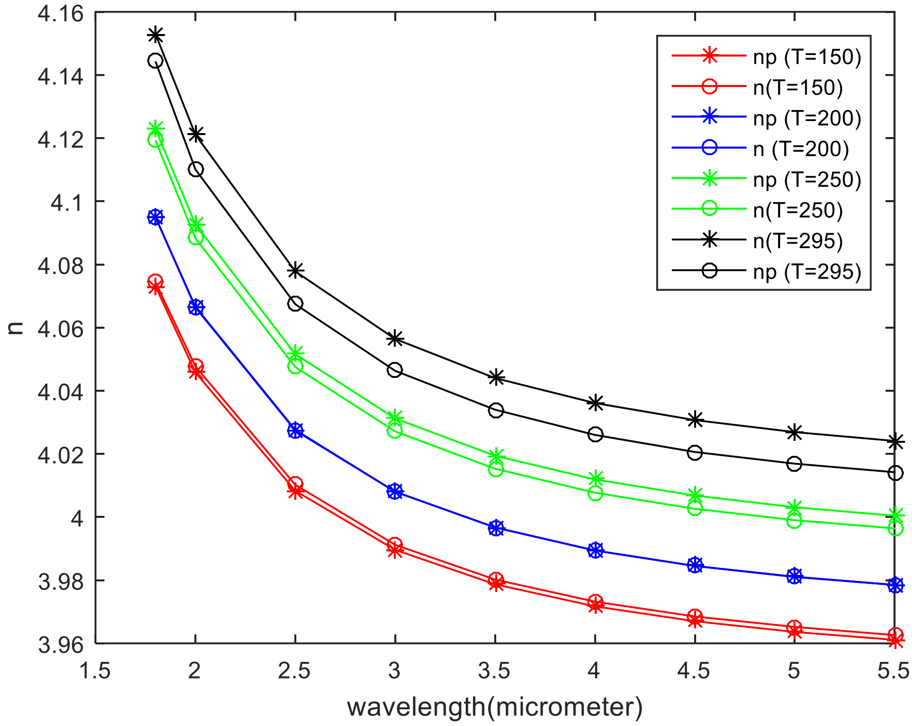

Fig. (4): Values of refractive index evaluated from Eq.(3), n, with the empirical values from ref.[3],np. For temperatures (150, 200, 250, 295)K.

Fig. (5) shows a 3-D plot represents the dependence of refractive index on wavelength and temperature using Eq.(3), and Fig.(6) represents the empirical values of ref. [3], the two figures are identical.

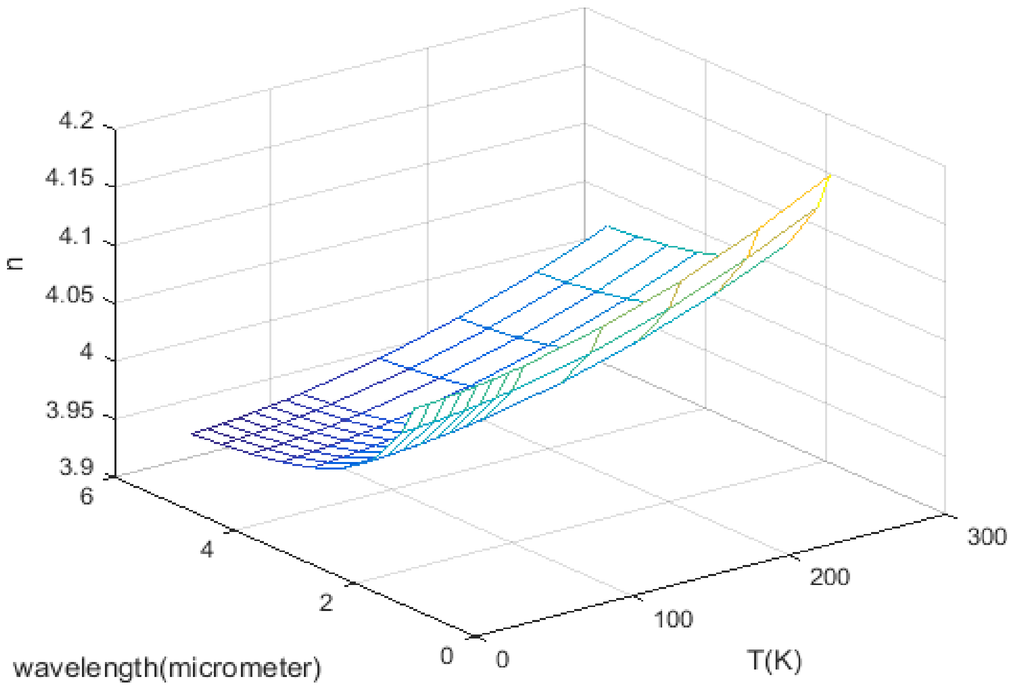

Fig. (5) 3-D plot represents the dependence of refractive index on wavelength and temperature using Eq. (3).

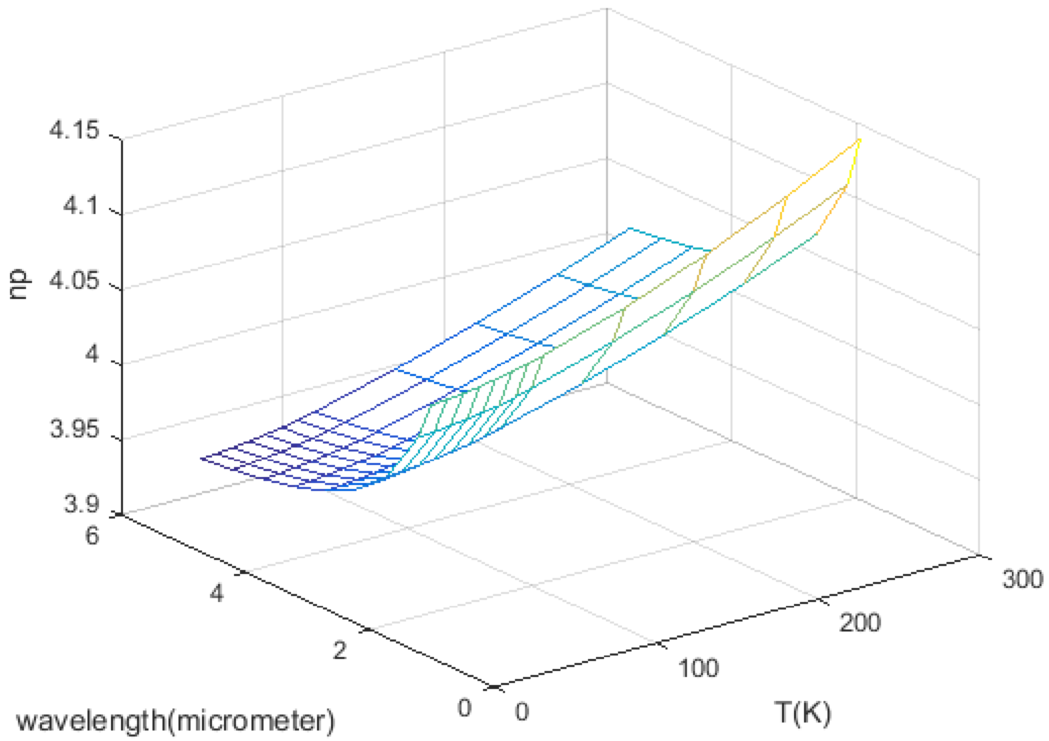

Fig. (6) 3-D plot represents the empirical valu s of refractive index depending on wavelength and temperature from ref.[3]

4- Phase velocity

The rate at which the wave cause to propagates in a medium is called the phase velocity, which is the velocity that the phase of travelling of any single frequency component of the wave. The phase velocity is given in terms of angular frequency ω and wave number k:

$vp = \omega/k = c/n$

where c is the velocity of light, $3*10^8$m/sec, then the phase velocity can be written as:

$$vp= \frac{c}{n(\lambda,T)} = \frac{c}{A(T)+\frac{B(T)}{\lambda^2}+\frac{C(T)}{\lambda^4}+\frac{D(T)}{\lambda^6}}$$

This equation shows that the phase velocity is directly proportional with the wavelength and inversely proportional with temperature, as illustrated in Fig.(7).

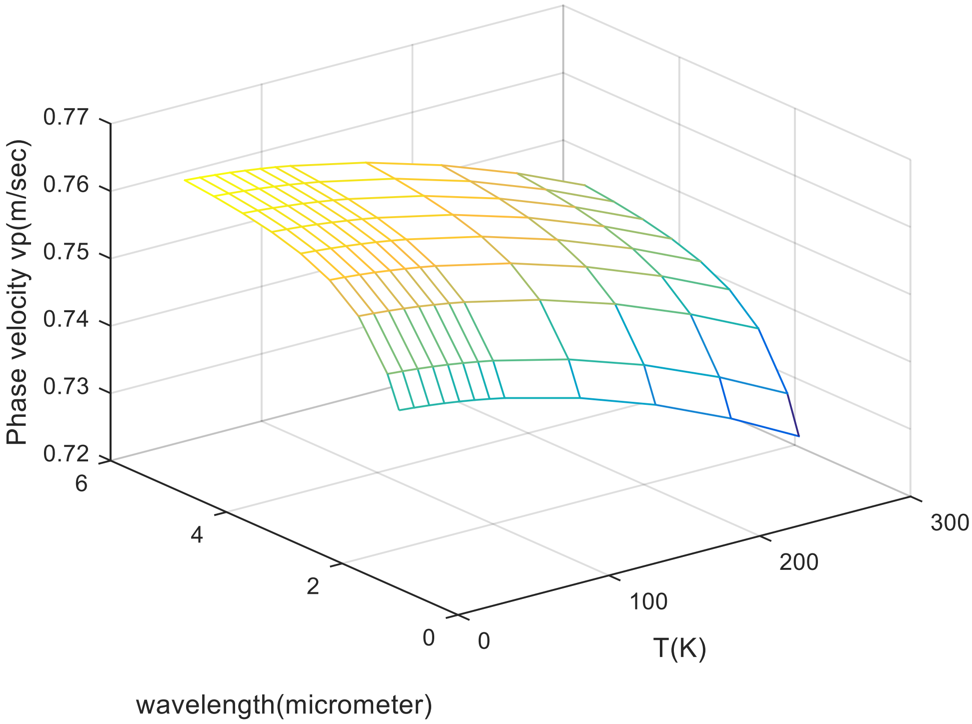

Fig. (7) 3-D plot represents the phase velocity (Eq. (3)) depending on wavelength and temperature from ref.[3].

5- Conclusions

Refractive index of Germanium material were found in this work using Caushy's equation in the IR wavelength region and Cryogenic temperatures. Several conclusions can be constructed as follows:

1- Cauchy's equation is a good method to predict the refractive index values to an accuracy of three digits.
2- Cauchy's equation need to know several empirical values of refractive index, and the number of these values depend on the number of coefficients in the equation used, and this in turn affect the accuracy of equation.
3- The coefficient were used for Cauchy equation in this work were suitable for IR wavelength and cryogenic temperatures. So the results accuracy for the temperatures T>200K were not as good as those of Crygenic temperatures.
4- Refractive index of Germanium, in this region of interest, is inversely proportional with wavelength and directly proportional with temperature.
5- Phase velocity of the propagating wave in Germanium in this region of interest, is directly proportional with wavelength and inversly proportional with temperature.

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