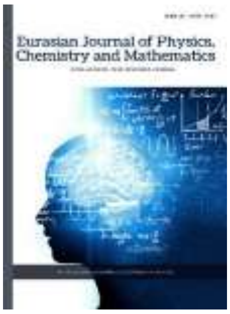

An Integrated Production Inventory Model for Repairable Items with Uncertain Lead time

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ABSTRACT

The integrated Much focus has been placed on inventory policy recently as a means of preserving a competitive edge. The integrated inventory model has been the subject of numerous studies.

Keywords: Integrated inventory, competitive edge

Introduction:

The integrated Much focus has been placed on inventory policy recently as a means of preserving a competitive edge. The integrated inventory model has been the subject of numerous studies. Banerjee devised a model in 1986 that considers the economics of a two-tier supply chain with a single supplier, a single customer, and a maximum output. After the complete lot has been produced, Goyal (1988) supposing the manufacturing run is sent out in a series of consignments of about the same size. Other studies did away with the requirement that shipments begin when the production lot was finished. Lu (1995) presented a lot-splitting model with single set-up and many delivery by relaxing the model of Goyal (1988). According to Goyal (1995), a different strategy comprising Tayal et al. established a unified approach for the production and inventory management of perishable goods, including a trade credit term and investments in preservation technologies (2015). Hill (1997) expanded on Goyal (1995)'s initial generalisation by treating the geometric growth factor as a decision variable as opposed to a fixed value. Hence, the decision factors are the sum of all shipments that came from a single manufacturing run, the volume of the initial shipment, and the geometric growth factor. In Goyal and Nebebe, a particular instance of the problem was considered (2000). They suggested a strategy that calls for a tiny shipment to be sent first, then several larger, equal-size shipments. In their 2009 work, Singh and Diksha presented a cooperative model between vendors and buyers that included a graduated credit system, where demand was seen as a multivariate function. A cost-integrated model for optimum maintenance and production scheduling was developed by Hadidi et al. in 2011. In this analysis, task order scheduling and preventive maintenance choices for a single machine are taken into account simultaneously. Singh and Singh proposed a trade credit policy integrated supply chain model for volatile goods (2012). A two-tiered supply chain with a single seller and customer was the focus of Tayal et al(2014) .'s inventory model, as well as multiple lead-time and storage space limitations. In addition, Tayal et al. (2014) created a model for integrated inventories that could account for seasonal product lead times and the existence of a secondary market. Using a trade credit term and expenditures in preservation technologies, Tayal et al. introduced a comprehensive model of production and stockpiling for perishable goods (2015).

The conventional inventory model considers the ideal scenario where things have an endless lifespan, but in reality, all products experience some level of deterioration over time. Tayal et al. offer a complex inventory model that takes into consideration perishable goods, their expiration dates, and the amount of stock that may be lost before the goods spoil (2014). In this model, the degradation of the products is thought to occur in a linear fashion. Inventory models for two-tier supply chains were examined by Tayal et al. (2014) that are decaying with efficient preservation technology investment. To slow down the pace of deterioration, the model is described under two distinct lead-time and preservation technique conditions.

Remanufacturing and reparability are key factors in inventory modelling when taking environmental issues related to waste disposal into account. People are increasingly highly aware about the consumption of natural resources. So, after using the returned and used goods from the market, the enterprises are compelled to take the initiative for remanufacturing/repair ability. This allows for the reuse of waste products that damage the environment while conserving natural resources. The idea of remanufacturing and repairability has received significant attention from scholars over the previous few decades. It was Schrady who first utilised "repairing/remanufacturing" in inventory modelling (1967). A known and predetermined percentage of the gathered objects can be used for remanufacturing, according to an inventory model Teunter (1998) presented. According to Dobos and Richter (2000, 2004), who relaxed the premise of a constant return rate, a pure strategy—in which every returned item is either being remanufactured or promptly discarded—will always be the best course of action. A higher return rate may be achieved via the use of promotional techniques that entice buyers to resale the old items. Repair was then described by King et al. (2006) as "the improvement of particular product faults," with the caveats that the quality of fixed goods is lower than that of remanufactured products and that these things can only be sold in secondary markets. To complement the work of Chung et al. Reverse logistics for a multi-tiered supply chain was created in 2008. To date, Singh et al. have developed the most sophisticated model to account for reverse logistics (2013), who includes flexible production and stock out situations. Yang and coworkers construct a multi-retailer closed-loop inventory model for the supply chain (2013). There's consideration of price-sensitive demand in this article.

This study presents a a supply chain that continuously recycles materials for consumables that can be repaired.

Products that have already been used are gathered from the marketplace and, in certain, fixed and remanufactured. A lead time is considered while delivering these goods to the retailer. The integrated system's overall average cost has been computed. A numerical example is used to demonstrate the theoretical results.

Assumptions:

1. The integrated model for the development of new products and the remanufacturing of collected objects is presented.
2. The relationship between a product's selling price and its popularity among buyers.
3. It is assumed that the remanufactured products are identical to new products.
4. Assuming a correlation between the two, we may say that the demand rate and the output rate are connected to each other.
5. The used things are gathered regularly of $\frac{b\alpha}{{p_2}^{\beta}}$.
6. Just a specific proportion of the gathered items—those with a quality level suitable for remanufacturing—are employed in production; the remainder is recycled.
7. It is expected that the things are decomposing naturally.

8. The shop takes the lead time into account.

Notations:

θ	deterioration rate
α, β	demand parameters
p_1, p_2	selling price per unit for the producer and the retailer
a	production parameter, $a \geq 1$
b	collection parameter, $b<1$
t_1	time for remanufacturing
t_2	time at which inventory level for remanufactured items becomes zero
t_3	time up to which production of fresh items occur
T	length of the complete cycle
$I_r(t)$	At any one time, the remanufactured product inventory level
$I_m(t)$	inventory level for fresh produced items at any time t
$I_R(t)$	inventory level of collected items at any time t
$I_s(t)$	retailer's inventory level at any time t
y	the lead time
v	the time at which inventory level becomes zero
n	number of replenishment cycles for the retailer
c_m	procurement cost per unit
c_R	acquisition cost per unit
s_m	production cost per unit
s_r	remanufacturing cost per unit

h_r, h_m, h_R, h_s holding cost per unit for remanufactured items, produced items, collective items and for the retailer

O	ordering cost per order
c_1	production cost per unit
c_4	deterioration cost per unit
K_1, K_2, K_3	set up cost for remanufacturing, fresh production and for the collection

Mathematical Modeling:

Figures 1 and 2 show how the system's inventory time behavior changes throughout the remanufacturing cycle, production cycle, and collection cycle. The system's differential equations are provided as follows:

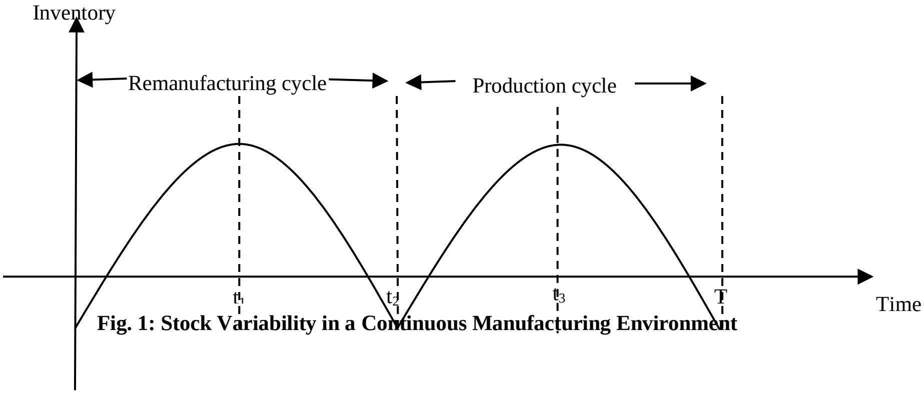

Fig. 1: Stock Variability in a Continuous Manufacturing Environment

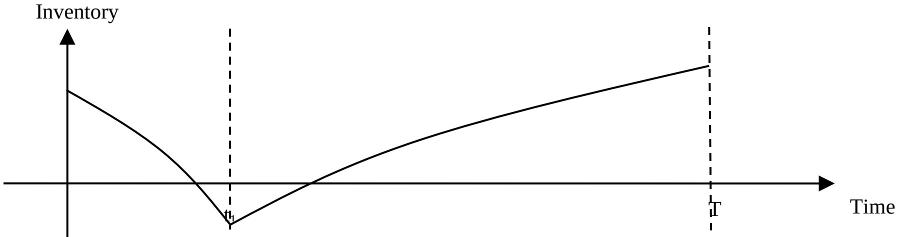

Fig. 2: Behaviour of the returned and collected items

$$\frac{dI_r(t)}{dt} = \frac{\alpha}{p_1^{\beta}}(a-1) - \theta I_r(t) \qquad 0 \le t \le t_1 \qquad (1)$$

$$\frac{dI_r(t)}{dt} = -\frac{\alpha}{p_1^{\beta}} - \theta I_r(t) \qquad t_1 \le t \le t_2 \qquad (2)$$

$$\frac{dI_m(t)}{dt} = (a-1)\frac{\alpha}{p_1^{\beta}} - \theta I_m(t) \qquad t_2 \le t \le t_3 \qquad (3)$$

$$\frac{dI_m(t)}{dt} = -\frac{\alpha}{p_1^{\beta}} - \theta I_m(t) \qquad t_3 \le t \le T \qquad (4)$$

$$\frac{dI_R(t)}{dt} = (b-a)\frac{\alpha}{p_2^{\beta}} - \theta I_R(t) \qquad 0 \le t \le t_1 \qquad (5)$$

$$\frac{dI_R(t)}{dt} = b\frac{\alpha}{p_2^{\beta}} - \theta I_R(t) \qquad t_1 \le t \le T \qquad (6)$$

With boundary conditions:

$$I_r(0) = 0, \quad I_r(t_2) = 0, \quad I_m(t_2) = 0, \quad I_m(T) = 0, \quad I_R(t_1) = 0 \qquad (7)$$

The following is a list of the solutions to the differential equations listed above:

$$I_r(t) = (a-1)\frac{\alpha}{p_1^{\beta}\theta}(1-e^{-\theta t}) \qquad 0 \le t \le t_1 \qquad (8)$$

$$I_r(t) = \frac{\alpha}{p_1^{\beta}\theta}(e^{\theta(t_2-t)} - 1) \qquad t_1 \le t \le t_2 \qquad (9)$$

$$I_m(t) = (a-1)\frac{\alpha}{p_1^{\beta}\theta}(1 - e^{\theta(t_2-t)}) \qquad t_2 \le t \le t_3 \qquad (10)$$

$$I_m(t) = \frac{\alpha}{p_1^{\beta}\theta}(e^{\theta(T-t)} - 1) \qquad t_3 \le t \le T \qquad (11)$$

$$I_R(t) = (b-a)\frac{\alpha}{p_2^{\beta}\theta}(1 - e^{\theta(t_1-t)}) \qquad 0 \le t \le t_1 \qquad (12)$$

$$I_R(t) = \frac{b\alpha}{p_2^{\beta}\theta}(1 - e^{\theta(t_1-t)}) \qquad t_1 \le t \le T \qquad (13)$$

Figure 3 below illustrates the retailer's inventory-time behaviour. In this scenario, the retailer obtains the stock at time t=y. where the lead time (y) is. Inventory levels decrease during [y,v] as a result to the degree of demand and the rate of decline. When time elapses, all available stock is gone t=T/n.

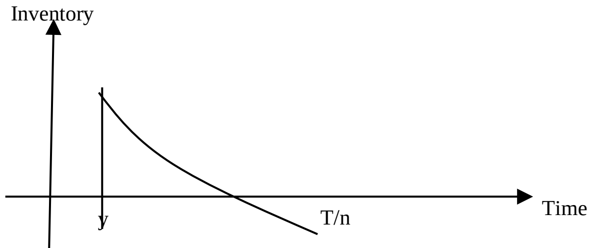

Fig. 3: Timely actions of retailers regarding

Now, differential equations for the retailer are stated as follows if the manufacturer needs y days to get the goods to the retailer:

$$\frac{dI_s(t)}{dt} = -\frac{\alpha}{p_2^{\beta}} - \theta I_s(t) \qquad y \le t \le T/n \qquad (14)$$

With boundary condition $I_s(T/n) = 0$ (15)

This set of equations can be solved by:

$$I_s(t) = \frac{\alpha}{p_2^{\beta}\theta}(e^{\theta(v-t)} - 1) \qquad y \le t \le T/n \qquad (16)$$

T.A.C_m =1/t [Obtaining cost + Gaining cost + Manufacturing cost + Remanufacturing cost + Property cost + Set up cost + Recover cost]
gives the manufacturer's total cost. (17)

Retailer's total cost is calculated as:

T.C_s = $\frac{n}{T}$ [Purchasing cost + Holding cost + Ordering cost + Deterioration cost] (18)

These are components of manufacturing costs within the designated inventory cycle:

Cost of acquisition and procurement = $c_m\int_{t_2}^{t_3}\frac{a\alpha}{p_1^{\beta}}dt + c_R\int_0^T R dt$ (19)

Production and remanufacturing cost = $S_m\int_{t_2}^{t_3}\frac{a\alpha}{p_1^{\beta}}dt + S_R\int_0^{t_1}\frac{a\alpha}{p_1^{\beta}}dt$ (20)

Holding cost $= h_r[\int_0^{t_1} I_r(t)dt + \int_{t_1}^{t_2} I_r(t)dt] + h_m[\int_{t_2}^{t_3} I_m(t)dt + \int_{t_3}^{T} I_m(t)dt] + h_R[\int_0^{t_1} I_R(t)dt + \int_{t_1}^{T} I_R(t)dt]$ (21)

Set up cost = $K_1 + K_2 + K_3$

(22)

Salvage cost = $S_R\{(1-\gamma)b\frac{\alpha}{p_2^{\beta}}\}T$ (23)

These are the cost elements that the store will incur during the specified inventory cycle:

Purchase price = $(I_s(y)+Q_2)c_1$ (24)

Where

$$Q_2 = \int_v^{T/n}\frac{\alpha}{p_2^{\beta}}\eta dt \quad (25)$$

Holding cost= $h_s\int_y^v I_s(t)dt$ (26)

Ordering cost= O (27)

Deterioration cost= $c_4\{I_s(y) - \int_y^{T/n}\frac{\alpha}{p_2^{\beta}}dt\}$ (28)

As a result, the above inventory model's total cost per unit time is given by the product of a, t1, t2, t3, y, and T. For all given values of t1, t2, t3, and y, and any constant T, we have

$$\text{T.C.}= \frac{1}{T}[\frac{a\alpha}{p_1^{\beta}}(t_3-t_2)c_m + \frac{b\alpha}{p_1^{\beta}}Tc_R + s_m\frac{a\alpha}{p_1^{\beta}}(t_3-t_2) + s_r\frac{a\alpha}{p_1^{\beta}}t_1 + h_r\{(a-1)\frac{\alpha}{p_1^{\beta}\theta}(t_1+\frac{e^{-\theta t_1}-1}{\theta})$$

$$+\frac{\alpha}{p_1^{\beta}\theta}(\frac{e^{\theta(t_2-t_1)}-1}{\theta}+t_1-t_2)\} + h_m\{(a-1)\frac{\alpha}{p_1^{\beta}\theta}((t_3-t_2)+\frac{e^{\theta(t_2-t_3)}-1}{\theta})$$

$$+\frac{\alpha}{p_1^{\beta}\theta}(\frac{e^{\theta(T-t_3)}-1}{\theta}-(T-t_3))\} + h_R\{\frac{\alpha(b-a)}{p_1^{\beta}\theta}(t_1+\frac{1-e^{\theta t_1}}{\theta}) + \frac{\alpha b}{p_1^{\beta}\theta}((T-t_1)+\frac{1}{\theta}(e^{\theta(t_1-T)}-1)\}$$

$$+K_1+K_2+K_3 - s_{av}\{(1-\gamma)b\frac{\alpha}{p_2^{\beta}}\}T] + \frac{n}{T}\{c_1(\frac{\alpha}{p_2^{\beta}\theta}(e^{\theta(v-y)}-1)+\frac{\alpha}{p_2^{\beta}}\eta(\frac{T}{n}-v))$$

$$+h_s\frac{\alpha}{p_2^{\beta}\theta}(\frac{(e^{\theta(v-y)}-1)}{\theta}+(y-v)) + O \qquad + c_4(\frac{\alpha}{p_2^{\beta}\theta}(e^{\theta(v-y)}-1) - \frac{\alpha}{p_2^{\beta}}(\frac{T}{n}-y))\} \quad (29)$$

The cost function of the system is shown in Equation (29) as a function of t1, t2, y, and T. For this system to give us its optimum response, we need to find the optimal values of t1, t2, t3, y, and T. These variables and our interactions between them.

$$0 \le t_1 \le t_2 \le t_3 \le T \tag{30}$$

$$(a-1)(1-e^{-\theta t_1}) = (e^{\theta(t_2-t_1)}-1) \tag{31}$$

$$(a-1)(1-e^{\theta(t_2-t_3)}) = (e^{\theta(T-t_3)}-1) \tag{32}$$

$$(b-a)(1-e^{\theta t_1}) = b(1-e^{\theta(t_1-T)}) \tag{33}$$

Equations (30) demonstrates the necessary condition for this model's existence. The inventory levels Ir(t) and Im(t) at t=t1 and t=t3 are shown in equations (31) and (32). Equation (33) shows that at both t=0 and t=T, the inventory level of things that have been gathered and returned will be the same.

The values of t1, t2, and t3 may be found in the form of T using the equations (31) and (33); hence, it can be argued that $t_1 = f_1(T)$, $t_2 = f_2(T)$, $t_3 = f_3(T)$
Thus, the relationship between T, y, and v will be the total average cost function.

Numerical analysis:
Here we use the following input data to quantitatively illustrate the aforementioned inventory model:
$\alpha = 2500\,units,\quad p_1 = 30\,Rs,\quad p_2 = 40\,Rs,\quad a = 1.5,\quad \gamma = 0.6,\quad \delta = 0.4,\quad b = 0.8,\quad n = 4,$
$\beta = 2.5,\quad h_r = 0.5\,Rs/unit,\quad h_m = 0.5\,Rs/unit,\quad h_R = 0.3Rs/unit,\quad \theta = 0.08,\quad c_m = 10\,Rs/unit,$
$c_R = 8\,Rs/unit,\quad s_{av} = 8.5\,Rs/unit,\quad K_1 = 1000Rs,\quad K_2 = 1200Rs,\quad K_3 = 1500Rs,\quad c_1 = 30Rs/unit,$
$c_4 = 30Rs/unit,\quad h_3 = 0.4\,Rs/unit,\quad O = 500Rs/order$
The ideal solution has been discovered for the parametric values listed above. The best values are t_1=12.2003, t_2=18.30045, t_3=21.35, T=22.8756,

Sensitivity analysis:
Limit variation and its effect on the optimal solution are studied by providing the model's output for a variety of limit settings.-20%, -15%, -10%, -5%, 5%, 10%, 15% and 20%.

Table 1: Analysis of the sensitivity to the production limit (α_1):

% variation in α	α	T	y	T.A.C.
-20%	2000	22.617	3.39401	520.069
-15%	2125	22.6843	3.39401	526.145
-10%	2250	22.7499	3.39401	532.223
-5%	2375	22.8136	3.39401	538.304
0%	2500	22.8756	3.39401	544.389
5%	2625	22.936	3.39401	550.476
10%	2750	22.9948	3.39401	556.567
15%	2875	23.0521	3.39401	562.66
20%	3000	23.1079	3.39401	568.756

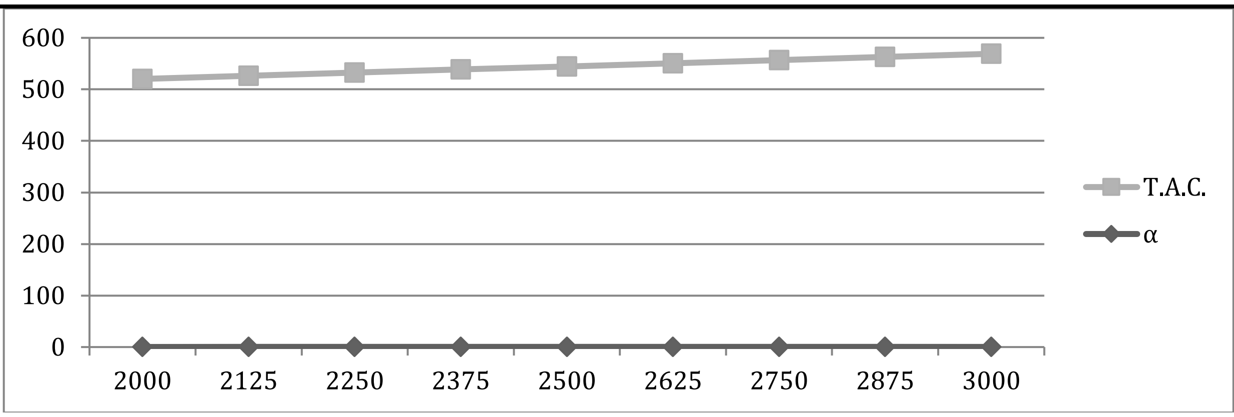

Fig 5: T.A.C. v/s α

Table 2: Analysis of the sensitivity to the production limit (p_1):

% variation in p_1	p_1	T	y	T.A.C.
-20%	24	23.6357	3.39401	635.606
-15%	25.5	23.4159	3.39401	605.541
-10%	27	23.2168	3.39401	581.121
-5%	28.5	23.0372	3.39401	561.053
0%	30	22.8756	3.39401	544.389
5%	31.5	22.7306	3.39401	530.42
10%	33	22.6005	3.39401	518.61
15%	34.5	22.4838	3.39401	508.548
20%	36	22.3791	3.39401	499.915

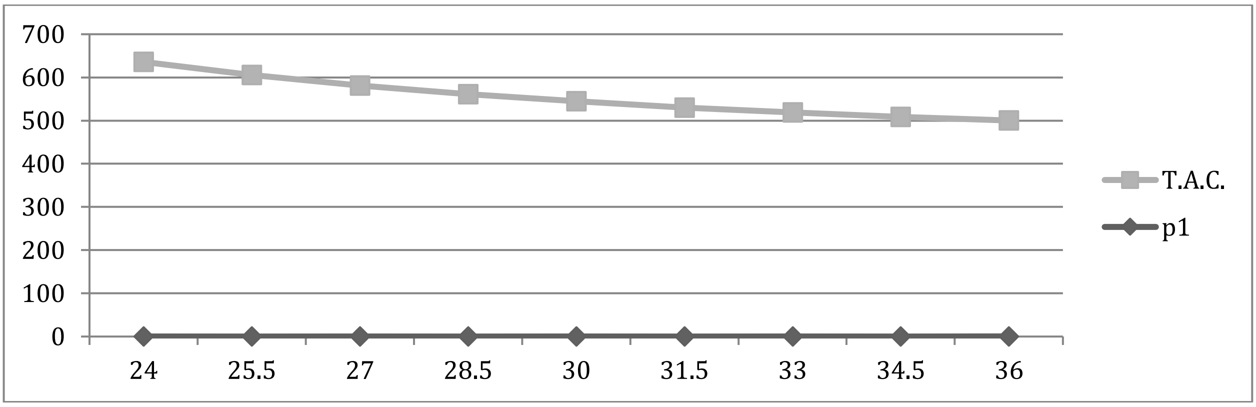

Fig. 6: T.A.C. v/s p_1

Table 3: Analysis of the sensitivity to the production limit (a):

% variation in a	a	T	y	T.A.C.
-20%	1.2	19.3571	3.39401	620.797
-15%	1.275	20.2956	3.39401	597.773
-10%	1.35	21.1953	3.39401	577.646
-5%	1.425	22.0556	3.39401	559.97
0%	1.5	22.8756	3.39401	544.389
5%	1.575	23.6548	3.39401	530.613
10%	1.65	24.3928	3.39401	518.404
15%	1.725	25.0892	3.39401	507.567
20%	1.8	25.7441	3.39401	497.936

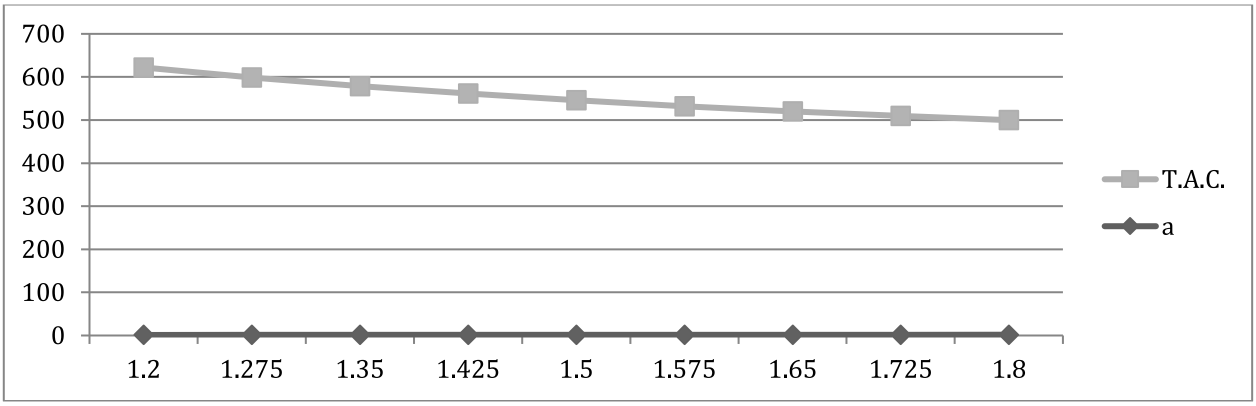

Fig. 7: T.A.C. v/s a

Table 4: examination of the sensitivity to a limit (γ):

% variation in γ	γ	T	y	T.A.C.
-20%	0.48	22.8756	3.39401	546.458
-15%	0.51	22.8756	3.39401	545.947
-10%	0.54	22.8756	3.39401	545.424
-5%	0.57	22.8756	3.39401	544.906
0%	0.6	22.8756	3.39401	544.389
5%	0.63	22.8756	3.39401	543.872
10%	0.66	22.8756	3.39401	543.354
15%	0.69	22.8756	3.39401	542.837
20%	0.72	22.8756	3.39401	542.32

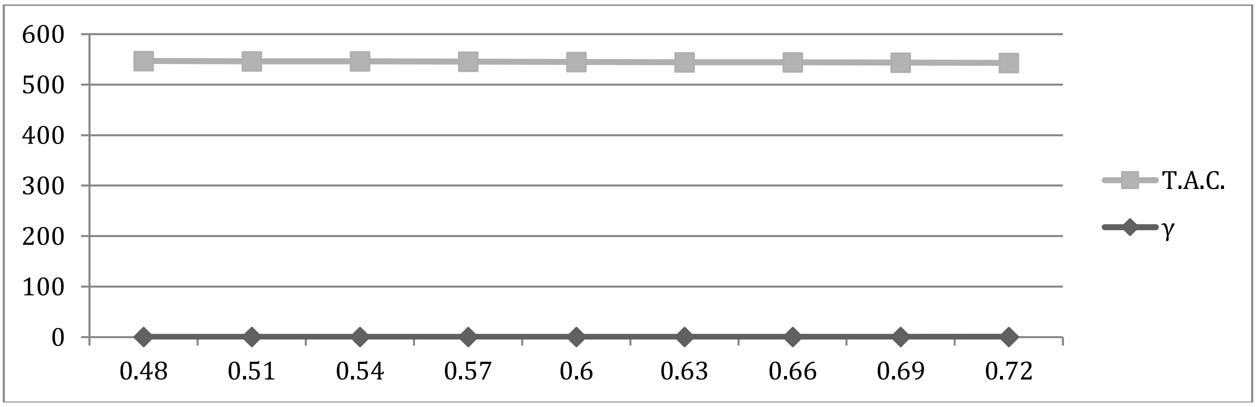

Fig. 8: T.A.C. v/s γ

Table 5: examination of the sensitivity to a limit (b):

% variation in b	b	T	y	T.A.C.
-20%	0.64	23.4885	3.39401	533.163
-15%	0.68	23.3302	3.39401	536.007
-10%	0.72	23.1753	3.39401	538.825
-5%	0.76	23.0239	3.39401	541.619
0%	0.8	22.8756	3.39401	544.389
5%	0.84	22.7305	3.39401	547.135
10%	0.88	22.5885	3.39401	549.857
15%	0.92	22.4493	3.39401	552.557
20%	0.96	22.313	3.39401	555.234

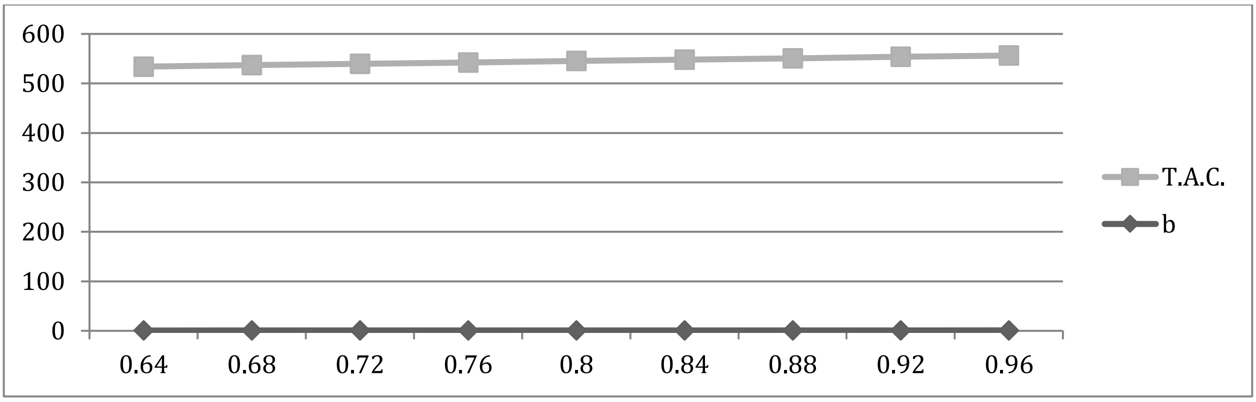

Fig. 9: T.A.C. v/s b

Table 6: Analysis of the sensitivity to the demand limit (β):

% variation in β	β	T	y	T.A.C.
-20%	2	7.73101	3.39401	977.848
-15%	2.125	9.62876	3.39401	945.924
-10%	2.25	12.4651	3.39401	874.028
-5%	2.375	16.6838	3.39401	713.568
0%	2.5	22.8567	3.39401	544.389
5%	2.625	31.7754	3.39401	400.461
10%	2.75	44.2321	3.39401	290.034
15%	2.875	61.1727	3.39401	209.75
20%	3	83.5909	3.39401	152.799

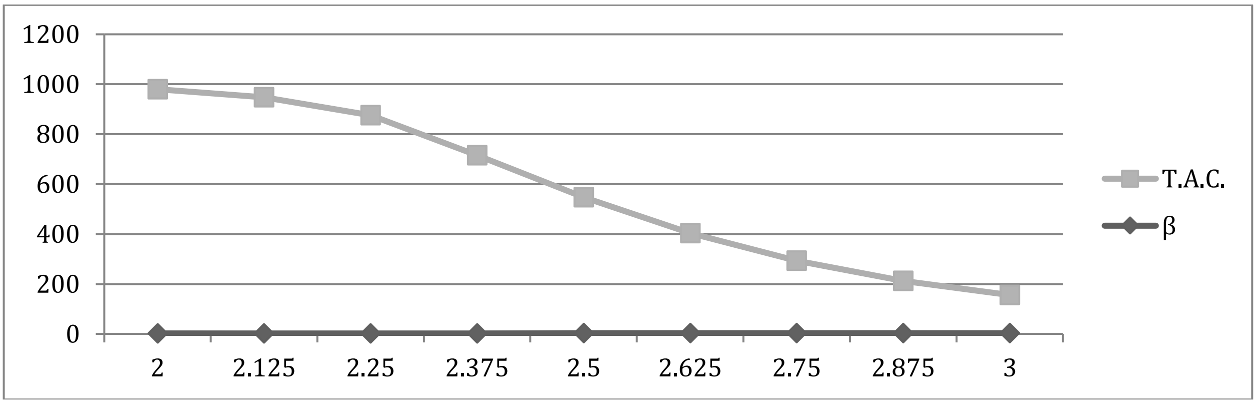

Fig. 10: T.A.C. v/s β

Table 7: Analysis of the degradation limit's sensitivity (θ):

% variation in θ	θ	T	y	T.A.C.
-20%	0.008	22.473	3.61478	581.185
-15%	0.0085	22.5325	3.55569	570.084
-10%	0.009	22.6257	3.49931	560.433
-5%	0.0095	22.7425	3.44547	551.94
0%	0.01	22.8756	3.39401	544.389
5%	0.0105	23.0197	3.34478	537.613
10%	0.011	23.1707	3.29765	531.486
15%	0.0115	23.3257	3.25248	525.909

20%	0.012	23.4824	3.20916	520.802

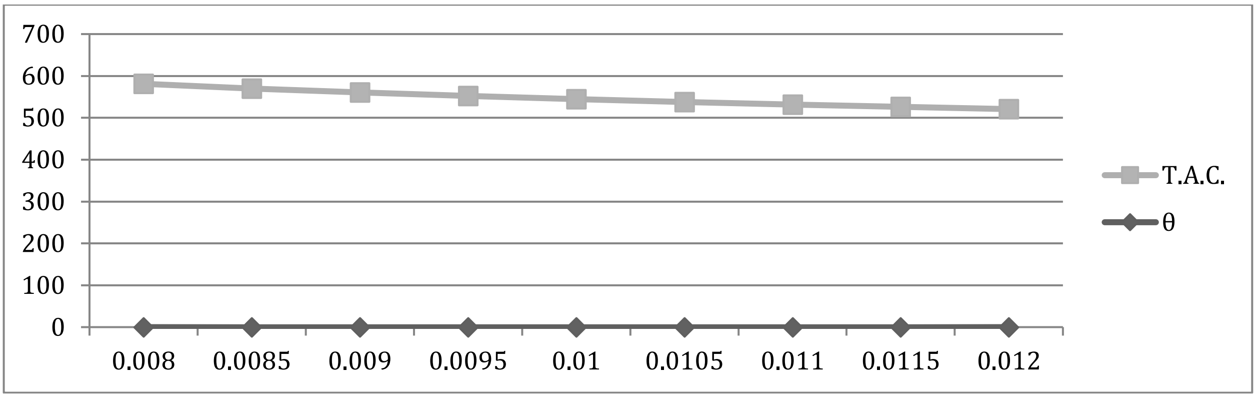

Fig. 11: T.A.C. v/s θ

Table 8: A sensitivity study of the holding cost (h_s):

% variation in h_s	h_s	T	v	y	T.A.C.
-20%	0.32	22.8422	2.68362	3.16517	561.95
-15%	0.34	22.8505	2.76115	3.22772	557.148
-10%	0.36	22.8589	2.834	3.28649	552.638
-5%	0.38	22.8673	2.90259	3.34183	548.392
0%	0.4	22.8756	2.96727	3.39401	544.389
5%	0.42	22.884	3.02837	3.44331	540.609
10%	0.44	22.8924	3.08619	3.48996	537.032
15%	0.46	22.9008	3.14097	3.53415	533.645
20%	0.48	22.9093	3.19295	3.5761	530.431

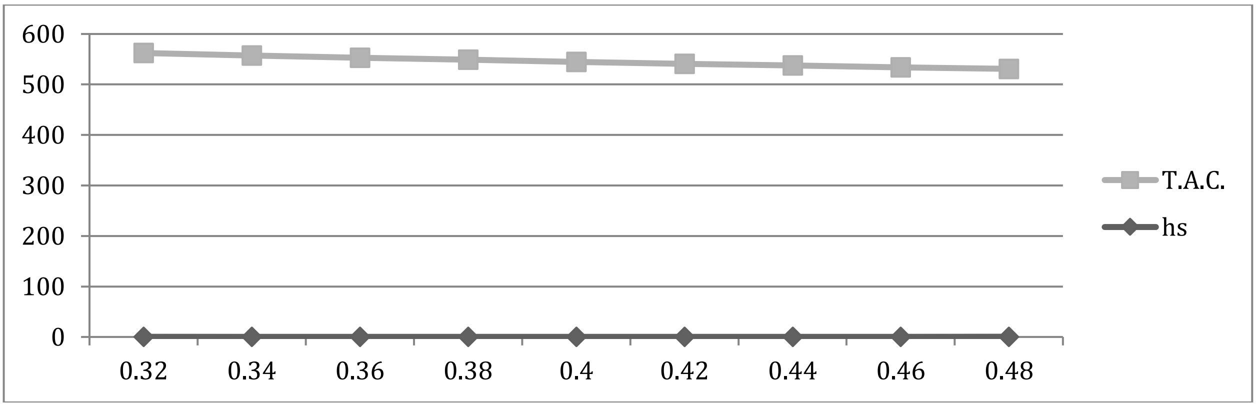

Fig. 12: T.A.C. v/s h_s

Concluding remarks:

1. Tables 1 and 2 show that the changes in the demand limit 1 and the cycle length T are favourably and negatively responsive to changes in total average cost and p1, respectively.

2. The tables 3 and 4 make it abundantly evident that, with an When the production limit (a) rises, the system's T.A.C. and cycle time (T) rise along with it. exhibits the opposite behaviour, while an increase in limit causes the cycle time

to remain constant and the system's T.A.C. to slightly decline.

3. Table 5 displays the impact of changes in returning limit b. As limit b is increased, the cycle time T somewhat reduces and the system's T.A.C. rises.

4. We can see from tables 6 and 7 that when the demand limit and degradation rate rise, the cycle time T lengthens and the total available capacity (TAC) decreases.

5. Table 8 demonstrates that T and T.A.C. cycle times have increased somewhat constantly reduces with the increase in holding cost hs.

Conclusion:

In this study, we developed a comprehensive inventory model for repairability and remanufacturing. The production pace and the rate at which money is collected are both thought to be a function of the true demand. The shop thinks about shipping times, too. A numerical example is provided to illustrate the theoretical findings. To further confirm the model's accuracy, we conduct a sensitivity analysis about a variety of system factors. The model can expand to account for inflation and the effects of education.

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