
# Pannal Data

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ABSTRACT

The study of any phenomenon, such as an economic phenomenon, a social phenomenon, a medical phenomenon, or any other phenomenon by researchers, requires the availability of DataThis is done by collecting data from its original sources for the purpose of Data analysisUsing one of the various statistical methods and obtaining conclusions that facilitate the role of the decision maker in reaching a conclusion Best decisionabout the phenomenon studied.
The data collection mechanism differs from one phenomenon to another, which highlights different types of data. Studying a particular phenomenon for a period of time requires collecting data. Time series dataThe study of a specific phenomenon for several groups or sectors that differ from one another requires a collection cross section data. Thus, the emergence of these differences in the quality of the data leads to the emergence of differences in the mathematical model. The one who represents it, if any, would be a model linear regression, or be a model Experimental design Accordingly, differences appear in the appropriate statistical analysis method in terms of the process Estimation For the parameters of the model and the direction in which the estimation process takes place, it would be classical or Bayesian As well as the process of testing hypotheses about the parameters of the model or hypotheses about the model used according to the two directions above.
Many researches have dealt with types of such data by estimating and testing the parameters of the mathematical model represented by them. Panel data which appeared for the first time in 1960 in surveys conducted as a study[29][14] National longitudinal surveys of labor market experience(NLSSince that time, researchers have addressed its subject by analyzing the mathematical models that represent it, which are special regression models panel dataIn terms of the process of estimating the parameters of the models and testing the hypotheses of the parameters, and the models used in the research, and most of the research dealt with the traditional method in the process of estimation and testing, and focused on two methods, namely the general least squares method (GLS) when the covariance matrix and covariances known The generally accepted least squares method (FGLS), when the covariance matrix and covariances unknown As for the research that agreed with the Bayesian method, it is very few and at the country level. The research that dealt with the subject of the research is very rare and was limited to the general least squares method.(GLS)The generally accepted least squares method(FGLS)It also did not address the BES method in the assessment and testing process.

**Keywords:** Pannal, Data, Mathematics, BES method

## Search objectives

Use method Maximum likelihood(ML) as a traditional estimation method and estimation Bayesian based on the elementary distribution on informative prior and the primary distribution is normal Natural conjugate priorwithweighted square loss function To estimate some special regression models panel data As well as conducting tests related to regression models and comparing the estimation methods used in the estimation process Based on data on investment at the level of economic sectors in the Republic of Iraq for the period of time2006- 2021, according to the data available for that period.

## Chapter II

## The theoretical side

### 2-1 An introduction

In this chapter, the researchers' views of the concept of (panel data) With which point of view does the research agree, as it reviews some types of models (panel data) in some detail through the other names of the models, using the methodMaximum liklihoodTo estimate the parameters of the models, and then test them in the case ofclassical analysis, with a proposal for a mechanism for estimating parametersRandom effect modelWhen the covariance and covariance matrix is unknown (unknown. . will also be usedBayesian analysisTo estimate the parameters of the same models by adopting functions of prime distributions (prior pdf) is the non-informational primary distribution, and the conjugated normal distribution withweight loss square error function.

## 2-2 Panel data Model

Since paired data consists of cross-sectional data and time series data, these data can have group effectsGroup effectsor own Time effectsor have both, and these effects may be of the type Fixed effector type Random effectAs a result of these types of effects, the analysis process has to follow the type of model representative of the data, namely:

**- Fixed effect model**. 1
**2-Random effect model**.

The fixed effect model assumes that the fixed limits differ(intercepts)Through the aggregates (cross-sections)(Groups)or through units(Time)While the random effect model is assumed(Random effect model)difference(error variances)In light of the foregoing, we may have One way modelContains one set of Dummy variablesFor example using dummy variables in groups (cross-sections) only or we may have Two way modelIt contains two sets of dummy variables, an example of which is its use in cross sections and time at the same time. It should be noted that the one-way model is the most common and applied model among researchers, and therefore the research focuses on a one-way model in the analysis process.

## 2-3 Classical Analysis

The basic idea of the traditional analysis for estimating any parameter is based on the assumption that the value of the parameter to be estimated is proven. This type of estimation may beUniqueAs estimations of the ordinary least squares method(OLS)weighted least squares estimators(WLS)and others, it may be not uniqueAs in the way Maximum likelihood (ML)In this study, this analysis will be adoptedOne-way panel data modelsAs follows:

### 2-3-1 Fixed effect model

As we have seen before, the fixed-effect model assumes that the fixed limits vary duringCross-sectionsover time units, so it appears Dummy variablesWith the fixed limits of the fixed effect model Through the following, suppose the regression model:

$Y_{it} = (\alpha + \mu_i) + \beta' X_{it} + u_{it}$ ... (1)
t=1,2, ..., T

which can be rewritten as follows:-

$Y_{it} = \alpha_i + \beta' \ X_{it} + u_{it}$ (2) ...
t=1,2, ..., T

Since:
Yit:Represents the view(t)From the observations of the dependent variable of the cross section(i).
$\alpha_i$: represents the constant limit parameter of the cross-section regression model(i).
$\beta'$: class prompt(1*k)Featuring regression parametersfor model. slope cross section$(\beta_1, \beta_2, \dots, \beta_k)$(i).
Xit:ordered matrix(k*1)Include views(t)From the observations of the explanatory variables of the cross-section regression model(i).
uit:represents the random error limit(t) (error term)From the Random Error Limits to the Cross Section Regression(i).
and that
$u_{it} \sim i.i.d\ N(0, \sigma_u^2) \quad \forall t = 1,2,\dots,T$

Using matrices, the model (2) can be developed as follows[13]:-

(3) ...$Y_i = J\,\alpha_i + X_i\,\beta + u_i$
Since:

Yi: ordered vertical wave(T*1)From the observations of the dependent variable of the cross section(i).
Xi: an ordered array(T*k)From the observations of the explanatory variables of the cross section(i).
: ordered vertical wave(k*1)For the gradient $\beta$ parameters of the cross section(i).
J: a single vertical wave of a mattress(T*1).

: Parameter of the constant limit of the cross-$\alpha_i$ section regression model(i).
ui: a vertical wave of a mattress(T*1)for random errors of the cross section(i).

Model (3) can be generalized for(n)of cross sections(Cross-sections)as follows [1][13]:-

$$\begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \\ \vdots \\ Y_n \end{bmatrix} = \begin{bmatrix} J & 0 & 0 & \cdots & 0 \\ 0 & J & 0 & \cdots & 0 \\ 0 & 0 & J & & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & J \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \vdots \\ \alpha_n \end{bmatrix} + \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ \vdots \\ X_n \end{bmatrix} \beta + \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ \cdots \\ u_n \end{bmatrix} \quad \ldots \quad (4)$$

and the model(4)he is called Fixed effect ModelAnd the number of viewsin it becomes(N=nT).

And according to the strategies for estimating the parameters of the fixed effect model(4)Other names for the model have appeared[1]:-

**1-Dummy variables model**
**2-Analysis of covariance model.**
While the researcher mentioned[21] (Park)Other designations are:

**1-Least squares dummy variable model (LSDV)**

The formula of this model is completely similar to Model (4) as it uses dummy variables in it, but a problem arises with it when the number of cross-sections is very large and the time is fixed(Time=fixed, n→∞)So, the estimations of the regression parameters of the model areConsistenWhereas the estimators for the parameters of the dummy variables are the parameters of the fixed limits not consistentAs their number increases with the increase in cross-sections, the researcher has launched[7] (Baltagi)The problem is called a problem Incidental parametersThe passerby mentioned in the study (2-2)In this case, it becomes a method(LSDV)useless.

**2- Within effect model**

When the problem of the parameters of the cross sections appears, this model can be used, as dummy variables are not used in it, but the deviations of the observations from the arithmetic means of the cross sections are used. Therefore, the formula of the model is accessed by writing the model (2) in terms of the deviations of the observations.(Yit)Its arithmetic mean is as follows:-

$$\left(Y_{it} - \bar{Y}_{i}\right) = \beta '\left(X_{it} - \bar{X}_{.}\right) + \left(u_{it} - \bar{u}_{i.}\right) \quad \ldots \quad (5)$$

t=1,2,..,T

By circulating form (5) fornFrom the cross sections, as we have seen before, we get the effect model within the cross section within effect modelThis model has downsides Because it does not use dummy variables, including having a large degree of freedom for random error, which leads to the value of the mean squares error(Mse).

**3-Between effect model**

It is a model that does not use dummy variables, but rather uses the arithmetic mean of the cross sections as observations of the dependent variable, so it is calledCross-section mean regression modelTherefore, the number of observations (sample size) is equal to(n)It should be noted that the dummy variables model is the least squares model for the dummy variable itself.

The process of estimating the parameters of a fixed-effect model depends on the number of cross-sections(n)In terms of whether they are large or small[14] [13] [1].

## 2-4 Bayesian Analysis

The basic idea of traditional analysis in terms of assuming that the value of the parameter to be estimated is held does not apply in Bayesian analysis because it assumes that the value of the parameter is not proven, that is, it is a random valueIt may have a preliminary distribution as it assumes the availability of prior information or Initial InformationThe parameter to be estimated can be set as an initial distribution Prior distributionand combine it withLikelihood Functionwhich expresses the new sample data we get Posterior Probability Functionwhich carries all the available information about the parameter to be estimated. Joint Posterior Probability function: This can be explained as follows:-

(6)..... $P\ (\underline{\Theta}\, l\, Y) \propto P\ (\underline{\Theta})\ .P(Y\ l\ \Theta)$

In order to obtain the capabilities by adopting the Bayes theorem, it is necessary to provide Loss Function[1]And it is the determinations of Bayes that reduce the expectation of loss, that is:-

(7) ...

$$Min_{\hat{\theta}}\ E\left(L(\hat{\theta},\theta)\right) = Min_{\hat{\theta}} \int L(\hat{\theta},\theta)\, P(\theta\, l\, Y)\, d\theta$$

On the basis of the type of loss function, estimators of the parameters can be found. The most common types of loss functions when we have a vector of parameters areWeighted Squared error LossFunctionIt is known that the basing estimators of the parameter vector according to the loss function above represent meanfor the common post-probability function[30]Therefore, in this topic, the parameters of the fixed effect models will be estimated And the random effect with the presence of the weighted error squared loss function is as follows:-

### 2- 4- 1 Fixed effect Model:-

In order to estimate the fixed effect model Bayes MethodWe rewrite the form as follows:- $Y = Z\, h + u$

And the Likelihood FunctionFor model variables:

[1]For more information, see Resource (1).

$$L(Y|h,g) = \pi^{-0.5N}(0.5)^{0.5N} g^{-0.5N} \exp[-0.5g(Y-Zh)'(Y-Zh)]$$

It can be written as follows:-

(8).... $L(Y|h,g) \propto g^{0.5N} \exp[-0.5g(Y-Zh)'(Y-Zh)]$

$$L(Y|h,g) \propto \exp[-0.5g(Y-Zh)'(Y-Zh)]$$

$$\propto exp\left[-0.5g\left(\nu S^2 + (h-\hat{h})'Z'Z(h-\hat{h})\right)\right]$$

(9).... $\propto \exp\left[-0.5g(h-\hat{h})'Z'Z(h-\hat{h})\right]$

Since:-

$\propto$ means that the quantity is proportional.

$\hat{h}$ Known in advance.

and that :-

$$\nu s^2 = (Y - Z\hat{h})'(Y - Z\hat{h}) \quad \nu = N - n - k$$

As for(Prior dist.)It depends on the availability of prior information about the parameters to be estimated. In this research, we assume the availability of a non-informational primary distribution non imformative PriorThe primary distribution is normal (Natural Conjugate Prior)As a result of the foregoing in paragraph (2-3-1) taking into account the number ofNO.of Cross SectionsWhether it is small or large, the assumed initial distribution will be of two types accordingly, as follows[12].

**First Type Non hierarchical Prior**

This type is used when the number of cross-sections is n: small, i.e. deals with parameters $(\alpha, \beta)$ As an initial distribution at once, so the initial distribution is not informative for the parameters, which is determined according to my rule[1] (Jeffery)When the parameter $\sigma$ Knownbe as follows:-

(10)....Constant

$$P(\alpha, \beta|\sigma) = P(h|\sigma) \propto$$

[1]For an accurate understanding of the two rules, see the source[30]

By integrating the possible function defined by the formula(9)With the non-informational distribution function defined by the formula(10)We get the subsequent probability function of the parameter vector(h)as follows:-

(11).... $p(h|Y,\sigma)\propto \exp\left[-0.5g(h-\hat{h})'Z'Z(h-\hat{h})\right]$

$-\infty\langle h\langle\infty \qquad ,0\langle\sigma\langle\infty$

By integrating the function(11)For the teacher $\sigma$ Equating it to one, we get the following[ 30]:-

$$p(h|Y,\sigma)=\pi^{-0.5(n+k)}(0.5)^{0.5(n+k)}g^{0.5(n+k)}\exp\left[-0.5g(h-\hat{h})'Z'Z(h-\hat{h})\right] \qquad ...(12)$$

and the formula(12)representMultivariate – Normal distIt represents the boundary post-probability function of the parameter vector(h) mean $(\hat{h})$ Represents a piez estimator for a parameter vector $h$ and matrix(ver)And the((cov $g^{-1}(Z'Z)^{-1}$ , and it is noted that the Bayes estimator is equivalent to the estimators ofMaximum liklihood.

The initial non-informational distribution according to the first type when the parameter $\sigma$ unknownbe as follows:-

$$P(\alpha,\beta)=P(h)\propto \quad \text{constant}$$

$$P(\sigma)\propto \quad g^{0.5}$$

(13).... $P(\alpha,\beta,\sigma)=P(h,\sigma)\propto g^{0.5}$

And by integrating the possible function(8)With a prime distribution function defined by the formula(62)We get the common post-probability function as follows:-

$P(h,\sigma|Y)\propto g^{0.5(N+1)}\exp\left[-0.5g\ (Y-Zh)'(Y-Zh)\right]$

(14).... $\propto g^{0.5(N+1)}\exp\left[-0.5g(\nu S^2+(h-\hat{h})'Z'Z(h-\hat{h})\right]$

By integrating the function(14)For the teacher $\sigma$ We get the subsequent probability function of the parameter vector(h)As follows:-

$$P(h|Y)=\int_0^\infty P(h,\sigma|Y)\,d\sigma$$

(15).... $P(h|Y)\propto\left[\nu S^2+(h-\hat{h})'Z'Z(h-\hat{h})\right]^{-0.5N}$

and the function(15th)represent distributionmean (Multivariate -t) $(\hat{h})$ RepresentBayes estimatorWith a weighted error squared loss function and a matrix(ver) ,((covIt is known as:-

(16).... $V-\hat{C}OV(\hat{h})=\nu(\nu-2)^{-1}S^2(Z'Z)^{-1}$

As for the probability density function subsequent to the parameter $(\sigma)$ It can be obtained by integrating the function(14)For the parameter vector(h)To be as follows:-

(17).... $P(\sigma|Y)\propto g^{0.5\nu}\exp\left(-0.5(\nu g S^2)\right)$

As the function(17)represent distribution inverted gammaHe who has an eccentric torque is defined as:[30]:-

$$\mu_r' = \Gamma\left[(\nu - r)0.5\right]\left[\Gamma(0.5\nu)\right]^{-1}(0.5\nu s^2)^{0.5r} \qquad \nu > r$$

and that:-

(18).... $\mu_2' = \nu(\nu-2)^{-1}S^2 = E\left[\sigma^2 \mid Y\right] \qquad \nu > 2$

The initial distribution of the parameters is normalhwhich is determined according to the distributions of the possible function when $\sigma$ knownIt has a normal distribution as follows[30]:-

(19).... $P(h) \propto \exp\left[-0.5g(h-\tilde{h})'\sum(h-\tilde{h})\right]$

$-\infty < hi < \infty \qquad i = 1,2\ldots(n+k)$

Since:-

$\tilde{h}$ :-meanfor distribution.

$(g\sum)^{-1}$ :-MatrixverAnd thecovfor teachersh.

By integrating the possible function(9)With the initial distribution function normal(19)We get the subsequent probability function of the parameter vectorhAs follows:-

$$P(h|Y,\sigma) \propto \exp\left[-0.5g(h-\hat{h})'Z'Z(h-\hat{h})\right].\exp\left[-0.5g(h-\tilde{h})'\Sigma(h-\tilde{h})\right]$$

(20) ... $p(h|Y,\sigma) \propto \exp\left[-0.5g\left((h-\bar{h})'(Z'Z+\Sigma)(h-\bar{h}\right)\right]$

Since:-

(21).... $\bar{h} = (Z'Z + \Sigma)^{-1}(Z'Z\hat{h} + \Sigma\tilde{h})$

and the function(20)It represents a normal multivariate distribution with an arithmetic mean $(\bar{h})$ Represents a piez estimator for a parameter vectorhWhen $(\sigma)$ Information about the existence of a weighted error squared loss function.

In the case of $(\sigma)$ It is not known, so the initial distribution normal is a function (Normal Gamma p.dt)because ofMaximum liklihood (8) It can be written in the form of a normal distributive product - sham, where it can be written as follows [30] :-

(22).... $P(h|\sigma) \propto g^{0.5(n+k)} \exp\left[-0.5g(h-\tilde{h})'\Sigma(h-\tilde{h})\right]$

$-\infty < h_i < \infty \quad i = 1,2\ldots(n+k)$

and that :-

(23).... $a \rangle 0 \quad p(\sigma) \propto g^{0.5(a+1)} \exp\left[-0.5gaS^2\right]$

And by merging the two functions(22) (23)We get the normal contiguous primary distribution function as follows:-

(24).... $p(h,\sigma) \propto g^{0.5(n+k+a+1)} \exp\left(-0.5g(aS^2 + (h-\tilde{h})'\Sigma(h-\tilde{h})\right)$

And by merging the function(24)With the possible function(8)We get the common posterior probability function for the parameters $(h,\sigma)$ As follows:-

$$P(h,\sigma|Y)\propto g^{0.5(N+n+k+a+1)}\,exp\left[-0.5g\left(aS^2+(h-\tilde{h})'\Sigma(h-\tilde{h})\right)+(Y-Zh)'(Y-Zh)\right] \quad ...(25)$$

By taking some steps to simplify the function (25), we get the following:-

$$P(h,\sigma|Y)\propto g^{0.5(N+n+k+a+1)}\exp\left[0.5g(aS^2+(L-Bh)'(L-Bh))\right]$$
$$\propto g^{0.5f}\,exp\left[-0.5g\left(as^2+(L-B\bar{\bar{h}})'(L-B\bar{\bar{h}})+(h-\bar{\bar{h}})'B'B(h-\bar{\bar{h}})\right)\right] \quad ...(26)$$

Since:-

$$L=\begin{bmatrix} Y \\ \Sigma^{o.5} \end{bmatrix} \quad ,B=\begin{bmatrix} Z \\ \Sigma^{0.5} \end{bmatrix} \quad ,\ f=N+n+k+a+1$$

and that :

(27).... $\bar{\bar{h}}=(B'B)^{-1}B'L$

By integrating the function(26)For the teacher$(\sigma)$We get the subsequent probability function of the parameter vector(h)As follows:-

(28).... $P(h|Y)\propto\left[fS^2+(h-\bar{\bar{h}})'B'B(h-\bar{\bar{h}})\right]^{-0.5(N+n+k+a)}$

and the function(28)represents a multivariate functiont-in the middle of my account $(\bar{\bar{h}})$ Represents a piez estimator for a parameter vectorhWhen the parameter$(\sigma)$ It is not known whether the weighted error squared loss function exists[16][30][1].

**Second type Hierarchical Prior**

This type is used when the number Cross-sectionvery big$(n\to\infty)$ Here, a separation is made between the fixed limits of the cross sections and the regression parameters for all the cross sections.(Jeffery)When the parameter$(\sigma)$ knownas follows[30] [1]:-

$P(\alpha|\sigma)\propto$ Canstant

$P(\beta|\sigma)\propto$ Canstant

(29).... $P(\alpha,\beta|\sigma)=P(\alpha|\sigma).P(\beta|\sigma)\propto \text{Canstant}$

It is noted that the hierarchical non-informative initial distribution defined by the formula(29)It is the same as the identifier in the formula(10)So, the mechanism for extracting the common post-probability function for parameters$(\alpha,\beta)$ It is the same and can then be written in the form(61)Which represents Multi variate –Normal distAnd in the middle of my account$(\hat{h})$ Represents a piez estimator for a parameter vector(h)which contains parameters $(\alpha, \beta)$ covariance and covariance matrix $\left(g^{-1}(Z'Z)^{-1}\right)$ [30]That is:-

(30).... $h \sim MVN\left(\hat{h}, g^{-1}(Z'Z)^{-1}\right)$

Notes from the formula(30)that destined biz $(\hat{h})$ It is equivalent to method estimatorsMaximum liklihood.

### 2- 4- 2 Bayes test for testing Of Structural hom

Here a mechanism can be set for selecting the structural homogeneity hypothesis defined in the formula(30)using the bis method(Bayes Method)as follows[30]:-

1-Finding the marginal post-probability function of a parameter vector $\alpha$ .

-Assuming linear constraints(Linear Constraints)2

(31)....
$$\left.\begin{aligned} \phi_1 &= \alpha_1 - \alpha_2 \\ \phi_2 &= \alpha_1 - \alpha_3 \\ &\vdots \\ \phi_{(n-1)} &= \alpha_1 - \alpha_n \end{aligned}\right\}$$

and that :- $\phi_1 = 0 \quad , \phi_2 = 0 \quad ,...., \phi_{n-1} = 0$

It means that:- $\alpha_1 = \alpha_2 = \alpha_3 = ..... = \alpha_n$

3 -write variables $\phi_i$ looks likeLinear Combinationas follows:-

$$\phi_i = \sum_{i=1}^{n} a_{ji}\,\alpha_i = a'_j\,\alpha \quad , j = 1,2,...,n-1$$

Since:- $\sum_{i=1}^{n} a_{ji} = o$

and that :-

$a_{n-1} ,...., a_2 \ , a_1$ vectors.Linearly independent

4- Setting the formula(31)As follows:-

$A\alpha = 0$

Since:-

A: an array defined in the test hypothesis defined by the formula(30).

and that $\alpha' = (\alpha_1 \ \alpha_2 \ \cdots \alpha_n)$

5-Through the foregoing in the paragraph1-4-2Show that the boundary posterior probability function of a vector $\alpha$ is a normal multivariate distribution when the parameter $\sigma$ Multivariate information and distributiontWhen the parameter $\sigma$ unknown, and in both cases, theQuadratic formIt is distributed as a chi-square distribution with a degree of freedomnThat is:-

$$Q(\alpha)=(\alpha-\tilde{\alpha})'C(\alpha-\tilde{\alpha})\sim\chi^2_{(n,\delta)}$$

Since: -

Non-informative initial distribution

Primary distribution natural facilities

$$C=\begin{bmatrix} R'R \\ (\sum_\alpha + R'R) \end{bmatrix}$$

and that :
R: matrix defined.

6-Finding the value:

(32) $\ldots\ldots\gamma=\dfrac{Q(\alpha)|H_0}{(n-1)S^2}$

(non-informative prior)for parameter prompt(h)When the variance parameters are known as:

(33).... $P(h)\propto \text{Constant}$

$-\infty<hi<\infty \quad i=1,2,\ldots..,(n+k)$

By integrating the initial distribution(33) (Likelihood Function ) We get the common posterior probability function(Posterior Function)for parameter vector (h) meaning that:-

wafor formula(32)distributed distributionFtwo degrees of freedom(n-1)to simplify and(Nnk)for the denominator, which is the Bayes test formula for the structural homogeneity hypothesis[30].

**3-4-2 Random effect Model:-**

To estimate the random effect modelusingBayes MethodAccording to the types of primary distribution functions used in the research, we assume the availability of a non-informational primary distribution or

(34).... $P(h|Y,\sigma_u^2\,\sigma_\mu^2)\propto exp\left[-0.5(Y-Zh)'\Omega^{-1}(Y-Zh)\right]$

$$\propto exp\left[-0.5\left[(Y-Z\hat{h})'\Omega^{-1}(Y-Z\hat{h})+(h-\hat{h})'Z'\Omega^{-1}Z(h-\hat{h})\right]\right]$$

(35).... $$P(h\,|\,Y,\sigma_u^2,\sigma_\mu^2)\propto exp\left[-0.5\left[(h-\hat{h})'Z'\Omega^{-1}Z(h-\hat{h})\right]\right]$$

$-\infty < hi < \infty \quad i=1,2,\ldots..,n+k$

Since:-

$$\hat{h}=(Z'\Omega^{-1}Z)^{-1}Z'\Omega^{-1}Y$$

and the formula(35)It represents a normal multivariate distribution(Multivariate normal)in the middle of my account $\hat{h}$ Represents a piez estimator(Bayes Estimator)for parameter prompthcovariance and covariance matrix $(Z'\Omega^{-1}Z)^{-1}$ .

**Chapter III**

Application side

**3-1 Introduction:-**

In this chapter, the application will be carried out on the analysis of statistical data related to investment functions for the economic sectors in the Republic of Iraq, as a typical fixed effect will be estimated (fixed effect model) and random effect (random effect model(using the prepared statistical program)gretl) This falls within the traditional analysis of the above-mentioned models. For the same data, a Bayesian analysis of the models themselves will be conducted and then a comparison between the two methods to determine the optimal method for estimation.

**3-2 What data was used in the research**

Data on investment at the level of economic sectors in the Republic of Iraq for the period of time2006-2021 .

**3-3 Analyzing the data used in the research**

It concerns investment data at the level of the nine (nine) economic sectors in the Republic of Iraq for the period of time2006-2021 and the nine economic sectors are the agriculture, forestry and fishing sector, the mining and quarrying sector, the manufacturing sector, the electricity and water sector, the building and construction sector, the transportation and storage sector, the wholesale and retail trade, hotels and the like, the finance, insurance and real estate services sector, and the social and personal development services sector .

To analyze the data, the total fixed capital formation was adopted at current prices for the years2006- 2021 for each sector as approved variables and total fixed capital formation at current prices for the years2006-2021 for a previous year and GDP at current prices for the years
2006-2021As illustrative variables, using a programgretlCapabilities have been obtainedMLFor the parameters and the statistical indicators of each regression model for the main crops individually, the following has been assumed:

The first model: - means the regression model of the agricultural, forestry and fishing sectors.

The second model: - means the regression model of the mining and quarrying sector.

The third model: - means the regression model of the manufacturing sector.

The fourth model: - means the regression model of the electricity and water sector.

The fifth model: - means the regression model of the building and construction sector.

The sixth model: means the regression model of the transportation and storage sector.

The seventh model: It means the regression model and the wholesale and retail trade sector, hotels and the like.

The eighth model: - means the regression model of the financial sector, insurance and real estate services.

$Y_i = \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + U_i \quad , \quad i = 1,2,3,\cdots,9$

Joule (1) explains that:

The ninth model: means the regression model of the social and personal development services sector.

As for the mathematical model representing each of the above models, each individually, it is as follows:-

Schedule (1)

It shows the values of the estimators in a wayMLIts standard deviation and test values -t for the parameters of each of the nine models.

| form | capacity | estimator value | standard deviation | *a test-t* | *Values-p* | morale |
|---|---|---|---|---|---|---|
| the first | $b_0$ | 1.82336e+07 | 1.42573e+08 | 0.1279 | 0.90242 | |
| | $b_1$ | 1.36931 | 0.913484 | 1.4990 | 0.18453 | |
| | $b_2$ | 2.52723 | 13.8067 | 0.1830 | 0.86079 | |
| Second | $b_0$ | -5.6608e+08 | 7.57475e+08 | -0.7473 | 0.48311 | |
| | $b_1$ | 0.776935 | 0.417437 | 1.8612 | 0.11204 | |
| | $b_2$ | 29.5802 | 25.9415 | 1.1403 | 0.29764 | |
| Third | $b_0$ | -4.5782e+08 | 5.83239e+08 | -0.7850 | 0.46233 | |
| | $b_1$ | -0.674832 | 1.40394 | -0.4807 | 0.64778 | |
| | $b_2$ | 1762.15 | 640,596 | 2.7508 | 0.03326 | ** |
| the fourth | $b_0$ | -9.7732e+07 | 1.55226e+08 | -0.6296 | 0.55215 | |
| | $b_1$ | 1.79798 | 0.671957 | 2.6757 | 0.03674 | ** |
| | $b_2$ | 486.73 | 1508.24 | 0.3227 | 0.75786 | |
| Fifth | $b_0$ | 2.65126e+07 | 1.69135e+07 | 1.5675 | 0.16803 | |
| | $b_1$ | 2.14111 | 1.09546 | 1.9545 | 0.09844 | * |
| | $b_2$ | -63.5185 | 40.0339 | -1.5866 | 0.16370 | |
| VI | $b_0$ | -3.4478e+08 | 1.39947e+08 | -2.4637 | 0.04887 | ** |
| | $b_1$ | 0.386542 | 0.298304 | 1.2958 | 0.24266 | |
| | $b_2$ | 164.676 | 43.4294 | 3.7918 | 0.00905 | *** |
| seventh | $b_0$ | 9.11214e+06 | 3.06567e+07 | 0.2972 | 0.77631 | |
| | $b_1$ | -0.0356475 | 0.368786 | -0.0967 | 0.92614 | |
| | $b_2$ | 12.7271 | 9.6788 | 1.3149 | 0.23655 | |
| VIII | $b_0$ | 1.1852e+07 | 8.53799e+06 | 1.3881 | 0.21444 | |
| | $b_1$ | 0.319273 | 0.32312 | 0.9881 | 0.36126 | |
| | $b_2$ | 6.63288 | 2.82703 | 2.3462 | 0.05735 | * |

| | | | | | | |
|---|---|---|---|---|---|---|
| **ninth** | $b_0$ | **-6.3744e+07** | **4.55193e+08** | **-0.1400** | **0.89321** | |
| | $b_1$ | **0.0399152** | **0.436826** | **0.0914** | **0.93017** | |
| | $b_2$ | **379,477** | **152.74** | **2.4845** | **0.04752** | ** |

Schedule (2)
Shows the values of the determination coefficients, the corrected determination coefficients, and the value ofFThe Durban-Watson and Testwhiteand testVIFor each of the three models.

| form | R2 | R2* | F | DW | W | VI |
|---|---|---|---|---|---|---|
| **the first** | **0.27347** | **0.03129** | **1.12922** | **0.794443** | **6.834112** | **1.038** |
| | | | | | | **1.038** |
| **Second** | **0.71584** | **0.62113** | **7.55758** | **2.03642** | **8.635270** | **1.955** |
| | | | | | | **1.955** |
| **Third** | **0.57813** | **0.43751** | **4.111118** | **1.81735** | **7.555765** | **1.014** |
| | | | | | | **1.014** |
| **the fourth** | **0.94974** | **0.93299** | **56,689** | **2.47739** | **6.295757** | **12,709** |
| | | | | | | **12,709** |
| **Fifth** | **0.45247** | **0.26996** | **2.47915** | **2.03071** | **8.587187** | **13.273** |
| | | | | | | **13.273** |
| **VI** | **0.85296** | **0.80394** | **17,402** | **2.08764** | **5.916218** | **1.581** |
| | | | | | | **1.581** |
| **seventh** | **0.22884** | **-0.02821** | **0.890262** | **2.16075** | **5.616602** | **1.062** |
| | | | | | | **1.062** |
| **VIII** | **0.73306** | **0.64408** | **8.23841** | **1.93392** | **6.521218** | **1.735** |
| | | | | | | **1.735** |
| **ninth** | **0.72993** | **0.63990** | **8.10807** | **2.69066** | **8.965370** | **2.483** |
| | | | | | | **2.483** |

It is noted from Table (2) the following:-
1- Finding the values,dL=0.824,ά=0.05) (dU=1.320It is noted that the Durban–Watson test failed because there is a problem of autocorrelation of all models due to the occurrence of a value ofDWIn areas of test failure, therefore, it is not possible to say anything about the problem for all models.
2- Based on test valuesWhiteand evaluatePProbability Note that there is no variance heterogeneity problemHeteroskedasticityFor all models where:-
**p-value = P(Chi-square(5) > 6.834112) = 0.233273**
**0.124527 p-value = P(Chi-square(5) >8.635270 ) =**
**p-value = P(Chi-square(5) > 7.555765) = 0.182478**
**p-value = P(Chi-square(5) >6.295757) = 0.278495**
**0.126707 p-value = P(Chi-square(5) > 8.587187) =**
**p-value = P(Chi-square(5) > 5.916218) = 0.314457**
**p-value = P(Chi-square(5) > 5.616602) = 0.345329**
**P(Chi-square(5) > 6.521218) = 0.258751 p-value =**
**p-value = P(Chi-square(5) > 8.965370) = 0.110453**

3- Depending on the values ofVariance Inflation Factors
(VIF )It is noted from the table that there is no problem of polylinearityMulticollinearityfor all models.
4. Depending on the valuesFCalculated and valuesPThe following possibility:-

**F-statistic (2, 6) = 1.12922 (p-value = 0.383)**
**F-statistic (2, 6) = 7.55758 (p-value = 0.0229)**
**F-statistic (2, 6) = 4.111118 (p-value = 0.0751)**
**F-statistic (2, 6) = 56.689 (p-value = 0.000127)**
**F-statistic (2, 6) = 2.47915 (p-value = 0.164)**
**F-statistic (2, 6) = 17.402 (p-value = 0.00318)**
**F-statistic (2, 6) = 0.890262 (p-value = 0.459)**
**F-statistic (2, 6) = 8.23841 (p-value = 0.019)**
**F-statistic (2, 6) = 8.10807 (p-value = 0.0197)**

It is noted that the models (2,4,6,8,9) are significant, and this indicates the existence of a relationship between the total fixed capital at current prices as an approved variable and the total fixed capital at current prices for a previous year and the gross domestic product at current prices as illustrative variables and thus the adoption of the estimates shown in the table (1) for these models, while models (1,3,5,7) were not significant.

The insignificance of the models (1,3,5,7) is in fact inconsistent with the economic theory as it does not clarify the relationship between total fixed capital at current prices as an approved variable and total fixed capital at current prices for a previous year and GDP at current prices as illustrative variables, so the method of b is adopted Combine cross-section and time-series data(panel data)And then we have (81) singular and the model representing it is either the model of the fixed effect as follows:-

$$Y_{ij} = \beta_{0i} + \beta_1 X_{ij1} + \beta_2 X_{ij2} + U_{ij} \qquad i = 1,2,\cdots,9$$

Or the following combined form:

$$Y_j = \beta_0 + \beta_1 X_{j1} + \beta_2 X_{j2} + U_j \qquad j = 1,2,\cdots,81$$

Or the following random effect model:

$$Y_{ij} = \beta_0 + \beta_1 X_{ij1} + \beta_2 X_{ij2} + U_{ij} \qquad i = 1,2,\cdots,9$$

The choice of the fixed effect model or the combined model is based on the decision to accept or reject the pre-defined structural homogeneity hypothesis using the traditional pre-defined test or the Bayesian test defined by formula (32) according to the method used in the assessment and testing process. Either the choice of the fixed effect model or the random effect model is according to the decision to accept or reject the hypothesis of the Hassmann test.(Hausman)described in a previous study and by applying the estimation method ((mlwhich is defined and quantified by basing by adopting a non-informational function ((bnThe identifier is within the function (15) and is estimated by basing by adopting the natural conjugate function ((bcThe identifier within function (28) for the parameters of the fixed-effect model and the . methodmlTo estimate the parameters of the built-in model and the methodmlTo estimate the parameters of the random effect model and estimate it by adopting a non-informational distributionbnThe base estimate is based on the distribution of natural facilitiesbcThe results shown in Table (3) were obtained as follows:

**Table (3)**

Builds the capabilities of the method of the greatest possibility(ml)For parameters of fixed, random, and combined effect models and basing estimators(bn) (bc)Parameters of the fixed and random effect models

| form | $b_1$ | $b_2$ | $S^2$ |
|---|---|---|---|
| **fixed effect(ml)** | **0.934731 (0.152562)** | **27.7695 (14.9245)** | **4.16e+017** |
| **fixed effect(bn)** | **0.934731 (0.15478 )** | **27.7695 (15.1423)** | **4.282e+017** |

| | | | |
|---|---|---|---|
| **fixed effect(bc)** | **1.90541 2.78658** | **29.56213 (18.55341)** | **4.945e+017** |
| **random effect(ml)** | **1.13225 (0.128924)** | **7.35611 (7,28267 )** | **4.161e+017** |
| **random effect(bn)** | **2.12093 (0.19760)** | **9.78621 (9.11126)** | **4.6931e+017** |
| **random effect(bc)** | **4.33289 (0.23416)** | **10.98701 (10.9076)** | **4.9931e+017** |
| **built(ml)** | **1.13225 0.128924** | **7.35611 7.28267** | **4.118e+017** |

Note: Numbers in parentheses mean the standard deviation.

To test the random effects by testing the following hypothesis:

$H_0 : \sigma_\mu^2 = 0$

$H_1 : \sigma_\mu^2 \neq 0$

We find that the value of the test statistic is as follows:-

Chi-square(1) = 1.03296 , p-value = 0.309464

What is the value of the Hassmann test?(Hausman)They are as follows:-

Chi-square(2) = 5.86576 ,( p-value = 0.0532435 )

Finally, the following structural homogeneity hypothesis is tested:

$$H_0 : \beta_{01} = \beta_{02} = \cdots = \beta_{09}$$

$$H_1 : \beta_{01} \neq \beta_{02} \neq \cdots \neq \beta_{09}$$

we find that :-

F(8, 70) = 0.903496 , (p-value = P(F(8, 70) > 0.903496) = 0.518656)

F(8, 70) = 0.787422, (p-value = P(F(8, 70) > 0.903496) = 0.615278)

Based on the results of Table (3) and the subsequent tests, we note the following:

1- The capabilities of the methodmlThe estimation of the random effect model is better than the same estimators for the fixed effect model because it has the least variance.

2- The bis estimations are based on a non-informational initial distribution(bn)It is better than the bis estimations by adopting a normalized initial distribution for both the fixed and random models.

3- That capabilitiesmlThe parameters of the random effect model are better than the bis estimators based on a non-informational distribution because they have lessmse.

4- That abilitiesmlThe parameters of the built-in model are equal to the parameters of the same method of the random effect model.

5- Having less built-in model $S^2$ from the rest of the models.

6- Depending on the random effects test, we accept the null hypothesis and reject the alternative hypothesis.

7- Depending on the Hassmann test, we accept the null hypothesis and reject the alternative hypothesis, that is, the effects are constant.fixed effectsIt is not random and therefore the fixed effect model is preferred to represent the data of the total fixed capital formation according to the economic activities in Iraq.

8- Depending on the test of the structural homogeneity hypothesis in the traditional and Bayesian style, respectively, we accept the null hypothesis and reject the alternative hypothesis, that is, the fixed model has one fixed limit parameter, so that we have the built-in model the value of the parameter estimator1.24544e+08with standard deviation8.31126e+07As for the rest of the estimations of the parameters of the built-in model, they are shown in Table (3).

## The fourth chapter
## Conclusions and Recommendations
### 4-1 Introduction

This chapter includes the most important conclusions that we reached in this research, depending on what was presented in the theoretical side and the results of the work that were reached within the practical side, as well as the most important recommendations that we thought were necessary in the study of

Bayesian analysis of regression models for double data, as well as some future prospects for developing and expand this study.

**4-2 Conclusions**

1- The use of a non-informational initial distribution using the Bayesian method to estimate the parameters of the fixed effect modelFixed effect modelWhen the number of cross-sections is small and ( $\sigma$ ) unknown gives a suffix probability function for parameters having a multivariate distribution -tarithmetic mean represents an estimate (GLS(For the parameters, it is a piez estimator for the parameters with the presence of the weighted error squared loss function while using the same initial distribution when the number of cross sections is large) $\sigma$ ) unknown gives a suffix probability function for parameters having a multivariate distribution -tarithmetic mean represents an estimate (GLS) for the parameters and it is a piez estimator for the parameters with the presence of the weighted error squared loss function.

2- Using a normal conjugated primary distribution using the Bayesian method to estimate the parameters of the fixed effect modelFixed effect modelWhen the number of cross-sections is small and ( $\sigma$ ) unknown gives a posterior probability function for parameters having a multivariate distribution −tThe same distribution is also used when the number of cross-sections is large and ( $\sigma$ (unknown gives a posterior probability function for parameters having a multivariate distribution -t.

3- The use of a non-informational initial distribution using the Bayesian method to estimate the parameters of the random effect modelRandom effect modelIt gives a posterior probability function for parameters with a normal multivariate distribution after an asymptotic expansion(Asmptotic Expansion) meanAn identifier representing estimators of parameters with a weighted error squared loss function.

4- Using a normalized first distribution using the Bayesian method to estimate the parameters of the random effect modelRandom effect modelIt gives a posterior probability function for parameters with a normal multivariate distribution after an asymptotic expansion(Asmptotic Expansion) meanAn identifier representing a piez estimator for parameters with a weighted error squared loss function

5- Through what was presented in the practical side and for the first and second applications, it was concluded that the method of estimationmlIt is the best at the level of estimation of the parameters of the fixed effect model
(First practical application) when the number of cross-sections is small among the estimation methods adopted in the research, and the Bayes method based on a non-informational distribution comes in second place, while the results of (second practical application) showed the advantage of the methodAppreciationmlOn the rest of the methods at the level of the combined model (fixed effect model with only one fixed limit) when the number of cross-sections is large over the rest of the estimation methods, the BASE method comes in second place based on a non-informational distribution.

6- The practical results showed the agreement of the decision about the hypothesis of structural homogeneity in the fixed effect model using the traditional test previously defined and the Bayesian test defined by the formula (32).

7- The experimental results showed that the use of cross-sectional data and time-series data merging method to obtain paired data (panel dataRepresenting it with one of its models gives better results than using individual models for each of the cross-sections.

**4-3 Recommendations**

As a result of the foregoing, the researcher recommends the following:

1-Using the traditional method ofMLIn estimating the parameters of dual data models represented by the . methodMLIn the event that the number of cross-sections is small or large, instead of the Bayesian method, whether prior information is available or not, as well.

2- The use of a random effect model (Random effect Model(When the number of cross-sections in the paired data)panel data) Very large when choosing the alternative hypothesis

in the Hassmann test as a first choice to represent it or using the effect model within the cross section
(within effect mode) as a second test.
3-The traditional method is sufficient to test the structural homogeneity hypothesis in the fixed effect model, although we obtained a test using the BAES method, since the first method is easier in practical application.
5- Activate the use of the ready-made program (gretl) in paired data analysis(panel data)In particular, for cross-sectional data (Cross-section) and for time series data (time seriesIn general, because of the possibility of this program to analyze such data.
6- The researchers urged the use of double data (panel data) in the study of many problems as it provides a greater vision for the researcher about the phenomenon studied.

**References**


1- Al-Hasnawi, Amori Hadi and Al-Qaisi, Basem Shalebah (2002) "Advanced Economic Measurement (Theory and Application)" Dunya Al-Amal Library - Baghdad.
2- Al-Dulaimi, Nazem Abdullah (1994) "Methods of integrating time series and cross-sectional data in analyzing some economic phenomena" PhD thesis in Statistics, College of Administration and Economics, University of Baghdad.
3- Zaidi,Azhar Salman Zamel (1986) "Using the method of integrating time series and cross-sectional data to analyze the behavior of the Iraqi consumer in the light of the family budget research for the year 1984" Master's Thesis in Statistics, College of Administration and Economics, University of Baghdad.
4- Al-Samarrai, Ibtisam Karim Abdullah (1995) "Using error compounds and endocrine variables in estimating the production function of the general establishment for dairy products" Master's thesis, College of Administration and Economics, University of Baghdad.
5- Al-Abdali, Abed bin Abed (2005) "Estimating the impact of exports on economic growth in Islamic countries: an econometric study."
6- htmExports and Growth/Estimating the Impact of Exports on Economic Growth/I\\:file.
7- Al-Qaisi, Bassem Shaleba (1999) "Baes' method in testing and treating the problem of homogeneity of ionization in linear models (a comparative study), a master's thesis in Statistics, College of Administration and Economics, University of Baghdad.
8- Baltagi, badi (2005) "Econometric Analysis of panel data" John wily &sons inc.third edition.
9- Baltagi, badi (2006) "contributions to econometric analysis" Elserier B BV
10- Baltagi, badi (2006) "Forcasting with panel data" Econometric studies, series1, NO.25.
11- Beck, Nathaniel (2001) "time series-cross section data: what have we learned in post few years?" Annual reviews polit sei.4 pp271-293.
12- Beck, Nathaniel (2004) "Longitudinal (panel and time series-cross .section)"http://www.nyu.edu/gsas/dept/poliyics/facult/beck
13- Box, george EP & Tiao georgic (1973) "Bayesian inference in statistical analysis" addision-wesley publishing company.
14- Greene, Willian h. (2003) "Econometric analysis" pearson ducation, Inc.Fifth edition.
15- Hsiao, Cheng (2003) "Analysis of panel data" combridge university press,second edition.
16- Kementa, Jan (1971) "Element of Econometrics" Macmillan publishing . co.,Inc.
17- Koop, Gary (2003) "Bayesian Econometrics" John wily&sons ltd.
18- Lee cA (2008) "Using gretl for priciples Econometrics" 3rd Edition. Economometrics.com.http://www.learn
19- Maddala, GS (1971)" The use of variance components modelinpooling cross

section and time series data "Econometrica, vol.39, No2, p.341.

20- Mundlak, yair (1978) "On the pooling of time series and cross section data" Econometrica, vol.40, No.1, p.69.
21- Nuamah, NNN (1986) "pooling cross section and time series data" the statistician vol.35, No3 pp 345.
22- Park, HM (2005)" Linear regression models for panel data using SAS STATS, LIMDEP and SSPS"http://www.indiana.edu/~stat.math.
23- Resenbld.A (2008) "gretl" Journal statisticalsoftware, vol.25 htth://www Jstat soft.org.
24- Swamy, PAVB&Mehta, JS(1973) "Bayesian analysis of error components Regression models" JASA, vol.68, No343, pp648.
25- Swamy, PAVB&Mehta, JS (1975) "Bayesian and non BayesianAnalysis of switching regressions and of random coefficient Regression models" JASA, No.351, pp593.
26- Terry, ED&Neeley, M, J. (1988)"pooled cross-section and time Marcel Dekker INC.-London.
27- Vaid Yanathan, KE (1974) "The panel longitudinal opproach in Demographic Enquiries"sankhya vol.36,series B,pt.3,pp314.
28- Waclawiw, Myron.A&Liang, kung-yee (1993) "prediction of random effects in the generalized linear odels" JASA.Vol.88, No.421, pp. 171.
29- Ware.James.H(1985)"Linearmodels for the analysis of longitudinal Studies"the American statistician, Vol.39, No.2 pp95
30- Wooldridge, Jeffry M. (2002) "Econometric analysis of cross sectionalAnd panel data" combridge, MA:MIT press.
31- Zellner .A(1971) "An introduction to Bayesian inference in Econometrics "John Wily&sons,Inc.