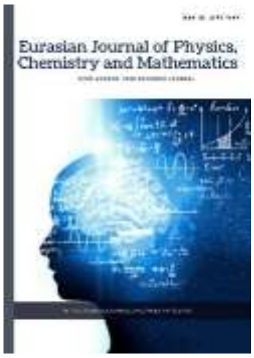

The Problem of Anomalous Filtration and Solute Transport in an Inhomogeneous Porous Medium

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ABSTRACT

The problem of anomalous transport and filtration of substance in a porous medium with a fractal structure is posed and numerically solved. The piezoconductivity equation proposed as fractional differential equations based on the anomalous Darcy law is proposed. The profiles of changes in the concentration of suspended particles, pressure and filtration velocity are determined. Various transport and filtration characteristics evaluated.

Keywords: anomalous Darcy's law, fractional derivative, solute transport, filtration, porous medium.

Introduction

In the process of filtration and solute transport in nonlinear media, as well as in the flow of rheologically complex media, the characteristics often exhibit scale invariance (fractality) both in space and time. This circumstance makes it possible to develop some general methods for modeling complex media and, in some cases, facilitates the description of the processes occurring in them [1]. In fractal objects, the usual quantitative characteristics (length, area, mass, etc.) are not applicable [2].

In [3], a fractional-differential modification of the model is presented, developed using a time-fractional generalization of Darcy's law, and based on this model, it is proposed to develope a hydrodynamic simulator of filtration flows in oil and gas reservoirs.

In [4], the issues of mathematical modeling of nonlinear processes of migration of polluted groundwater are considered, which cannot be described within the framework of the classical approach in the theory of mass transfer. To study the space-time regularities of nonlinear effects due to scale invariance, a non-local boundary value problem for the model differential equation of nonlinear migration is correctly posed.

In the classical case, the process of pollution transport by groundwater flow in natural porous media within the framework of simple idealized representations of pore space and the validity of the locality principle with a certain schematization is satisfactorily described by a system of differential equations [5]. In [6], a filtration model of flow through a porous medium is proposed. A porous medium is a fractal object whose structure is determined by the gap between the conjugate surfaces, consisting of pores and contact areas of wavy and rough conjugate surfaces. The methods of determining the fractal dimensions of the tortuosity and porosity of the medium are given. The leak dependences on the parameters of the porous and compacted medium, as well as the fractal dimension of the tortuosity and porosity of the compacted medium are obtained.

In [7], an equation describing the process of fluid filtration in a porous medium with fractal properties is derived. It is shown that the structure of the equation is identical to the structure of the known equations describing the processes of diffusion and random walks. The physical interpretation of the parameters included in the equation and the method of their experimental determination are given. Also note that a number of papers are devoted to the problem of modeling and solute transport in media with fractal structure, as well as the numerical analysis of equations with fractional derivatives [8–15]. The problems of modeling the processes of pollution and geomigration of groundwater are devoted to works [16–18]. Radon transport processes in media with fractal structure were analyzed in [19].

In this work, filtration and solute transport in a one-dimensional medium of fractal structure is considered.

Statement and numerical solution of the problem

Let the area of study of the problem consist of $R\{0 \le x < \infty\}$. Initially, the area is filled with a fluid without solute. The process of solute transport, taking into account anomalous effects, can be described by the equation [15]

$$\frac{\partial c}{\partial t} = D\frac{\partial^{\beta} c}{\partial x^{\beta}} - \frac{\partial (vc)}{\partial x}, \quad (1)$$

where c is the concentration of solid particles in the fluid, v is the filtration velocity, D is the diffusion coefficient, β is the order of derivative, t is the time, x is the coordinate.

The anomalous filtration velocity is defined as [3]

$$v = -\frac{k}{\mu}\frac{\partial^{\gamma} p}{\partial x^{\gamma}} \quad (2)$$

where p is the pressure, μ is the viscosity coefficient of the suspension, k is the permeability coefficient, and γ is the order of derivative. The continuity equation of the flow of a compressible fluid through a porous medium can be written as [20]

$$\frac{\partial(\rho m)}{\partial t} + div(\rho \vec{v}) = 0, \quad (3)$$

where m is the porosity coefficient, ρ is the density of the liquid. We use the equations of state of an elastic fluid and an elastic porous medium [20]

$$\rho = \rho_0(1 + \beta_l(p - p_0)),$$
$$m = m_0 + \beta_m(p - p_0), \quad (4)$$

where β_l is the volume compression coefficient of the liquid, β_m is the elasticity coefficient of the medium, ρ_0 is the initial density of the liquid, p_0 is the initial pressure.

Substituting (2), (4) into (3), we can obtain the piezoconductivity equation with a fractional derivative

$$\frac{\partial p}{\partial t} = \chi\frac{\partial^{\gamma+1} p}{\partial x^{\gamma+1}}, \quad (5)$$

where $\chi = k/\mu\beta^*$ is the piezoconductivity coefficient, β^* is the elastic compressibility coefficient of the medium.

So, we obtain a system of suspension filtration and solute transport equations consisting of the balance equation (1), Darcy's law (2) and the piezoconductivity equation (5)

$$\frac{\partial c}{\partial t} = D\frac{\partial^{\beta} c}{\partial x^{\beta}} - \frac{\partial (vc)}{\partial x},$$

$$v = -\frac{k}{\mu}\frac{\partial^{\gamma} p}{\partial x^{\gamma}},$$

(6)

$$\frac{\partial p}{\partial t} = \chi \frac{\partial^{\gamma+1} p}{\partial x^{\gamma+1}}.$$

The initial and boundary conditions of the problem have the following form

$$c(0,x) = 0, \qquad (7)$$

$$c(t,0) = c_0,\ c_0 = \text{const}, \qquad (8)$$

$$\frac{\partial c}{\partial x}(t,\infty) = 0, \qquad (9)$$

$$p(0,x) = p_0,\quad p_0 = \text{const}, \qquad (10)$$

$$p(t,0) = p_c,\qquad p_c > p_0,\ p_c = \text{const}, \qquad (11)$$

$$\frac{\partial p}{\partial x}(t,\infty) = 0. \qquad (12)$$

To solve the problem (6) — (12), we use the finite difference method. To accomplish this, we will construct a grid in the area R in the form

$$\omega_{h\tau} = \left\{ (t_j, x_i),\ t_j = \tau j,\ x_i = ih,\ j = 0,1,\ldots,J,\ i = 0,1,\ldots,\ \tau = T/J \right\}$$

, where h is the grid step in the direction of x, τ is the grid step in time, T is the maximum time during which the process is investigated.

Instead of the functions $c(x,t)$, $v(x,t)$ and $p(x,t)$, we will consider net functions, the values of which in the nodes (x_i, t_j), respectively, we denote c_i^j, v_i^j and p_i^j.

On the grid $\omega_{h\tau}$, we approximate the first equation of system (6) as follows [12,14,15]

$$\frac{c_i^{j+1} - c_i^j}{\tau} = \frac{D}{\Gamma(3-\beta)h^{\beta}} \sum_{l=0}^{i-1} \Big(c_{i-(l-1)}^j - 2c_{i-l}^j + c_{i-(}^j \ldots$$

$$- \frac{(v)_{i+1}^j c_{i+1}^{j+1} - (v)_{i-1}^j c_{i-1}^{j+1}}{2h},$$

(13)

where $\Gamma()$ is the gamma function.

For the filtration velocity, we use the following scheme

$$(v)_i^j = -\frac{k}{\mu}\frac{p_{i+1}^j - \gamma p_i^j}{\Gamma(2-\gamma)h^{\gamma}} \qquad (14)$$

The third equation of system (6) is approximated as

$$\frac{p_i^{j+1} - p_i^j}{\tau} = \frac{\chi}{\Gamma(3-\gamma)h^{\gamma}} \sum_{l=0}^{i-1} \Big(p_{i-(l-1)}^j - 2p_{i-l}^j + p \ldots$$

(15)

The initial and boundary conditions are approximated as

$$c_i^j = 0,\quad i = \overline{0,N},\ j = 0, \qquad (16)$$

$$c_i^j = c_0,\quad i = 0,\ j = \overline{0,J}, \qquad (17)$$

$$\frac{c_i^{j+1} - c_i^j}{h} = 0,\quad i = N,\ j = \overline{0,J}, \qquad (18)$$

$$p_i^j = p_0 = \text{const},\quad i = \overline{0,N},\ j = 0, \qquad (19)$$

$$p_i^j = p_c,\quad i = 0,\ j = \overline{0,J}, \qquad (20)$$

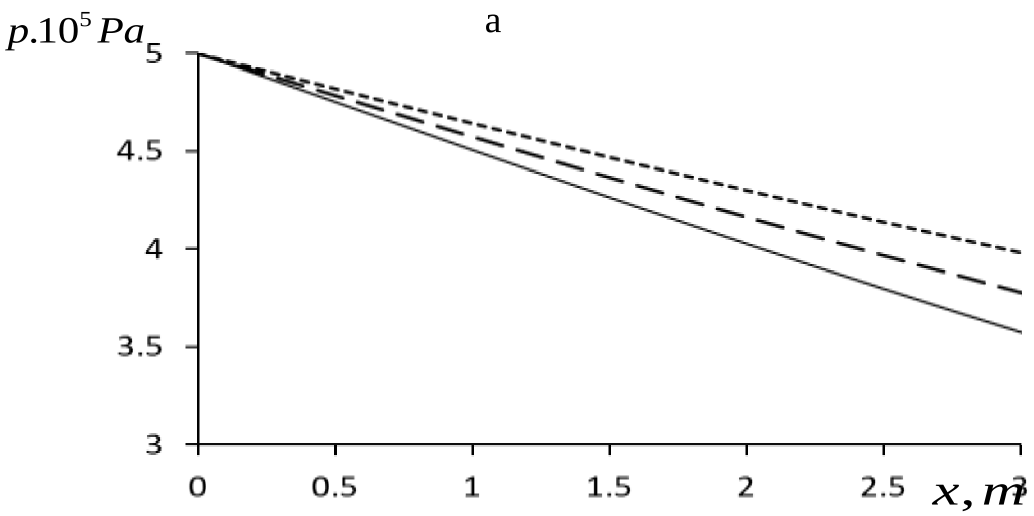
$$\frac{p_i^{j+1} - p_i^j}{h} = 0,\quad i = N,\ j = \overline{0,J}, \qquad (21)$$

where N is a sufficiently large number for which equation $c_N^j = 0$ is approximately satisfied.

The sequence of calculations is as follows: first, p_i^{j+1} is determined from the finite difference scheme (15), then the anomalous filtration velocity are calculated from (14), after that c_i^j is determined on the ($j+1$) layer from the difference equations (13). The following values of the initial parameters were used in the calculations: $k = 10^{-13}\, m^{1+\gamma}$, $\mu = 5 \cdot 10^{-3}\, Pa \cdot s$, $\beta^* = 3 \cdot 10^{-8}\, Pa^{-1}$, $p_c = 5 \cdot 10^5 Pa$, $p_0 = 10^5\, Pa$, $c_0 = 0{,}01$ and $D = 10^{-5}\, m^{\beta} / s$.

Some of the results are shown in Fig. 1-2. Pressure, concentration and filtration velocity profiles are determined. The obtained results show that a decrease in the values of γ from 1 leads to an increase in the effects of anomalous filtration and diffusion process. From this, it is possible to notice an increase in diffusion effects in the direction of x when γ and β takes values less than 1 and 2 (Fig.2). A decrease in the value of β from 2 leads to more intense dynamics (Fig. 2b). The effect of fast diffusion is especially noticeable in Fig.2b. In Fig. 3 shows the change in the concentration profiles at different values of γ in the case of $t = 1200$; 2400; 3600. By comparing Fig.3a and Fig.3b, one can observe a wider distribution of concentration profiles in Fig.3b.As can be seen from Fig. 4, a decrease in the order of the derivative of two will lead to a more diffuse distribution of the concentration profiles. Comparing Fig. 4 a, b, v, one can notice more advanced profiles in the direction of x in case b) and c). This corresponds to the case of "fast diffusion".

a



$v.10^{-4}\, m / s$

b

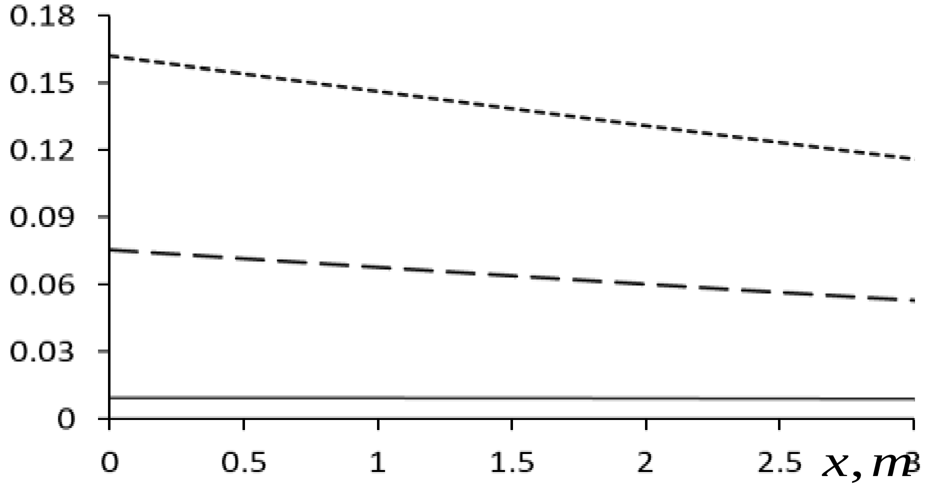
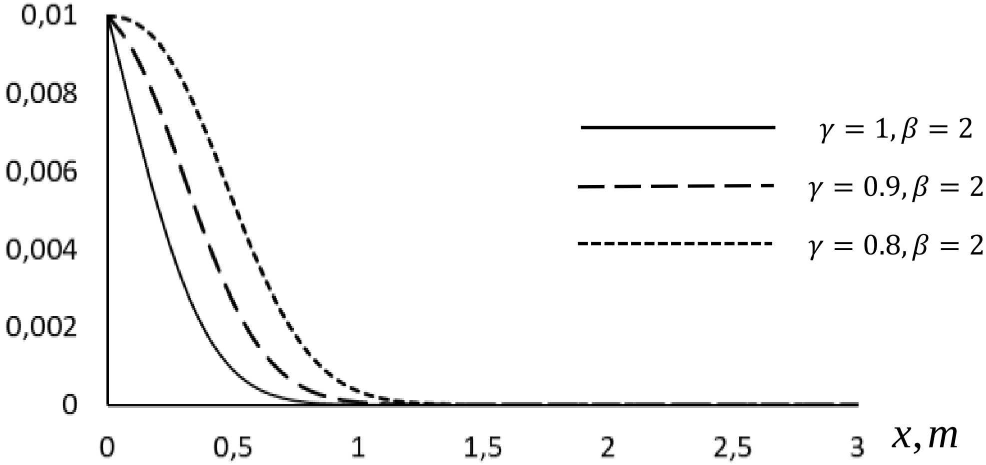
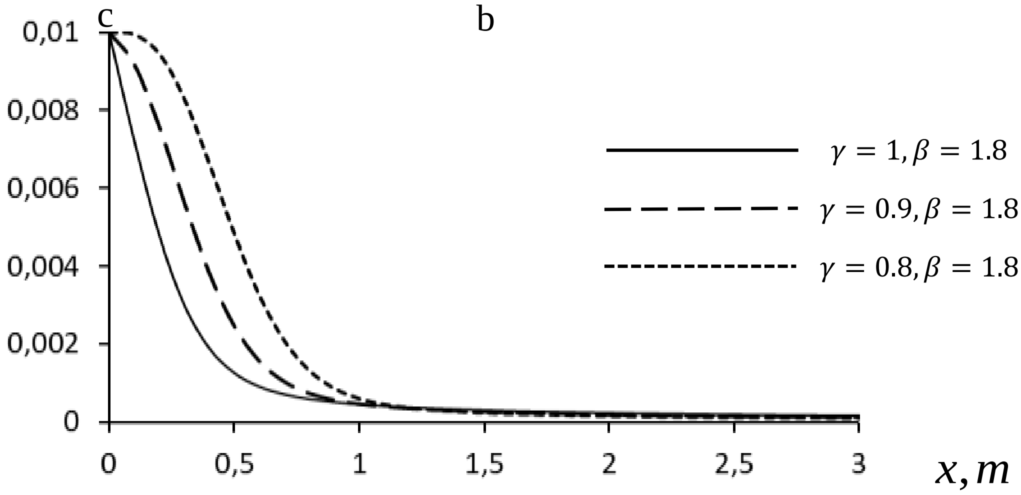


Fig.1. Changes p (a) and v (b) when $k = 10^{-13} m^{1+\gamma}$, $\beta^* = 3 \cdot 10^{-8} Pa^{-1}$, $\mu = 5 \cdot 10^{-3} Pa \cdot s$, $p_0 = 10^5 Pa$, $p_c = 5 \cdot 10^5 Pa$, $\beta = 2$, $t = 3600$ s,
—— $\gamma = 1$, — — — $\gamma = 0,9$, a ------ $\gamma = 0,8$.

c



c b



c v

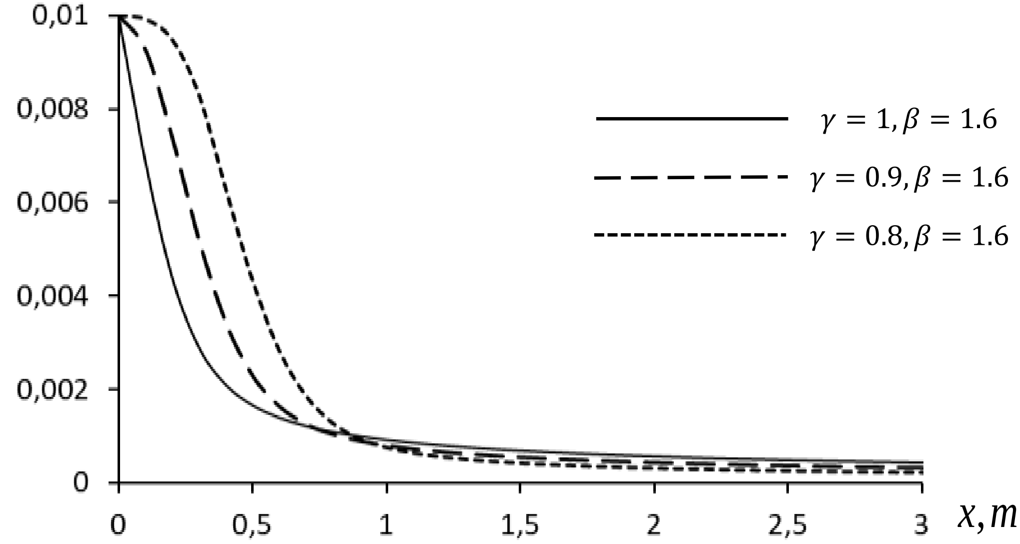
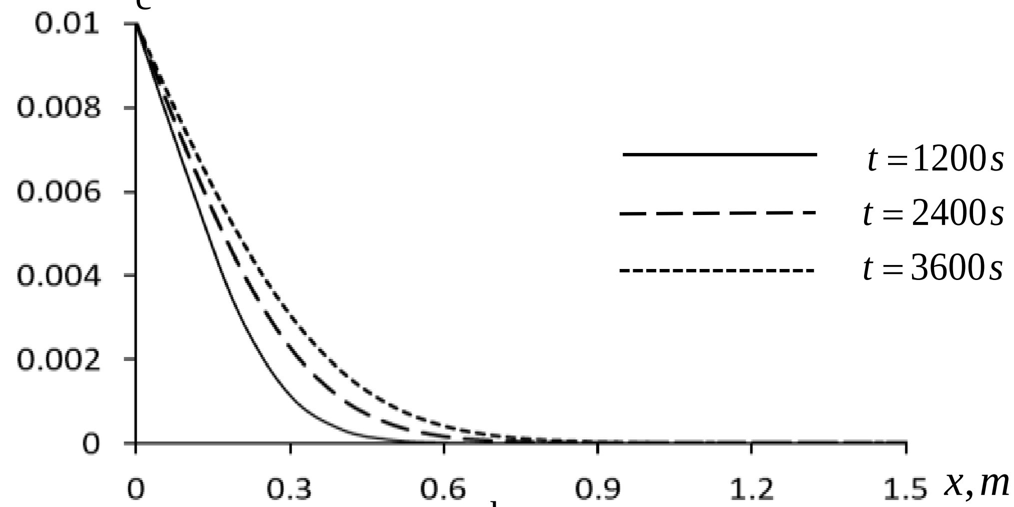


Fig.2. Profiles of concentration changes at $k = 10^{-13}\, m^{1+\gamma}$, $\beta^* = 3\cdot 10^{-8}\, Pa^{-1}$, $\mu = 5\cdot 10^{-3}\, Pa\cdot s$, $p_0 = 10^5\, Pa$, $p_c = 5\cdot 10^5\, Pa$, $t = 3600$ s.

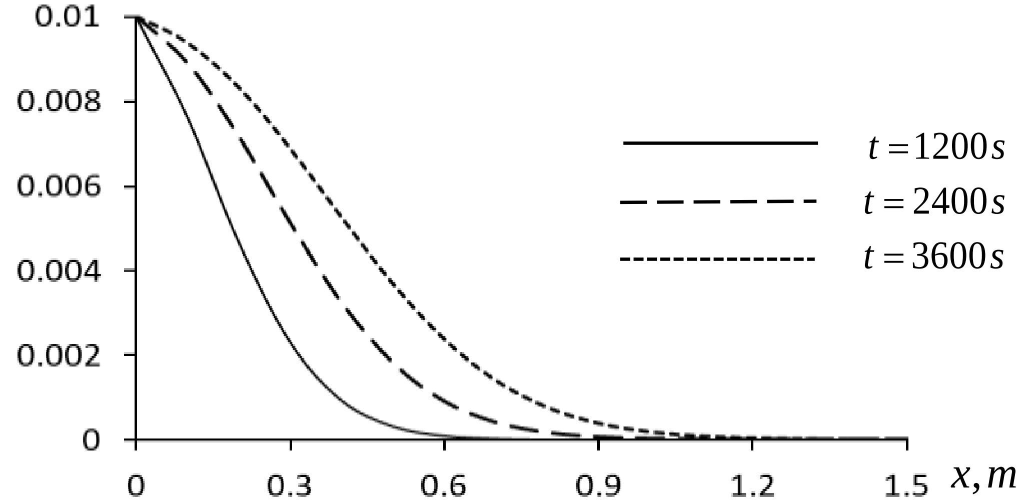
c

a



b

c



c

v

$t = 1200s$

$t = 2400s$

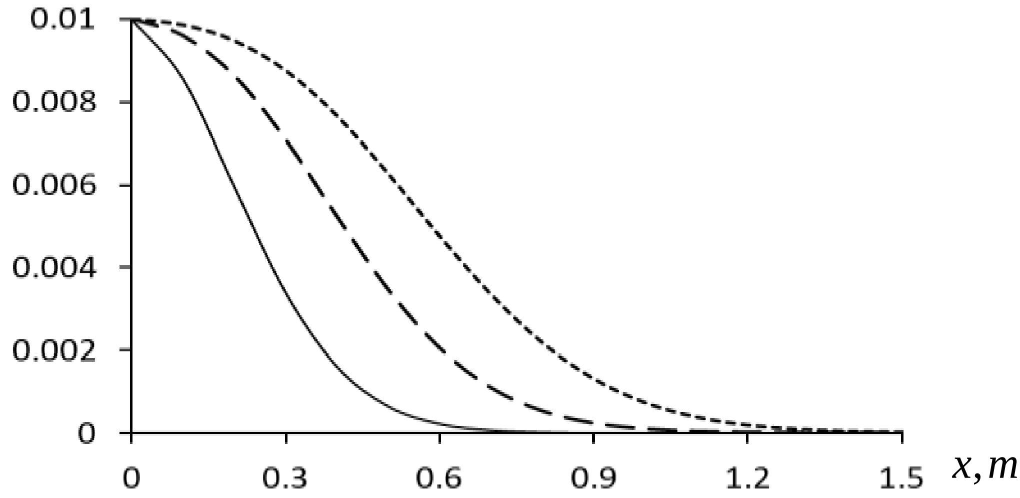


Fig.3. Profiles of concentration changes at $k = 10^{-13}\, m^{1+\gamma}$, $\beta^* = 3 \cdot 10^{-8}\, Pa^{-1}$, $\mu = 5 \cdot 10^{-3}\, Pa \cdot s$, $p_0 = 10^5\, Pa$, $p_c = 5 \cdot 10^5\, Pa$, $\beta = 2$, $\gamma = 1$ (a), $\gamma = 0,9$ (b), $\gamma = 0,8$ (v).

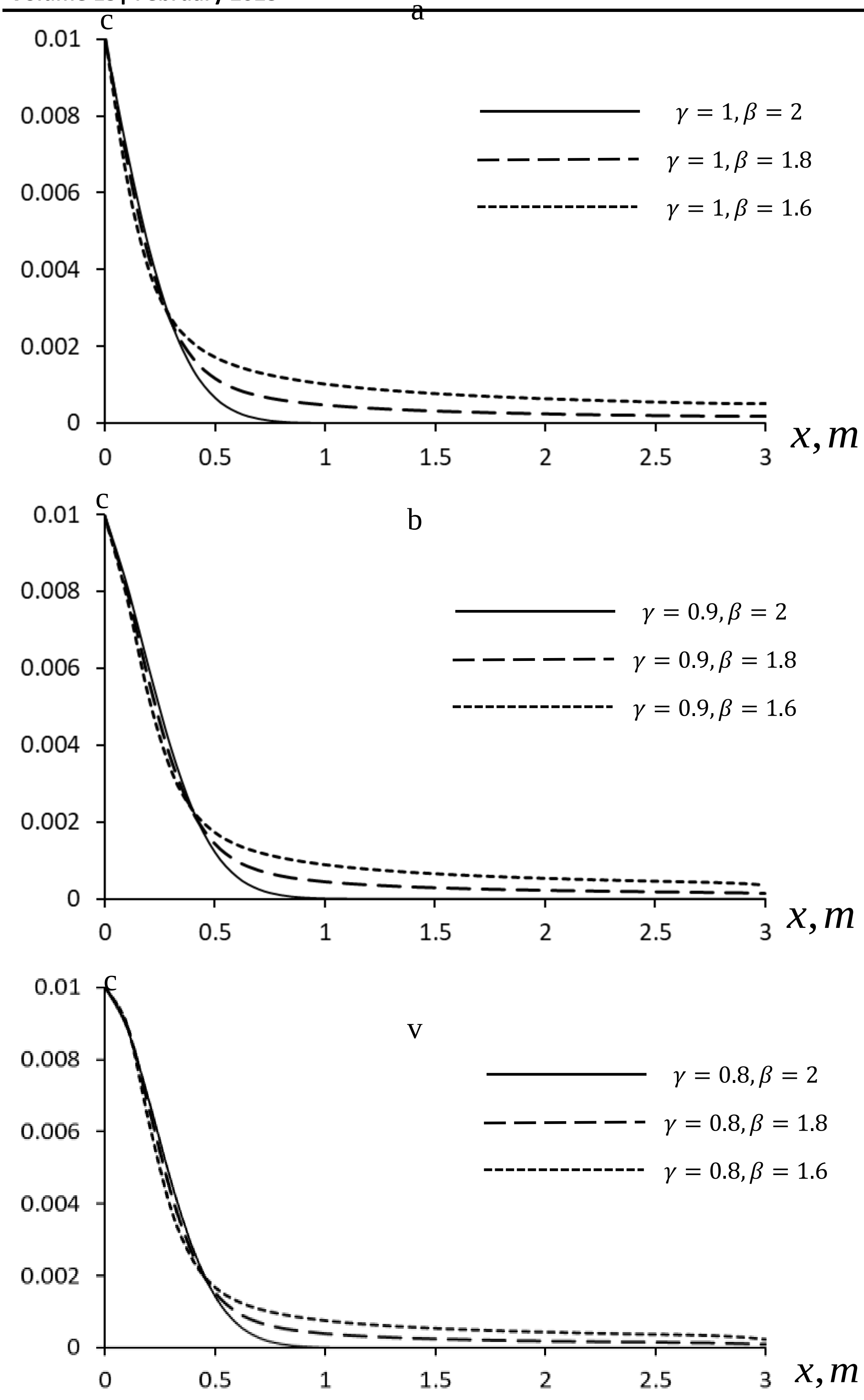


Fig.4. Profiles of concentration changes at $k = 10^{-13}\, m^{1+\gamma}, \beta^* = 3 \cdot 10^{-8}\, Pa^{-1}$, $\mu = 5 \cdot 10^{-3}\, Pa \cdot s$, $p_0 = 10^5\, Pa$, $p_c = 5 \cdot 10^5\, Pa$, $t = 3600$ s.

Conclusion. The problem of filtration and solute transport in a one-dimensional porous medium with a fractal structure is considered. The solute transport in such media is described by an equation with fractional derivatives with respect to the coordinate. The calculation results show that a decrease in the order of derivative in the filtration equation from 1 leads to an increase in pressure and filtration velocity. A decrease in the exponent of the derivative in the diffusion term from 2 leads to “acceleration” of the diffusion process. At the same time, with a decrease in the order of the derivative in the anomalous filtration equation from 1 and the order of the derivative in the diffusion term of the solute transport equation from 2, a wider distribution of concentration profiles can be observed.

Literature

1. Khasanov M.M., Bulgakova G. T. Nonlinear and nonequilibrium effects in rheologically complex media. - Moscow-Izhevsk: Institute of Computer Research. 2003. - 288 p.(in Russian)
2. Mirzajanzade A.H., Khasanov M.M., Bakhtizin R.N. Modeling of oil and gas production processes. Moscow. Izhevsk: Institute of Computer Research. 2004. - 368 p. (in Russian)
3. Belevtsov N.S. On one fractional-differential modification of the non-volatile oil model. Mathematics and mathematical modeling. 2020. No. 06. pp. 13 – 27. DOI: 10.24108/mathm.0620.0000228. (in Russian)
4. Vendina A.A. On mathematical modeling of the process of fractal migration of pollutants in natural porous systems// Vestn. Sam. gos. tech. un-ta. Ser. Phys.-mat. sciences. No. 3 (24). 2011. pp. 199-201 . (in Russian)
5. Bear J. Dynamic of fluids in porous media. Dover, New York. 1972. 761 p.
6. Izmerov M.A., Tikhomirov V.P. Filtration model of flow through a fractal porous medium // Fundamental and applied problems of engineering and technology. 2014. №3. C.7-14(in Russian)
7. Bagmanov V. H., Baykov V. A., Latypov A. R., Vasiliev I. B. Methods of interpretation and determination of parameters of the filtration equation in a porous medium with fractal properties // Bulletin of the Ufa State Aviation University. 2006. pp.146-149. (in Russian)
8. Bazzaev A.K. A locally one-dimensional scheme for a fractional order diffusion equation with a fractional derivative in the lower terms with boundary conditions of the first kind // Vladikavk. matem. Journal. 2014. Vol. 16. No. 2. pp. 3-13. (in Russian)
9. Bazzaev A.K., Tsopanov I.D. Locally one-dimensional difference schemes for the fractional order diffusion equation with a fractional derivative in the lower terms // Sib. electron. math. izv. 2015. Volume 12. pp. 80-91. (in Russian)
10. Bazzaev A.K., Shkhanukov-Lafishev M.H. A locally one-dimensional scheme for a fractional order diffusion equation with boundary conditions of the third kind. matem. and math. phys. 2010. Vol. 50. No. 7. pp.1200–1208. (in Russian)
11. Beibalaev V.D. Mathematical model of heat transfer in media with fractal structure // Matem. modeling. 2009. Vol. 21. No. 5. pp. 55-62. (in Russian)
12. Beibalaev V.D., Yakubov A.Z. Analysis of the difference scheme of the analog of the wave equation with the fractional differentiation operator // Vestn. Sam.state. tech. un-ta. Ser. Phys.-mat. sciences. 2014. Issue 1(34). pp. 125-133. (in Russian)
13. Beibalaev V.D., Shabanova M.R., Numerical method for solving the initial boundary value problem for a two-dimensional heat equation with fractional derivatives // Vestn. Sam. gos. tech. un-ta. Ser. Phys.-mat. sciences. 2010. Issue 5(21). pp. 244-251. (in Russian)
14. Khuzhayorov B.Kh., Djiyanov T.O., Yuldashev T.R. Anomalous Nonisothermal Transfer of a Substance in an Inhomogeneous Porous Medium // J. Eng. Phys. Thermophys. 2019, 92. Pp. 104–113.

15. Khuzhayorov B., Usmonov A., Nik Long NMA, Fayziev B. Anomalous solute transport in a cylindrical two-zone medium with fractal structure // Applied Sciences (Switzerland), 10, 2020, 5349.
16. Afonin A. A. Linear two-dimensional models of geofiltration in porous media with fractal structure // Izvestiya SFU. Technical sciences. Section II. Mathematical modeling of ecosystems. C.150-154. (in Russian)
17. Bulavatsky V. M. Fractional-difference mathematical models of dynamics of uneven geomigration processes and tasks with non-local boundary conditions // Dopovidi of the Nazi Academy of Sciences of Ukraine. Informatics is cybernetics. 2012, №12. C.31-40. (in Russian)
18. Serbina L.I., Vendina A.A. An asymptotic method for solving the fractional equation of migration of groundwater pollution // Vestn. SamGU. Natural Science ser. 2011. Issue 5(86). pp. 104-108. (in Russian)
19. Parovik R.I., Shevtsov B.M. Radon transport processes in media with fractal structure // Mathematical modeling 2009. Vol. 21. No. 8. pp. 30-36
20. Barenblatt G.I., Entov V.M., Ryzhik V.M. Theory of Fluid Flows Through Natural Rocks. Kluwer Academic Publisher, 1990. – 395 pp.