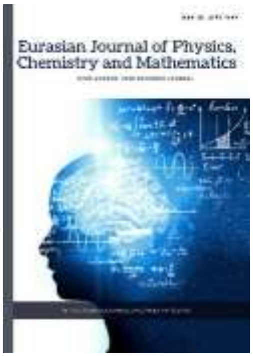

Study the Nuclear Structure of 42Ca and 42Ti Mirror Nuclei by Applying the Nuclear Shell Model Using Modified Surface Delta Interaction

Ali K. Hasan[1]
[1,2]Department of physics, College of Education for Girls, University of Kufa, Najaf, Iraq.
[1]E-mail: alikh.alsinayyid@uokufa.edu.iq

Shaima M. Saleh[2]
[1,2]Department of physics, College of Education for Girls, University of Kufa, Najaf, Iraq.
[2]E-mail: shaymaamahdi2023@gmail.com

ABSTRACT

The nuclear shell model was applied using the modified surface delta interaction MSDI to study the nuclear structure of the mirror nuclei (^{42}Ca, and ^{42}Ti) in 1f7/2 model space, where each of them contains two nucleons outside the closed core ^{40}Ca by calculating angular momentum J, parity π, the energy levels E_x and the reduced transition probability of electric quadrupole B(E2) and through the results of energy levels calculated from the angular momentum and the allowed parity, it was found that there is a perfect agreement for the values of the ground state and an acceptable agreement for the excited state with the experimental values.

Keywords: Nuclear shell model, Excitation energies E_x, 1f7/2 model space, Reduced transition probability of electric quadrupole B(E2), Modified surface delta interaction MSDI

1. Introduction

The nuclear shell structure is basis of particles multiplicity theory in the nuclei, as the nucleus is divided into a closed core and valence nucleons (particles or holes) that occupy the outer orbits. One of the important advantages of the nuclear shell model is the periodic and continuous change of nuclear properties [1-3]. According to the nuclear shell model, the nucleons are distributed in separate shells in the nucleus, and the capacity of each shell is determined by the largest number of nucleons, in line with the rules of quantum mechanics and Pauli's principle of exception, as each nucleon moves in its own orbit, independent of any other nucleons, this is due to the reason why the shell model is called the independent particle model [4-6].

Bartlett proposed the independent particle model in 1932 [7], in which he assumed that the nuclei that possess the stable structure such as the nuclei (^{4}He and ^{16}O) are the result of arrangement of nucleons in shells similar to those observed for electrons in atomic physics. And during 1934 this idea was expanded to include heavy nuclei by Elsasser when he presented evidence for his preoccupation with neutrons or protons in the numbers represented by {50, 82 and 126} [8], but at that time the available experimental data were very limited to support the nuclear theory, which led to the rejection of shell model has been severely criticized by many scientists for several years. In 1948, Mayer presented convincing evidence using measurements of nuclear binding energies and the abundance of isotopes of nuclei filled with neutrons or protons in the numbers {20, 50, 82, and 126}, which pushed the nuclear shell model again to nuclear physics research [9]. Despite the success of the shell model in the low orbits, it was unable to achieve this in the higher orbits when using a well nuclear potential, and after scattering studies of neutrons and protons proved that the nucleons are arranged in sequential energy levels according to the quantum numbers [10]. According to the nuclear shell model, the angular momentum and parities of the nuclear energy levels were predicted, as each level was named by the radial quantum number (n), the value of the orbital angular momentum (ℓ), and value of the angular momentum of the single particle (j). This type of level is called the single particle level in the nuclear shell model.

The first improved step in shell model is to choose more realistic potentials, as the basic premise of the shell model is that the effect of interactions between nucleons can be represented by the single particle potential. over the course of a single particle. But according to Pauli's exclusion principle in which nucleons are restricted to only a limited number of allowed orbitals, the typical shell model potential can be expressed [11]:

$$V(r) = -\frac{V_0}{1 + e^{(r-R)/a}} \tag{1}$$

The Wood-Saxon potential parameters in Eq. (1) R, a, V_0 and r are the average radius of nucleus, diffuseness parameters, potential depth and radial coordinate respectively where the typical values of the parameters are $V_0 \approx 57\,MeV$, $R \approx 1.25\sqrt[3]{A}$ fm and $a \approx 0.65$ fm. Whereas, the eigenvalues of the total energy E_n in the presence of a Wood-Saxon potential is according to the following mathematical expression:

$$E_n = \hbar\omega\left(2n + \ell - \frac{1}{2}\right) \tag{2}$$

where $\hbar$ is reduced Planck's constant, ω is the natural angular frequency of the simple harmonic oscillator, n is principal quantum number and ℓ is orbital angular momentum number and takes the values $\ell = 0,1,2,3,\dots$, in addition, we can add corrections to the depth of the well arising from the energy of symmetry for an unequal numbers of neutrons and protons, with the ability of the neutron to interact with the proton in more than one way, including a neutron with a neutron (n-n) and a proton with a proton (p-p), which represents Coulomb repulsion and the last interaction is a proton and a neutron (n-p) and therefore the (n-p) force is stronger than (n-n) and (p-p) [9].

The shell model relies on another justification, in particular using modified surface delta interaction (MSDI). Surface delta reaction (SDI) is known to give a good description of energy spectra, although it results in a number of methodological inconsistencies with regard to the generation of a number of experimental levels. This discrepancy is particularly noticeable in the nuclear binding energies and it has been shown that this description can be greatly improved upon changing the position of the central point of energy with isotope spin. The modification of SDI interaction led to the appearance of the surface delta interaction [12-16]:

2. Theory

$$\begin{aligned}\langle j_a j_b | V_{MSDI}(\vec{r_1},\vec{r_2}) | j_c j_d \rangle_{J,T} = -\frac{A_T}{2(2J+1)}\left\{\frac{(2j_a+1)(2j_b+1)(2j_c+1)(2j_d+1)}{(1+\delta_{ab})(1+\delta_{cd})}\right\}^{\frac{1}{2}} \\ \times\left[(-1)^{\ell_a+\ell_b+j_c+j_d}\left\langle j_a\frac{1}{2}j_b-\frac{1}{2}\middle|J0\right\rangle\left\langle j_c\frac{1}{2}j_d-\frac{1}{2}\middle|J0\right\rangle\{1-(-1)^{\ell_c+\ell_d+J+T}\}\right. \\ \left.-\left\langle j_a\frac{1}{2}j_b\frac{1}{2}\middle|J1\right\rangle\left\langle j_c\frac{1}{2}j_d\frac{1}{2}\middle|J1\right\rangle\{1+(-1)^T\}\right]+\{[2T(T+1)-3]B+C\}\delta_{ac}\delta_{bd}\end{aligned} \tag{3}$$

Whereas A_T, B, C represent the interaction strength and are determined from the experimental spectrum and given according to the following relations [17]:

$$A_T \approx B \approx \frac{25}{A}\ MeV \tag{4}$$

We can get the values of the elements of the matrix H_{ij} from the following equation [12,16]:

$$H_{ij} = (\varepsilon_i + \varepsilon_j)\delta_{ij} + \langle j_a j_b | V_{MSDI}(\vec{r_1}, \vec{r_2}) | j_c j_d \rangle_{J,T} \tag{5}$$

where ε_i and ε_j are the single particle energies of the two nucleons outside the closed core, δ_{ij} is a Kronecker Delta and energy matrix elements H_{ij} can be expressed by the following [17]:

$$\begin{bmatrix} H_{11} & H_{12} & \cdots & H_{1n} \\ H_{21} & H_{22} & \cdots & H_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ H_{n1} & H_{n2} & \cdots & H_{nn} \end{bmatrix} \begin{bmatrix} a_{1p} \\ a_{2p} \\ \vdots \\ a_{np} \end{bmatrix} = E_p \begin{bmatrix} a_{1p} \\ a_{2p} \\ \vdots \\ a_{np} \end{bmatrix} \tag{6}$$

and through the properties of the Hermitian operator H, which is characterized as being real for the matrix of elements, we get the following mathematical formula [18]:

$$H_{ij} = H_{ji} \tag{7}$$

since the H_{ij} matrix is symmetric, the only mathematically acceptable case is that value of the resulting determinant is equal to zero [18]:

$$\begin{vmatrix} H_{11} - E_p & H_{12} & \cdots & H_{1n} \\ H_{21} & H_{22} - E_p & \cdots & H_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ H_{n1} & H_{n2} & \cdots & H_{nn} - E_p \end{vmatrix} = 0 \tag{8}$$

by solving Eq. (8), we get (n) from the roots for the values of E_p. If these roots are different from each other, we have (n) energy levels after correction, and this corresponds to (n) of the eigenstate. The reduced transition potential is related to a single-particle electromagnetic multipolar operator $\hat{O}^{\sigma\lambda}$ [19]:

$$B(\sigma\lambda\,;\, J_i \rightarrow J_f) = \frac{1}{2J_i + 1} \left| \langle J_f || \hat{O}^{\sigma\lambda} || J_i \rangle \right|^2 \tag{9}$$

the operator $\hat{O}^{\sigma\lambda}$ is of order λ, the reduced transition matrix elements of the one-particle operator can be decomposed into the elements of the single-particle transition matrix [20]:

$$\langle J_f || \hat{O}^{\sigma\lambda} || J_i \rangle = \sum_{j_4, j_2} c(j_3, j_4, J_f, j_1, j_2, J_i) \langle j_3, j_4 || \hat{O}^{\sigma\lambda} || j_1, j_2 \rangle \tag{10}$$

Where $c(j_3, j_4, J_f, j_1, j_2, J_i)$ represent the linear correlation coefficients of the wave functions. The reverse (absorbent) transport can be expressed by the equation [21]:

$$B(\sigma\lambda\,;\, J_f \rightarrow J_i) = \frac{2J_i + 1}{2J_f + 1} B(\sigma\lambda\,;\, J_i \rightarrow J_f) \tag{11}$$

The transition probability of an electric quadrupole is given by [22]:

$$B(E2\,;\,J_i \longrightarrow J_f) = \frac{1}{2J_i+1}\sum_{M_i,\mu,M_f} \left|\langle J_i M_i|\hat{O}_{2\mu}|J_f M_f\rangle\right|^2 \tag{12}$$

whereas, M_i and M_f represent the projections of total angular momentum for initial and final states, respectively, while $\hat{O}_{2\mu}$ represents the transition operator of electric quadrupole and is given by [23]:

$$\hat{O}_{2\mu} = \tilde{e}\sum_i r_i^2\, Y_{2\mu}(\theta_i,\phi_i) \tag{13}$$

where $\tilde{e}$ represents the effective charge of nucleons outside the closed core and $Y_{2\mu}(\theta_i,\phi_i)$ represents the spherical harmonic function where (r_i,θ_i,ϕ_i) represents the coordinates of the spherical system, and $|J_i M_i\rangle$ and $|J_f M_f\rangle$ represent The initial and final transition states, respectively.

The reduced transition probability of electric quadrupole must be reduced by the total angular momentum projections using the Wikner-Eckart theorem, so it is [24]:

$$B(E2\,;\,J_i \longrightarrow J_f) = \frac{2J_f+1}{2J_i+1}\left|\langle J_i||\hat{O}_2||J_f\rangle\right|^2 \tag{14}$$

where $\langle J_i||\hat{O}_2||J_f\rangle$ represents the elements of the reduced transition matrix which is calculated from the sum of the elements of the reduced transition matrix for each allowed state in the initial level J_i and the final level J_f.

3. Results and discussion

The theoretical calculations of energy levels E_x and the reduced transition probability of electric quadrupole $B(E2)$ were carried out using the nuclear shell model and the modified surface delta interaction MSDI of the mirror nuclei (^{42}Ca, and ^{42}Ti) and within 1f7/2 model space that contains two nucleons outside the closed magic core represented by ^{40}Ca nucleus.

3.1 Energy levels

Eq. (5) was relied on to calculate the Hamiltonian matrix elements for two particles (two neutrons or two protons) and Eq. (3) for the modified surface delta interaction. The values of the coefficients were determined by the values ($A_T = 0.76\,MeV, B = 0.58\,MeV,$ $and\ C = -0.66\,MeV$) and through calculating the Hamiltonian matrix elements based on the values of the matrix elements for two particles as well as the energies of the particles within the 1f7/2 model space. The theoretical results of energy levels have been compared with the available experimental values [25] for each nucleus in Tables (1) and (2) and as follows:

Table (1): Comparison of the calculations of the excitation energy for the ^{42}Ca core with the values of the available practical energies.

Theoretical results		**Experimental values [25]**	
J^{π}	$E_x(MeV)$	$E_x(MeV)$	J^{π}
0^+	0.0000	0.0000	0^+
2^+	2.3140	1.5240	2^+
4^+	2.6820	2.7520	4^+
6^+	2.8606	3.1892	6^+

From Table (1), we note a good agreement between experimental value of 0^+ level and the theoretical value of 0^+ level, and we note an acceptable agreement between the experimental values of 2^+, 4^+, and 6^+ levels with the theoretical values of 2^+, 4^+, and 6^+ levels, respectively.

Table (2): Comparison of the calculations of the irritation energy for the ^{42}Ti core with the values of the available practical energies.

Theoretical results		Experimental values [25]	
J^π	$E_x(MeV)$	$E_x(MeV)$	J^π
0^+	0.0000	0.0000	0^+
2^+	2.1028	1.5546	2^+
4^+	2.4374	2.6700	4^+
6^+	2.6101	3.0400	6^+

From Table (2), we note a good agreement between experimental value of 0^+ level and the theoretical value of 0^+ level, and we note an acceptable agreement between the experimental values of 2^+, 4^+, and 6^+ levels with the theoretical values of 2^+, 4^+, and 6^+ levels, respectively.

3.2 The reduced transition probability of electric quadrupole

The reduced transition probability of electric quadrupole B(E2) was calculated for calcium isotope ^{42}Ca and the titanium isotope ^{42}Ti for the permissible transitions, where the eigenvectors of the energy matrix elements were calculated from Eq. (6) within the modified surface delta interaction, and then the calculations for the reduced transition probability of electric quadrupole B(E2) were derived. From Eq. (14), the calculations have been included in Tables (3) and (4) respectively, which show the theoretical values of B(E2) and compare them with the available experimental values for two nuclei.

Table (3): Comparison of B(E2) theoretical calculations for ^{42}Ca nucleus with available experimental values.

$J_i^\pi \longrightarrow J_f^\pi$	Theoretical results e^2fm^4	Experimental values e^2fm^4 [25]
$2^+_1 \longrightarrow 0^+_1$	82.31109	82.31179
$4^+_1 \longrightarrow 2^+_1$	74.20631	82.12514
$6^+_1 \longrightarrow 4^+_1$	30.51064	37.41447

Table (4): Comparison of B(E2) theoretical calculations for ^{42}Ti nucleus with available experimental values.

$J_i^\pi \longrightarrow J_f^\pi$	Theoretical results e^2fm^4	Experimental values e^2fm^4 [25]
$2^+_1 \longrightarrow 0^+_1$	138.57730	138.751856

$4^+{}_1 \longrightarrow 2^+{}_1$	74.20631	----
$6^+{}_1 \longrightarrow 4^+{}_1$	33.80681	27.75037

The comparison in Tables (3) and (4) between the theoretical results with the experimental values available at the time of conducting this study, we found a good agreement for value of $B\ (E2 : 2_1^+ \longrightarrow 0_1^+)$, and we note that the transition $B\ (E2 : 4_1^+ \ \longrightarrow 2_1^+)$ hasn't experimental value for ^{42}Ti nucleus.

4. Conclusions

From the study of the nuclear shell model of the nuclei ^{42}Ca and ^{42}Ti and through the results we obtained for excited nuclear energy levels and reduced transition probability of electric quadrupole using the modified surface delta reaction, it is clear that:

1- We found a great match for the values of energy levels between our theoretical calculations and experimental values, and small differences were due to the values of the coefficients A_T, B and C that were chosen.

2- The difference in values of the excitation energy levels in the two mirror nuclei studied is the difference in the type of particles outside the closed nuclear core, as the ^{42}Ca nucleus contains two neutrons outside the closed core represented by the ^{40}Ca nucleus, while the ^{42}Ti nucleus contains two protons outside the same closed core.

3- It was concluded that the nuclear shell model using modified surface delta interaction (MSDI) are a successful model and interaction for calculating energy levels and reduced transition probability of electric quadrupole within 1f7/2 model space in the current study.

References

1. Burger, "Nuclear Structure of Light Ca and Heavy Cr Isotopes", Ph.D. Thesis, Bonn university, Germany (2007).
2. W. E. Meyerhof, "Elements of Nuclear Physics", Mc Graw Hill book Company, London (1967).
3. S. Hilaire,"Level Densities, Lecture on Nuclear Data and Nuclear Reactors, Commissariat an Atomic Energy and Nuclear Physics", France (2000).
4. J.W. Negele and E. Vogt, "Advances in Nuclear Physics", Kluwer Academic Publishers, New York (2002).
5. T. Otsuka, A. Gadek, O. Sorlin, T. Suzuki and Y. Utsuno ,"Evolution of shell structure in exotic nuclei", Reviews of Modern Physics 92 (1), pp: 1-52 (2020).
6. E. Green, D. L. Sellin, and A. S. Zachor, "Analytic independent-particle model for atoms" Physical Review, 184 (1), pp: 1-9, (1969).
7. B. Wieckowski, "Periodic table of nuclides based on the nuclear shell model" Romanian Journal of Physics 64, pp: 1-19 (2019).
8. O. Sorlin, S. Lunardi, P.G. Bizzeti, C. Bucci, M. Chiari, A. Dainese, P. Di Nezza, R. Menegazzo, A. Nannini, C. Signorini and J. J. Valiente-Dobon, "Shell evolutions and nuclear forces", EPJ Web of Conferences 66, pp: 1-10 (2014).
9. M. G. Mayer, "On closed shells in nuclei", Physical Review 74 (3),pp:235-239 (1948).
10. R.B Bruce, P. Navratil and J. P. Vary,"Ab initio no core shell model", Progress in Particle and Nuclear Physics 69, pp:131-181, (2013).
11. J. Varma, R. C. Bhandari and D. R. S. Somayajulu," Fundamentals of Nuclear Physics", New Delhi, Bsngalore, India (2010).
12. Heusler, R.V. Jolos and P Von Brentano, " Excitation energies of particle-hole states in ^{208}Pb and the surface delta interaction ",Physics of Atomic Nuclei 76 (7), pp: 807-827 (2013).
13. K. Hasan, and F. M. Hussain, "Nuclear Structure Study of ^{62}Ni, ^{62}Zn and ^{62}Cu Nuclei by using SDI and MSDI Interactions", International Journal of Science and Research, 3(10), pp: 1450-1454 (2014).
14. D. N. Hameed, and A. K. Hasan, "Energy levels of isobaric nuclei (^{16}N, ^{16}F) within

the modified surface delta-interaction model", Ukrainian Journal of Physics, 63(7), pp: 579-579. (2018).
15. K. Hasan, and D. N. Hameed, "Energy Levels of ^{50}Ca Nucleus as a Function of the Classical Coupling Angle within MSDI", NeuroQuantology, 19(5), 61-68 (2021).
16. D. N. Hameed, and A. K. Hasan, "Determining the excitation energies of ^{68}Ni Nucleus a Function of the Coupling Angle, $\theta_{a,b}$ By Means of Modified Surface Delta-Interaction", Journal of Physics: Conference Series, 1963(1) pp: 1-9 (2021).
17. M .Waroquier and K. Heyde,"On quasiparticle interactions in the spherical N= 82 doubly even nuclei", Nuclear Physics A,164(1), pp:113-139 (1971).
18. R. Arvieu and S. A. Moszkowski, "Generalized seniority and the surface delta interaction", Physical Review 145 (3), pp: 830-837 (1966).
19. Aa. Bohr and B. R. Mottelson, "Nuclear Structure", Benjamin, New York, 1, pp: 379-389 (1969).
20. H. Grawe , "Shell Modelfrom a Practitioner's Point of View", In The Euroschool Lectures on Physics with Exotic Beams ,Springer, Berlin, Heidelberg 1, pp: 33-75 (2004).
21. L .Cáceres," Nuclear structure studies in the vicinity of the double-magic ^{132}Sn nucleus" , Doctoral dissertation, Universidad Autónoma de Madrid. (2008).
22. R. Edmonds, "Angular Momentum in Quantum Mechanics" ,Princeton University Press, Third printing (1974).
23. E. M. Raheem, "Study of the Static and Dynamic Nuclear Properties and Form Factors for Some Exotic Nuclei" Ph.D. Thesis, University of Baghdad (2015).
24. E. B. Dally, M. G. Croissiaux and B. Schweitz ,"Scattering of High-Energy Electrons by Nitrogen-14 and -15", Phys. Rev., C2(6), pp: 2057-2068 (1970).
25. Singh and J. A. Cameron, "Nuclear Data Sheets For A = 42", 92(1), pp: 1–145 (2001).