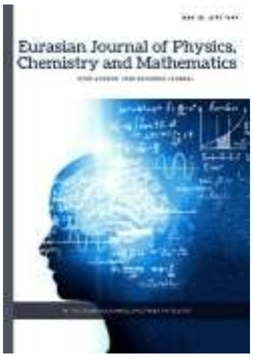

Auto-morphism of Finite Dimensional Leibniz Algebras Dimension (2) & (3)

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ABSTRACT

Discussing low-dimensional algebraic auto-morphism principles and looking at some new data to back them up are the goals of this study. These concepts have been revised to simplify computation for Leibniz algebra auto-morphism. The importance of these freshly published methods for determining auto-morphism is then illustrated by a number of fascinating findings and relevant examples.

Keywords: Leibniz Algebras; Auto-morphism; Finite Dimensiona

1. Introduction

There is 4 families of infinite simple lie algebra over C as the following :
A_n, where n >1 and B_n , where n $\geq$ 2, C_n, D_n where n $\geq 3,4$ respectively, Corresponding the groups: $L(n+1,C)$, $SO(2n+1,C)$, $Sp(2n,C)$, $SO(2n,C)$., G_2, F_4, E_6, E_7, E_8. denote five unique ones having dimensions of 14, 52, 78, 133, and 248 apiece. A Cartesian subalgebra (CSA), a certain maximum nilpotent sub algebra, unique up to congruency, is the essential structure block of the organization, as Chevalley proven much later. The constant (generalized) eigenvectors X are root vectors, the (generalized) Eigenspaces g is root spaces, and the (generalized) eigenvalues in the spectral decomposition of are specific linear forms on called Lie algebra encompasses more than Lie groups and differential geometry. Despite being a very complex algebraic structure with many different physics applications, we can study them as the tangent space to the identity of a Lie group. This area of physics uses Lie theory perhaps the least [10].

2 . Prefaces

Lie algebra includes considerably more than just Lie groups and differential geometry. They are a very complicated algebraic structure with many applications in physics, but we can think of them as the tangent space to the identity of a Lie group. This area of physics possibly employs the least amount of Lie theory [10].

Definition 2.1. A Field F is bilinear map and F-vector space by the Lie bracket $L \times L \longrightarrow L$, $(a,b) \longmapsto [a,b]$ are both components of Lie algebra on F. satisfying the following properties :

$[a,a]=0$, for all $a \in L$ (L1)

$[a,[b,c]]+[b,[c,a]]+[c,[a,b]]=0$ for each $a,b,c \in L$. (L2)

The bracket $[a, b]$ referred to the commutator of b and b and the Condition (L2) is the Jacobi identity. The Lie expression $[-,-]$ is bilinear, we require :

$$0 = [a+b, a+b]$$
$$= [a,a] + [b,b] + [a,b] + [b,a]$$
$$= [a, b] + [b, a].$$

Now (L1) denotes : $[a,b]$ = - $[b,a]$, for all $a, b \in L$ (L1')

If the field does not have typical 2, then placing $a = b$ in (L1') displays that (L1') that is mean (L1).

Definition 2.2. we said Leibniz algebra for the algebra $(L, [-,-])$ over the field F if : for all $a, b, c \in L$ is Leibniz identity $[a,[b,c]] = [[a,b],c] - [[a,c],b]$, then (2.1) clamps. for each $a \in L$, If $[a,a] = 0$ implies Lie algebra is called Leibniz algebra, with vice versa, all Lie algebra is Leibniz algebra when L^{ann} ideal $< [a,a]: a \in L >$, the factor algebra L/L^{ann} is also a Lie algebra. Consider a Leibniz algebra's resulting and minor central series as follows:

(i) $L^{(1)} = L,\ L^{(n+1)} = [L^{(n)}, L^{(n)}]\ where\ \ n > 1$;

(ii) $L^1 = L,\quad L^{n+1} = [L^n, L]\ \ where\ \ n > 1$.

Definition 2.3. A subspace $I \subseteq L$ is said to have a left (or right) ideal if it has $[a,x] \in I$ (resp.$[a,x] \in I$), for every possible value of $x \in I$ and $a \in L$; if I has both left and right ideals, it is said to have a 2-sided ideal, sub algebra H of Leibniz algebra is mentioned to as 2-sided ideal when $[L,H] \subseteq L$, $[H,L] \subseteq L$.. suppose H , K attend as L's dual ideals. The commutator-ideal for H & K, represented by [H,K] is 2-sided ideal of L enclosed in the comment $[k,h], [h,k],\ h \in H,\ k \in K$. . For exact Leibniz algebra L, now we can describe the minor principal and assume series to sequences with 2-sided ideals clear recursively. can be seen in [18], [19], [20]. $L^1 = L,\ L^{k-1} = [L^k, L],\ k \geq 1;$ $L^{[1]} = L,$ $L^{[s-1]} = [L^{[s]}, L^{[s]}],\ s \geq 1.$

3. Properties Of Auto-morphism Of Leibniz Algebras

This section demonstrates the lack of impact of a complex Leibniz algebra with limited dimensions and a non-degenerate derivation. Like in Lie example, the reverse of this avowal is untrue. The knowledge of a usually nilpotent Leibniz algebra, which is comparable to the Lie situation, is defined in [5], [6], [7], [8,] and [9].

Definition 3.1. we called right annihilator for the set $A_{nnR}(L) = \{a \in L | [L, a] = 0\} \in L(L)$ from Leibniz algebra L when $Ann_R(L)$ is an ideal of L.

Theorem 3.2. suppose the A be a (L.T) linear transformation to vector space V. Then V rots into the direct sumation of normal subspaces.

$V = V_{\lambda 1} \oplus V_{\lambda 2} \oplus \ldots \oplus V_{\lambda k}$ with reverence to A, where:

$V_{\lambda i} = \{x \in V: (A - \lambda_i I)k$, where $k \in N\}$ and λ_i , $1 \leq i \leq k$, are eigenvalues of A.

Theorem 3.3. let L be a Leibniz algebra then:

L is nilpotent $\Leftrightarrow$ R_x is nilpotent for any $x \in L$.

In the sense that H contains all of the solvable ideals of L where L is Leibniz algebra , let H be a maximal ideal. According to Ayupov and Omirov[4], the reality of a single maximal ideal, referred to as the fundamental of L, is implied by the fact that the sum of ideals is once more ideal. In a similar way. Suppose K be a maximum nilpotent ideal to Leibniz algebra L. According to Ayupov and Omirov[4], the fact that the summation of nil ideals also nilpotent ideal denotes the reality of solitary maximal nilpotent ideal, also known as the nilradical of L.

Definition 3.4. Let L be a Leibniz, $\sigma \in$ End(L). If $\sigma[a, b] = [\sigma(a), \sigma(b)]$ for all $a, b \in L$, and σ is bijective, then σ is called auto-morphism of Leibniz algebra L. We symbolize the auto-morphism group of Leibniz algebra as aut(L).

4 Application Of Low-Dimensional Leibniz Algebras

Let g be a function of morphism of a Leibniz algebra L to itself, and let

$$= \begin{vmatrix} A_{11} & A_{12} \ldots\ldots\ldots\ldots\ldots & A_{1n} \\ A_{21} & A_{22} \ldots\ldots\ldots\ldots\ldots. & A_{2n} \\ \vdots \ddots & & \\ \vdots & & \\ A_{n1} & A_{n2} \ldots\ldots\ldots\ldots\ldots. & A_{nn} \end{vmatrix} A$$

be the $a\ squre$ matrix and $[b_i, b_j]$ be the basis of L we defined

$g: L \times L \longrightarrow L$ as:

$(b_1, b_2, \ldots, b_n) \times$ $(A_{11}\ A_{12}\ \ldots\ldots.\ldots\ A_{2n}\ \vdots\ \vdots\ \ddots\ \vdots\ A_{n1} \ldots\ \ldots\ A_{nn}\ )$ = $(b_1', b_2', \ldots, b_n')$, such that:

$b_1' = A_{11}b_1 + A_{21}b_2 + \cdots + A_{n1}b_n$

$$b'_2 = A_{12}b_1 + A_{22}b_2 + \cdots + A_{n2}b_n$$
$$\vdots$$
$$b'_n = A_{1n}b_1 + A_{2n}b_2 + \cdots + A_{nn}b_n.$$

4.1. Two-Dimensional Leibniz Algebras

Loday [14] has provided a taxonomy of all two-dimensional Leibniz algebras. The following theorem provides a list of isomorphism classes.

Theorem 4.2. Any 2-d Leibniz algebra L is isomorphic to one from the following non-isomorphic Leibniz algebras

L: Abelian

Note: we will use some notations for the purpose of clarification and ease of delivery of the idea. For example: $[b_1, b_2] = \beta_{1,2}$ and $[b_3, b_2] = \beta_{3,2}$, and so on for the rest

$$L_1: \beta_{1,1} = b_2$$
$$L_2: \beta_{1,2} = b_2 = -\beta_{2,1}$$
$$L_3: \beta_{1,2} = b_1 = b_1, \beta_{1,2}$$

where $\{b_1, b_2\}$ is a basis of L. Based on the list above, there are two pure Leibniz algebras in dimension two. One of them has been disclosed by L_2 from above. The second is the two-dimensional Leibniz algebra L, which has the multiplication tables $[b_1, b_2] = b_1$, $[b_2, b_2] = b_1$. The derivations in this case have been given, and they are as follows.

Theorem 4.3 The auto-morphism group of 2-dimensional Leibniz algebra has the following form:

Table (1) Auto-morphism Group of 2-dimensional Leibniz algebra

Isomorphism class	**Aut** (G)	Dimension (D)
$L_1: \beta_{1,2} = b_2$	$\begin{pmatrix} \alpha_1 & 0 \\ \alpha_2 & \alpha_1^2 \end{pmatrix}$	3
$L_2: \beta_{1,2} = -\beta_{2,1} = b_1$	$\begin{pmatrix} 1 & 0 \\ 0 & \alpha_1 \end{pmatrix}$	1
$L_3: \beta_{1,2} = b_1, \beta_{2,2} = b_1$	$\begin{pmatrix} \alpha_1 + 1 & \alpha_1 \\ 0 & 1 \end{pmatrix}$	2

4.4. Tree-Dimensional Leibniz Algebras

We cite Casas [8], [16], [17], and [18] for the classification of all three-dimensional complicated Leibniz algebras. The following is a list of Lie algebra isomorphism classes.

Theorem 4.5 To get isomorphism, there is 3 parametric relations and 6 plain representatives of non-Lie complex Leibniz algebras of dimension (3):

$RR_1: \beta_{1,3} = -2. b_1,$ $\beta_{2,2} = b_1,$ $\beta_{3,2} = b_2,$ $\beta_{2,3} = -b_2.$

$RR_2: \beta_{1,3} = \alpha b_1,$ $\beta_{3,3} = b_1,$ $\beta_{3,2} = b_2,$ $\beta_{2,3} = -b_2\alpha \in C.$

$RR_3: \beta_{2,2} = b_1,$ $\beta_{3,3} = \alpha b_1,$ $\beta_{2,2} = \alpha b_1,$ $\alpha \in C /\{0\}.$

$RR_4: \beta_{1,2} = b_1,$ $\beta_{3,3} = b_1.$

$RR_5: \beta_{1,3} = b_2,$ $\beta_{2,3} = b_1.$

$RR_6: \beta_{1,3} = b_2,$ $\beta_{2,3} = \alpha b_1 + b_2, \alpha \in C.$

$RR_7: \beta_{1,3} = b_1,$ $\beta_{2,3} = b_2.$

$R R_8: \beta_{3,3} = b_1,$ $\beta_{1,3} = b_2.$

$RR_9: \beta_{3,3} = b_1,$ $\beta_{1,3} = b_1 + b_2.$

Theorem 4.6 The auto-morphism group of 3-dimensional non-Lie complex Leibniz algebras has the following form:

Table (2) Auto-morphism Group of 3-dimensional Leibniz algebra

Isomorphism class	Aut(G)	Dimension (D)
RR_1	$\begin{pmatrix} \alpha_1^2 & \alpha_1 & \frac{1}{2}\alpha_3^2 \\ 0 & \alpha_1 & \alpha_3 \\ 0 & 0 & 1 \end{pmatrix}$	4
RR_2	$\begin{pmatrix} -\alpha_1+1 & 0 & \alpha_1 \\ 0 & \alpha_2 & \alpha_3 \\ 0 & 0 & 1 \end{pmatrix}$	4
RR_3	$\begin{pmatrix} \alpha_1 & \alpha_2 & \alpha_3 \\ 0 & \alpha_1 & 0 \\ 0 & 0 & \alpha_1 \end{pmatrix}$	3
RR_4	$\begin{pmatrix} \alpha_3^2 & \alpha_1 & \alpha_2 \\ 0 & \alpha_3 & 0 \\ 0 & 0 & \alpha_3 \end{pmatrix}$	4
RR_5	$\begin{pmatrix} \alpha_1 & 0 & \alpha_2 \\ \alpha_3 & 0 & \alpha_4 \\ 0 & 0 & \alpha_5 \end{pmatrix}$	5
RR_6	$\begin{pmatrix} 0 & 0 & 0 \\ 0 & \alpha_1 & 0 \\ 0 & 0 & \alpha_2 \end{pmatrix}$	2
RR_7	$\begin{pmatrix} \alpha_1 & 0 & 0 \\ \alpha_2 & \alpha_3 & 0 \\ 0 & 0 & 1 \end{pmatrix}$	3
RR_8	$\begin{pmatrix} \alpha_2^2 & 0 & 0 \\ 0 & \alpha_2^3 & \alpha_1 \\ 0 & 0 & \alpha_2 \end{pmatrix}$	4
RR_9	$\begin{pmatrix} \alpha_1 & 0 & \alpha_1 \\ \alpha_1 & \alpha_2 & \alpha_3 \\ 0 & 0 & 1 \end{pmatrix}$	3

Corollary 4.7 The class RR_6 does not have automorphism since $A_{11} = 0$.

Theorem 4.8 Any complicated in three dimensions One of the Lie algebra isomorphism classes listed below includes Lie algebra L.

$RR_{10}: \beta_{1,2} = b_1$.

$RR_{11}: \beta_{1,3} = b_1 + b_2$, $\beta_{2,3} = b_2$.

$RR_{12}: \beta_{1,3} = 2b_1$, $\beta_{2,3} = -b_2$, $\beta_{1,2} = b_3$.

$RR_{13}: \beta_{1,3} = b_1$, $\beta_{2,3} = \alpha \in C/\{0\}$.

Theorem 4.9 The auto-morphism group of 3-dimensional complex Lie algebras has the following form:

Table (3) Auto-morphism Group of 3-dimensional Lie algebra

Isomorphism class	Aut (G)	Dimension (D)
RR_{10}	$\begin{pmatrix} \alpha_1 & 0 & 0 \\ 0 & \alpha_2 & \alpha_3 \\ 0 & \alpha_4 & \alpha_5 \end{pmatrix}$	5
RR_{11}	$\begin{pmatrix} \alpha_1 & 0 & \alpha_1 \\ \alpha_2 & \alpha_1 & -\alpha_1 \\ 0 & 0 & 1 \end{pmatrix}$	2
RR_{12}	$\begin{pmatrix} \alpha_1 & \alpha_2 & \alpha_3 \\ 0 & \alpha_4 & 0 \\ 0 & 0 & 1 \end{pmatrix}$	4

RR_{13}	$\begin{pmatrix} 0 & \alpha_1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & \alpha_2 \end{pmatrix}$	2

Corollary 4.10 The class RR_{13} does not has automorphism since $A_{11} = 0$.

5. Conclusion

The second and third dimensions have been proven and found through the use of the automorphism principles. A number of theories and axioms were used to find the dimensions referred to above, and the results were positive through research and were explained in tabular form.

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