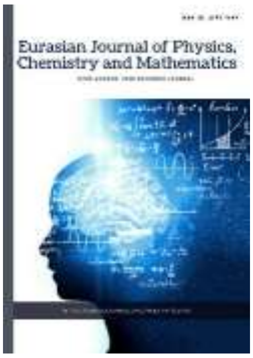

Using Real World Problems in Developing Students' Mathematical Skills

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ABSTRACT

In this article, the Diophantine equations, which can be solved in integers, and other vital and practical problems, aimed at making students interested in mathematics and developing their mathematical skills, are considered, and their solution is explained using interesting and simple methods.

Keywords: Diophantine equations, uncertain equation, Pythagorean theorem, Egyptian triangle, right triangle, Pythagorean triples, employee, salary, buyer, percentage, distance, time, speed.

Introduction: In the process of general secondary education, thorough knowledge, especially mathematical training, plays an important role in shaping students into qualified specialists. In many cases, it is observed that children who master mathematics well master other subjects easily. Also, mathematics, as a logical science, strengthens the thinking ability of students, develops their mind quickly, and as a result, creates a foundation for easy mastering of physics, engineering and other sciences. Studying practical and practical mathematical problems and methods of solving them in the classes of the mathematics club is the basis for the development of the students' mathematical abilities.

In this article, some interesting real-life problems that are solved in integers or Diophantine equations related to the development of students' mathematical abilities are considered, simple methods of solving them and a general formula that can be used for solving other problems of this type are presented. Also, problems related to the creation of equations and systems of equations, and several problems related to algebraic equations and geometric inequalities and their solution methods were considered in works [1-17].

According to historical data, it is known that the city of Alexandria, located on the shores of the Mediterranean Sea in Egypt, was one of the most developed cities of culture, trade and science in the ancient world. Alexandrian mathematician Diophantus lived in the 3rd century BC. In his book "Arithmetic", unknown equations and the problems related to them are studied. Diophantine equations are algebraic equations with two or more unknowns and whose coefficients are integers. The general theory of Diophantine equations of the first degree was created by the 17th century French mathematician Bachet.

Example 1. The store has nails in 16, 17 and 40 kg boxes. Can a shopkeeper deliver 100 kg of nails to a customer without opening the boxes?

Solution. We will try to solve the problem by making an equation as usual. Let's assume that the problem is solved, then there will be *x* boxes of 16 kg, *y* boxes of 17 kg, *z* boxes of 40 kg. Given the total of 100 kg of nails, we will create an equation of the form *16x+17y+40z=100.* Realizing that this equation is an indeterminate equation, it can be reasoned as follows:

1) There cannot be 3 boxes of 40 kg, because kg, which is more than necessary. It is not possible to have both, in this case it can be given only by opening one of the 20 kg boxes.

2) If one 40 kg box is taken, we will try to collect the remaining 60 kg through 16 and 17 kg boxes. If one 17 kg box is taken, 60-17=43 kg remains, and it cannot be taken with 16 kg boxes; if two 17 kg boxes are taken, we cannot collect them by 16 kg boxes; if 3 boxes of 17 kg are taken, then there will be 1 kg left, which cannot be filled without opening one of the boxes. As a result, a 40 kg box is not needed at all, and if the problem has a solution, then it is necessary to use 16 and 17 kg boxes.

Based on these considerations, this equation is formed. Now, taking into account that the number 100 is not divisible by 16 or 17, we should look at what happens if we subtract 100 from 100. In this case, if the difference is divisible by 16, the problem has a solution, if it is not divisible, the problem does not have a solution, that is, it is necessary to open one of the boxes in the store. And so,

the number is not divisible by 16; the number is not divisible by 16;

the number is not divisible by 16; number is divisible by 16,

and the matter is resolved. That is, it is necessary to give 4 boxes of 17 kg and 2 boxes of 16 kg. This solution is unique, that is, there are no other solutions.

There are many similar issues and many of them are practical in nature. Equations corresponding to them can be not only of the first degree $ax+by=c$. (where a,b,c are integers), but also of any other degree. Such equations are also called indeterminate equations, and special methods for solving them have been created in the number theory section of modern mathematics.

For independent solution, we present to the students the following problem related to uncertain equations.

Example 2. The boy had 50,000 soums, and he wanted to buy postage stamps with this money. There are stamps worth 4,000 soums and 3,000 soums in the press shop, but the seller does not have any small change. Help the boy and the newsagent to solve this problem.

Due to the simplicity of Diophantine equations, i.e., the life processes in problems leading to uncertain equations, and the fact that they are not based on any general theory, people were able to solve such problems in the same way as we solved the problems before Diophantus. We know that general theories can never be created in a vacuum. First of all, specific issues arise, and only then will people be found who understand that it is possible to switch to general methods and methods in such issues. For example, we will touch on another practical problem known to us regarding uncertain equations.

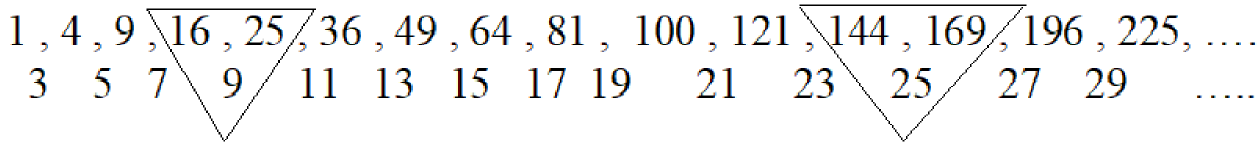
Example 3. Show that if the sides of a triangle are proportional to the numbers 3, 4, and 5, then such a triangle is a right triangle.

Solution. Since there were no optical instruments in ancient times, this fact was used in the construction of houses, palaces and other giant pyramids. With the help of ropes and stakes, they made a right-angled triangle with sides of 3, 4 and 5 units, and this triangle was also called the Egyptian triangle. It is known that such a construction is not a mistake because of the opposite theorem to the Pythagorean theorem: if the sum of the squares of two sides of a triangle is equal to the square of the third side, then such a triangle is a right-angled triangle.

In fact, in other words, the numbers 3, 4 and 5 are the roots of the equation.

A natural question arises: are there no other integer solutions to this equation? It is not difficult to understand that numbers 5, 12 and 13 are also solutions for this equation. But are there any more such threes? And is it possible to take an arbitrary one of these numbers and show two?

One of the ways to solve the equation $x^2+y^2=z^2$ in integers turned out to be simple enough. We write the squares of natural numbers (called "square numbers") in a row, separated by commas. We write down the difference of consecutive squares under each comma.



As a result, there are also square numbers in the bottom line, and we see that the first one is , and above it is . These are the 3, 4 and 5 triplets we are familiar with. If we take the next square number in the bottom line, the numbers and in the top line correspond to it. From this we make the second triple number 5, 12 and 13. If we continue the row of squares and calculate the corresponding differences, then in this row we find , and this number corresponds to the numbers and in the upper row. Indeed, and the numbers 7, 24 and 25 are third triplets.

Thus, the following statement can be made: **Any odd number is equal to the difference of two consecutive squares.**

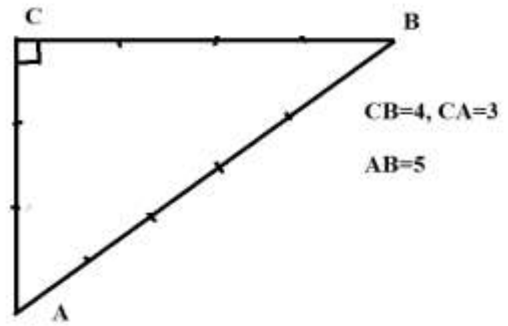
These three numbers can be found easily and quickly by formulas. If *x*- is an odd number, then it is possible to check that $y=\frac{x^2-1}{2}$ and $z=\frac{x^2+1}{2}$ and the equality $x^2+y^2=z^2$ is fulfilled. That is, the numbers found according to this rule satisfy the uncertain equation we are interested in. This equation is called "Pythagorean equation", and its solutions are called "Pythagorean triples". According to this rule, it is possible to form the above three known to us:

1). If *x=3*, then $y=\frac{3^2-1}{2}=4$ and $z=\frac{3^2+1}{2}=5$ are the first Pythagorean triples,

2). If *x=5*, then $y=\frac{5^2-1}{2}=12$ and $z=\frac{5^2+1}{2}=13$ is the second Pythagorean triple,

We don't know the rest yet, but the odd number after 7 is *x=9*, so $y=\frac{9^2-1}{2}=40$ and $z=\frac{9^2+1}{2}=41$. We will check. $9^2+40^2=41^2$ hence 81+1600=1681.

So 9, 40 and 41 are the next Pythagorean triples.



Now we look for and establish a rule for finding not only some Pythagorean triples, but all Pythagorean triples. We write the mathematical expression $x^2+y^2=z^2$ of the Pythagorean theorem as follows:

$$x^2=z^2-y^2 \quad \text{or}$$

$$x^2=(z-y)(z+y)$$

This equality means that x splits into two unequal multipliers *z+y* and *z-y.* We define them in such a way that the following system is formed as a result:

$$\begin{cases} z+y=2a^2 \\ z-y=2b^2 \end{cases}.$$

By removing this system We make $z=a^2+b^2$, $y=a^2-b^2$ and $x=2ab$ (*a*>*b*).

It follows that the smallest value of *b*- can only be 1, then the smallest value of *a*- is 2. Let's calculate *x, y* and *z*. *z=5, y=3* and *x=4* are formed.

This is the known "Egyptian triangle". Now, according to the above, we will make the following table:

b \ a	2	3	4	5	6
1	3.4.5	6.8.10	8.15.17	10.24.36	12.35.37
2	-	5.12.13	12.16.20	20.21.29	24.32.40
3	-	-	7.24.25	16.30.34	27.36.45
4	-	-	-	9.40.41	20.48.52

The table can be expanded to the right and down. The main thing is that the equation is solved and we know how to calculate all possible integer values of the lengths of the sides of right triangles.

Example 4. The buyer has 5 million soums. He bought a total of 100 cows, sheep and goats in the market. If one goat costs 10,000 soums, one sheep costs 100,000 soums, and one cow costs 500,000 soums, how much did the buyer get for cows, sheep, and goats?

Solution. Let the buyer buy x goats, y sheep and z cows. In that case, according to the condition of the matter

x+y+z=100 and *10000x+100000y+500000z= 5000000,*

x =100-y-z and *x+10y+50z=500* or *100-y-z+10y +50z=500,*

9y+49z=400. From this, *y=(400-49z)/9.*

Now we find the values of *y* by assigning natural values to the unknown *z*:

y=39 and *x=100-39-1=60* at *z=1; z =2, y =302/9* which is not an integer;

y =253/9 at *z =3, y =204/9* at *z =4; y = 155/9* at *z =5;*

y = 106 at *z = 6, y = 57/9* at *z = 7, y = 8/9* at *z = 8*

and *z* -variable cannot accept values greater than 8, because

y =400-441<0 at *z* =9.

So, *x* = 60 goats, *y* = 39 sheep, *z* = 1 cow.

Example 5. If the worker's salary was increased by 20% first and then by another 20%, by what percentage did his salary increase?

Solution: Let the worker's salary be equal to *x*. After his salary increases by 20%, the worker will receive a salary of *x+0.2=1.2x*. After the second increase of 20%, he will be paid that amount. So, his salary will increase by *1.44x –x = 0.44x*, i.e. by 44%.

Example 6. Father and son wanted to measure the distance between two trees by steps, and both of them simultaneously stepped from one tree to the other. The step of the father is 70 cm, and the step of the son is 56 cm. If it is known that their steps fell 10 times, find the distance between the trees.

Solution: The steps of father and son are 70 cm and 56 cm. We divide these numbers into prime multipliers: $70 = 2\cdot 5\cdot 7\,;56 = 2\cdot 7\cdot 4$. 1). Least common multiple LCM(70;56)=70 4=280. Every 280 cm, the steps of the father and son overlap.

2) 280 10=2800 cm or 2800 cm = 28 m.

Example 7. Two cars left cities A and B towards each other: "Nexia" from A at 7.20 am and "Damas" from city B at 7.00 am. If "Nexia" covers the entire road from A to B in 2 hours and 42 minutes, "Damas" in 3 hours and 36 minutes, then at what time did the cars meet?

Solution: 1) "Nexia" covers $1 : 2\frac{7}{10} = \frac{10}{27}$ part of the road from A to B in 1 hour; 2) "Damas" covers $1 : 3\frac{3}{5} = \frac{5}{18}$ part of the road from A to B in 1 hour; 3) Both of them together travel $\frac{10}{27}+\frac{5}{18}=\frac{35}{54}$ of the way from A to B in 1 hour; 4) "Damas" covers $\frac{5}{18}\cdot\frac{1}{3}=\frac{5}{54}$ of the way from A to B in 20 minutes;

5) Both cars covered 1-$\frac{1}{54}=\frac{49}{54}$ of the entire road from 7:20 a.m. to the time of meeting;

6) $\frac{49}{54}\div\frac{35}{54}=1{,}4$ clock. The cars met 1.4 hours after "Nexia" set off, that is, at 8:44 a.m.

Example 8. Several workers completed the work in 14 days. If they were more than 4 people and each of them worked 1 extra hour a day, the work would have been completed in 10 days. If there were more than 6 people and each of them worked 1 more hour a day, this work would have been completed in 7 days. How many workers were there and how many hours did each of them work per day?

Solution: let the number of *x*-workers be *t* - the number of working hours of one worker per day. Then, according to the conditions of the problem, the following system can be created:

$$\begin{cases} x\cdot t\cdot 14=(x+4)\cdot(t+1)\cdot 10 \\ x\cdot t\cdot 14=(x+6)\cdot(t+2)\cdot 7 \end{cases}$$

$$\Rightarrow \begin{cases} t=\frac{5x+20}{2x-20} \end{cases}$$

$$xt-2x-10t-20=0$$

$$\Rightarrow \begin{cases} t=\frac{5x+20}{2x-20} \end{cases}$$

$$x^2-30x+200=0$$

$$\Rightarrow \begin{cases} x_1=10 \\ x_2=20 \\ t=\frac{5x+20}{2x-20} \end{cases} \Rightarrow \begin{cases} t=6 \\ x=20 \end{cases}.$$

Example 9. The road from A to B is 3 km uphill, 6 km downhill and 12 km flat. The motorcyclist covered the entire road from A to B in 1 hour and 7 minutes. On the way back, the distance from B to A was covered in 1 hour and 16 minutes. If the motorcyclist's speed is 18 km/h on a flat surface, determine his speed when climbing a mountain?

Solution: Let the speed of the motorcyclist up the mountain be *x*- km/s and the speed of the descent *y*- km/s, then the motorcyclist spends $\left(\frac{3}{x}+\frac{12}{x}+\frac{6}{y}\right)$ hours on the road from A to B, which is the condition of the problem is equal to $1\frac{7}{60}$ hours, so $\frac{3}{x}+\frac{6}{y}+\frac{12}{18}=1\frac{7}{60}$. Returning from B to A, the motorcyclist spent $\left(\frac{12}{18}+\frac{6}{x}+\frac{3}{y}\right)$ hours on the road, which is $1\frac{14}{15}$ hours, i.e. $\frac{12}{18}+\frac{6}{x}+\frac{3}{y}=1\frac{4}{15}$.

The result is this $\begin{cases} \frac{3}{x}+\frac{6}{y}+\frac{12}{18}=1\frac{7}{60} \\ \frac{12}{18}+\frac{6}{x}+\frac{3}{y}=1\frac{4}{15} \end{cases}$ we will have a system of equations. Solving the system, we find *x=12* km, *y=30* km, that is, the speed of a motorcyclist is 12 km/s when climbing a mountain, and 30 km/s when descending a mountain.

Conclusion

1. Diophantus equations are of great theoretical and practical importance, and many practical and economic problems are solved by these equations. Therefore, in the following years, such equations and the problems solved by them were included in special school programs and in the series of Olympiad problems. From this point of view, the study of Diophantine equations is one of the relevant issues even today, and solving them requires special attention, skills and knowledge from students.

2. Knowing how to solve practical problems like the one above will help students to develop mathematical abilities and skills necessary for solving life problems.

3. That is why we recommend teaching Diophantine's equations and other practical problems to gifted students in general education schools. After all, human talent and ability is a divine blessing. This ability should be discovered, honed and serve the country.

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