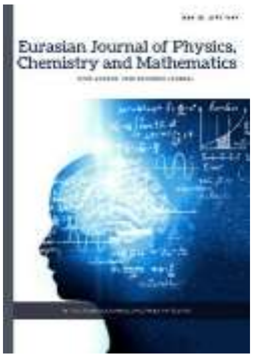

Applications of Ordinary Differential Equations to Problems of Physics and Biology in the Geogebra Program

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ABSTRACT

The article discusses the application of the GeoGebra system of dynamic mathematics for solving ordinary differential equations underlying many physical and biological processes

Keywords: computer modeling, ordinary differential equations, GeoGebra

Introduction

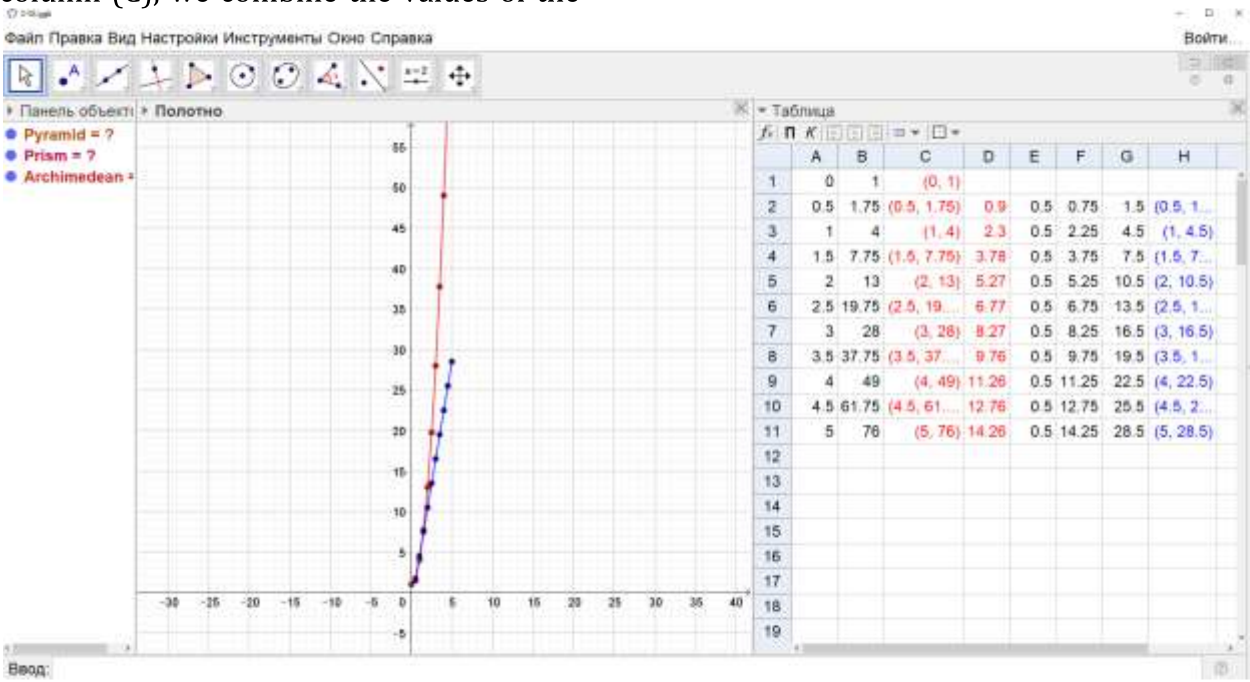
As part of the April profile program for schoolchildren in mathematics of the ANO "Kazan Open University of Talents 2.0", we have prepared a number of topics aimed at popularizing complex sections of mathematics. The section of ordinary differential equations (ODES) was chosen as one of such sections. The program is aimed at high school students and is useful in classes with a natural science and physics and mathematics profile. The purpose of the module of introduction to the ODE is to give students an idea of differential equations and their applications as the most important computing tool of classical physics, geometry, biology. This idea is well illustrated by the phrase of one of the creators of the mathematical analysis Isaac Newton: "The laws of nature are expressed by differential equations". The content of the module corresponds to the current trends in the development of the school mathematics course, the ideas of the development of interdisciplinary connections, deepening and expanding students' knowledge. This module gives students the opportunity to get acquainted with numerical methods for solving ODES using modern digital technologies.

The GeoGebra system of dynamic mathematics was chosen as a tool for composing, solving ODES and visualizing results [1]. This program is open, cross-platform and has the necessary mathematical functionality to solve computer modeling problems.

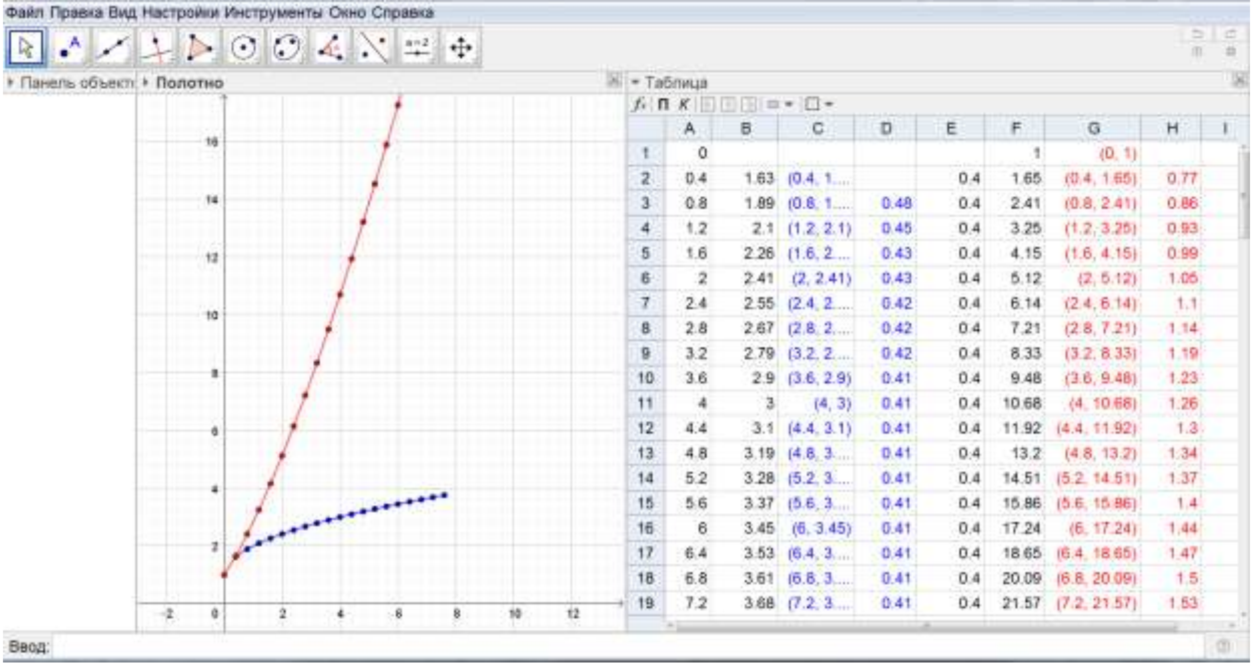
Computer modeling in the GeoGebra program

The GeoGebra program contains a number of convenient tools that allow to illustrate working with functions, for example, the ability to work with a tabular representation of functions. Numerical data can be entered into the table and calculations can be performed by referring to the contents of cells using absolute or relative addresses (similar to actions in Microsoft Excel).

Let's consider the sequence of actions for plotting a given function and a graph of a derived function. In the first column (A) of the table, we enter the values of the function argument for which calculations will be performed. It is enough to enter the necessary values in the first two cells, simultaneously select them and "stretch" the desired number of cells for the lower right corner of the selection. Next, in the second column (B), we place the values of the function using the absolute addresses of the cells of the argument. In the next column (C), we combine the values of the first two columns into the coordinates of the points lying on the graph of the function. And using these coordinates, we connect the segments in pairs in the next column (D). To visualize the process of differentiating the function, we calculate the increments of the argument and the function itself in the cells of the first and second columns, put the obtained values in the next two columns (E, F). Next, we find the ratio of the increment of the function to the increment of the argument (column G). Find the coordinates of the points of the derivative of the function and connect them in pairs by segments (columns H, I).



The samples of the GeoGebra program and the tabular method of specifying functions can be used to solve the Cauchy problem by numerical methods. For simplicity, consider the Euler method for solving ordinary differential equations. The following figure shows the program window, where the primitive of this function is approximately found in a tabular way and the corresponding graphs are constructed.

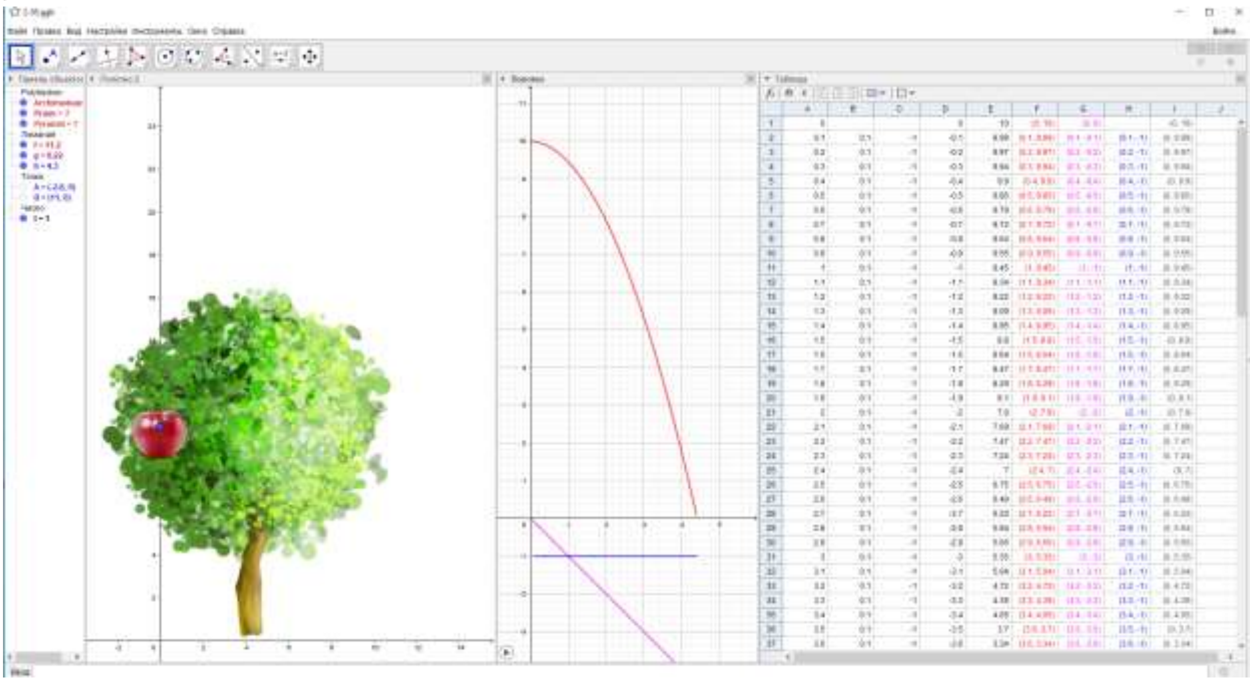


	A	B	C	D	E	F	G	H
1	0					1	(0, 1)	
2	0.4	1.63	(0.4, 1....		0.4	1.65	(0.4, 1.65)	0.77
3	0.8	1.89	(0.8, 1....	0.48	0.4	2.41	(0.8, 2.41)	0.86
4	1.2	2.1	(1.2, 2.1)	0.45	0.4	3.25	(1.2, 3.25)	0.93
5	1.6	2.26	(1.6, 2....	0.43	0.4	4.15	(1.6, 4.15)	0.99
6	2	2.41	(2, 2.41)	0.43	0.4	5.12	(2, 5.12)	1.05
7	2.4	2.55	(2.4, 2....	0.42	0.4	6.14	(2.4, 6.14)	1.1
8	2.8	2.67	(2.8, 2....	0.42	0.4	7.21	(2.8, 7.21)	1.14
9	3.2	2.79	(3.2, 2....	0.42	0.4	8.33	(3.2, 8.33)	1.19
10	3.6	2.9	(3.6, 2.9)	0.41	0.4	9.48	(3.6, 9.48)	1.23
11	4	3	(4, 3)	0.41	0.4	10.68	(4, 10.68)	1.26
12	4.4	3.1	(4.4, 3.1)	0.41	0.4	11.92	(4.4, 11.92)	1.3
13	4.8	3.19	(4.8, 3....	0.41	0.4	13.2	(4.8, 13.2)	1.34
14	5.2	3.28	(5.2, 3....	0.41	0.4	14.51	(5.2, 14.51)	1.37
15	5.6	3.37	(5.6, 3....	0.41	0.4	15.86	(5.6, 15.86)	1.4
16	6	3.45	(6, 3.45)	0.41	0.4	17.24	(6, 17.24)	1.44
17	6.4	3.53	(6.4, 3....	0.41	0.4	18.65	(6.4, 18.65)	1.47
18	6.8	3.61	(6.8, 3....	0.41	0.4	20.09	(6.8, 20.09)	1.5
19	7.2	3.68	(7.2, 3....	0.41	0.4	21.57	(7.2, 21.57)	1.53

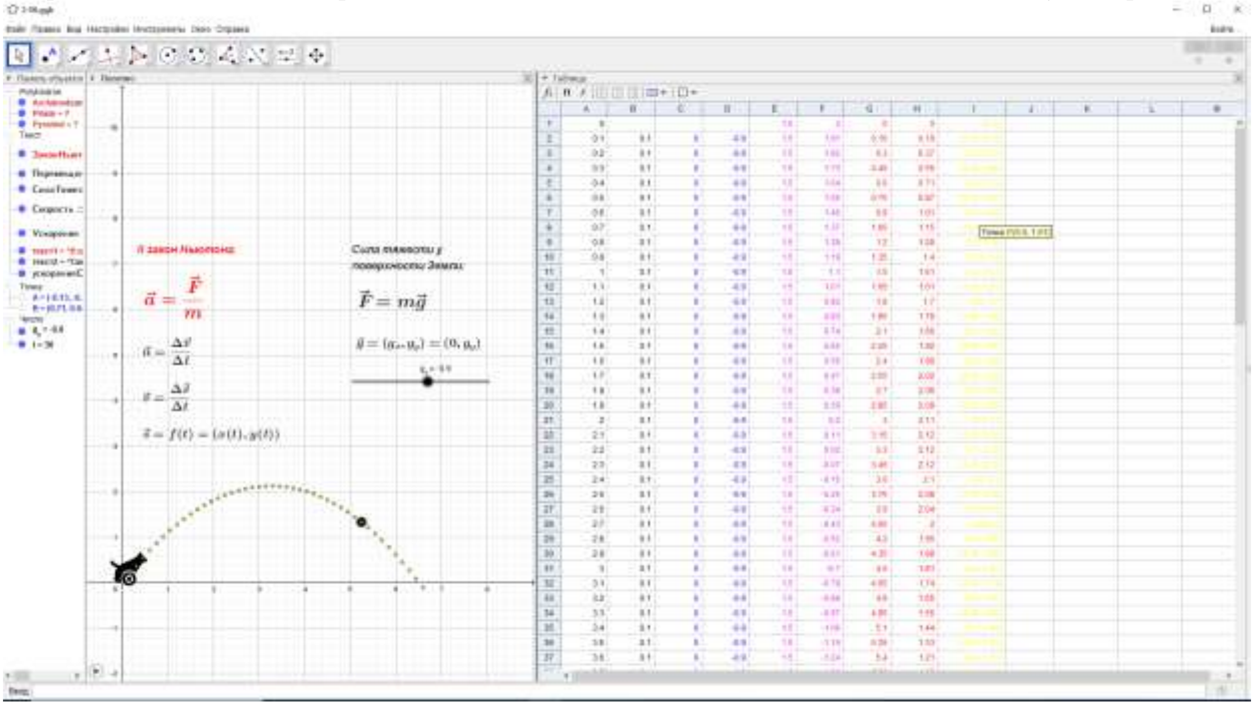
Modeling of physical problems

As an example of the application of computer simulation of solving ordinary differential equations in physics, let us consider the problem of the motion of a body in the gravitational field of the Earth without taking into account the force of air resistance.

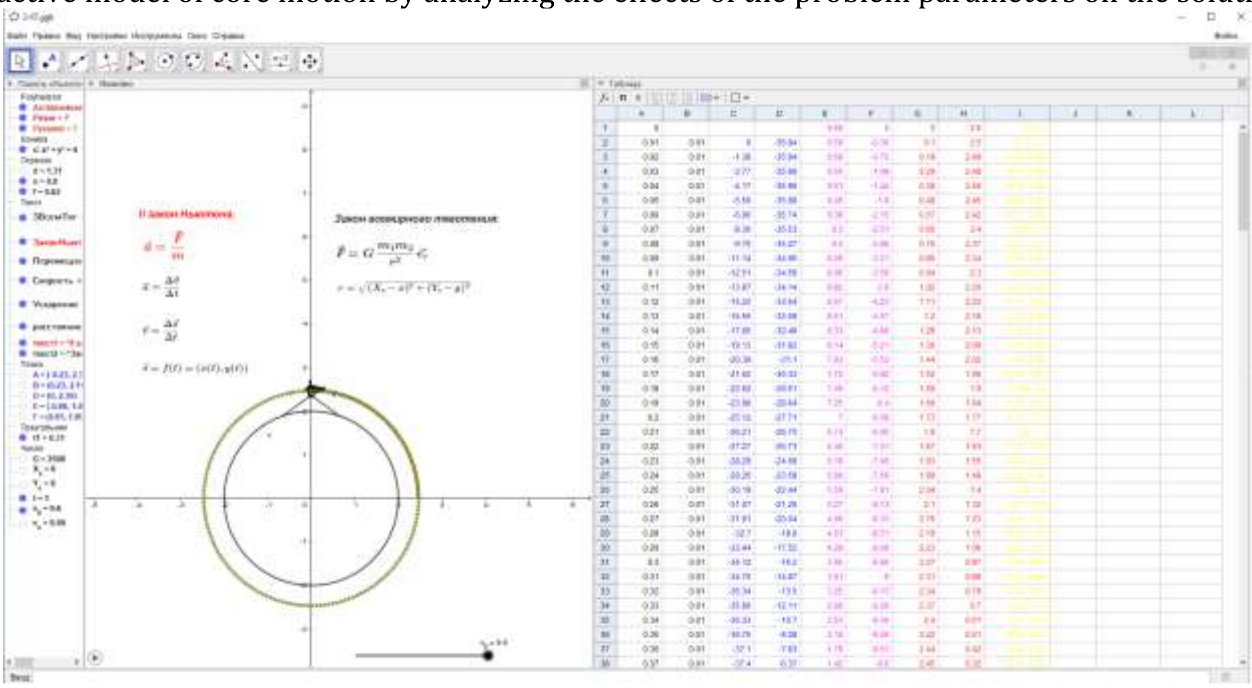
In the simplest case, the problem is reduced to one-dimensional motion in a homogeneous gravity field. The mathematical model of the apple falling process is described by one second-order differential equation. At the initial moment, the apple rests at some distance from the ground, under the influence of gravity after separation, the apple linearly increases its falling speed and reduces the distance to the ground according to the quadratic law. It is possible to visualize the acceleration, velocity and displacement functions using the tabular integration approach described above and integrating the acceleration function of the body twice using the Euler difference method. The functionality of the GeoGebra program allows you to build an animation of the movement of an object based on the values found.



In the case of two-dimensional motion under the action of homogeneous gravity, it is necessary to perform similar manipulations, but for two coordinate functions of the object's position.

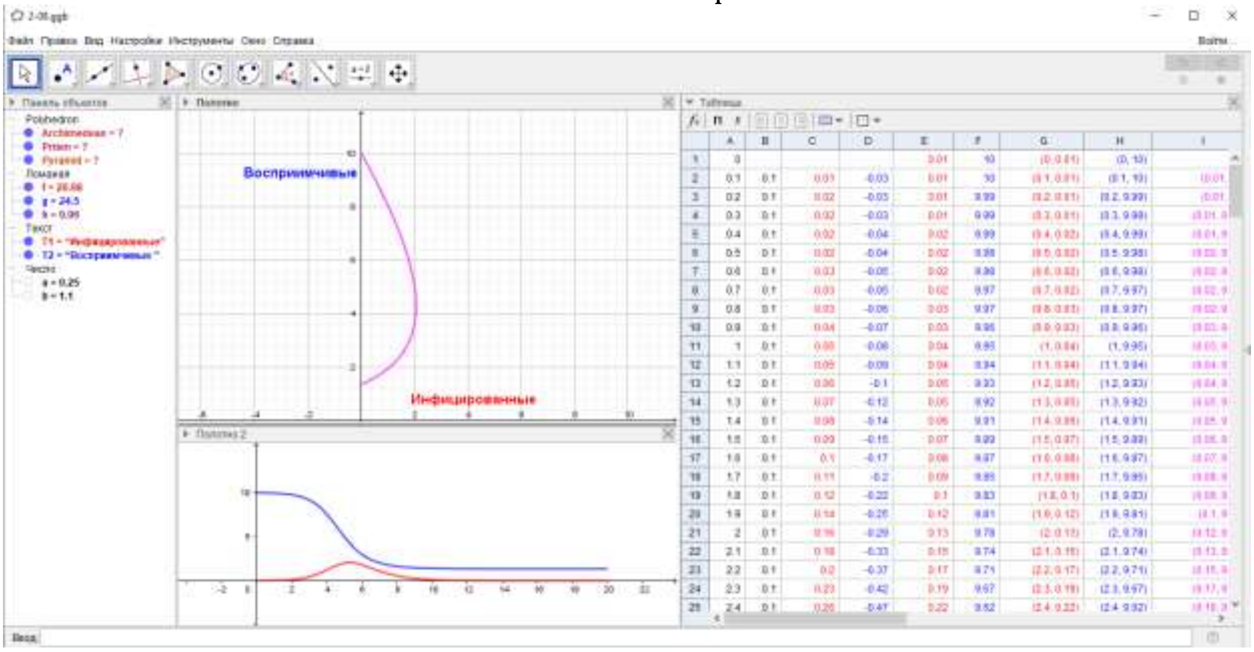


Consider the visualization of the famous thought experiment – "Newton's cannon": if you put the cannon on a high mountain, neglect the air resistance and the non-spherical nature of the Earth and shoot it strictly parallel to the surface, then the higher the velocity of the core, the farther the core will fly away; and at a certain value of the initial velocity, the core will return to its starting point, becoming artificial satellite of the Earth. GeoGebra allows to solve such a problem, as well as to build an animated interactive model of core motion by analyzing the effects of the problem parameters on the solution.

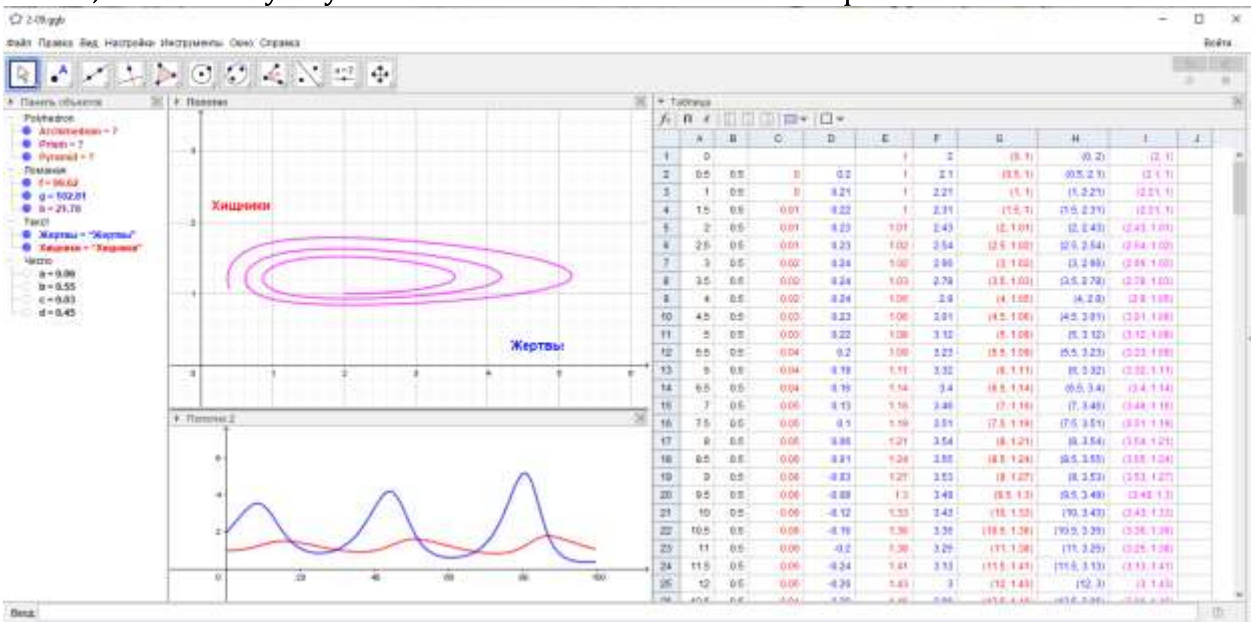


Modeling of genetic problems

To illustrate the application of ordinary differential equations in genetics, consider two problems. The first is a model of the spread of an infectious disease in an isolated population. The model consists of three first-order differential equations describing the change in the number of three classes of people: infected, susceptible to the disease and immune. The figure shows the peak of the growth in the number of infected individuals and the further exit to the plateau.



The second model is the classic Lotka–Volterra model of changes in predator and prey populations, described by a system of two first-order differential equations.



Conclusion

Using digital tools such as the GeoGebra program, it is possible to introduce students to methods of constructing mathematical models of natural science processes based on ordinary differential equations and techniques for solving differential equations that go beyond the school mathematics course.

References

1. Ziatdinov R., Rakuta VM., Dynamic Geometry Environments as a Tool for Computer Modeling in the System of Modernmathematics Education - European Journal of Contemporary Education, 2012, 1 (1), 93-100
2. Volterra, V., Mathematical Theory of Struggle for Existence. Nauka, Moscow, 1976, 288 p.

3. Sokolov S. V. Models of population dynamics: textbook. stipend. St. Petersburg: Publishing house of SPbSETU "LETI", 2018. 61 p.
4. Arnold V. I. Ordinary differential equations. –– New edition, corrected. –– Moscow: ICNMO, 2012. 344 p.

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