Variational method of solving fractional differential equation problems

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ABSTRACT

In this article, a new method for finding the solution of an integro-differential equation involving a fractional operator is proposed, with the help of which solutions of homogeneous and non-homogeneous equations are found. This method is based on the construction of a normal system of functions with respect to the fractional integro-differential operator.

Keywords: Variation of constant, Bernoulli's method, differential equations, algorithm, homogeneous

If it is required to solve a differential equation of first and fractional order, it is first necessary to check whether it is possible to separate the variables in this differential equation. If it is not possible to separate the variables in the differential equation, then the differential equation should be checked for homogeneity. In both cases, we know the algorithm for solving the differential equation.

If it is not possible to separate the variables in the first-order differential equation, as we mentioned above, and the equation is also not homogeneous, then in 90% of cases we may encounter a linear non-homogeneous first-order differential equation.

The general form of a linear inhomogeneous first-order differential equation is as follows:

$$a(x)\cdot y'(x)+b(x)\cdot y(x)=c(x)$$

and the default view in this case

$$y'(x)+p(x)\cdot y(x)=q(x)$$

We will consider solving a linear inhomogeneous first-order differential equation given in the standard form in two different ways:

1. The method of variation of the constant.
2. Bernoulli's method.

The algorithm of the method of variation of a constant is as follows:

1. $y'+p(x)\cdot y=0$ – a differential equation with separable variables is solved$\Rightarrow$ $y(x)=f(c,x)$ – a general solution is found.
2. Instead of **c** another ***u(x)*** – ***x*** we put the function of $\Rightarrow c=u(x)\Rightarrow$ $y(x)=f(u(x),x)$, This is where the name of the method comes from– variation of a constant
3. $y(x)=f(u(x),x)$ of $y'+p(x)\cdot y=q(x)$ into the differential equation.

Note: After this focus, the equation should come to a differential equation where the variables are separated!

4. Solving the resulting differential equation with separable variables, we find *u(x)*
5. Expression of *u(x)* put in $y(x) = f(u(x), x)$, we find a general solution.

Example 1. $y' + 2xy = x \cdot e^{-x^2}$

1. $y' + 2xy = 0 \quad \Rightarrow \dfrac{dy}{y} = -2xdx \quad \Rightarrow ln|y| = -x^2 + c \quad \Rightarrow y = C \cdot e^{-x^2}$
2. $y = C \cdot e^{-x^2} \quad \Rightarrow C = u(x) \Rightarrow \quad y = u(x) \cdot e^{-x^2}$
3. $(u \cdot e^{-x^2})' + 2x \cdot u \cdot e^{-x^2} = x \cdot e^{-x^2} \quad \Rightarrow$
   $u' \cdot e^{-x^2} + u \cdot (-2x) \cdot e^{-x^2} + 2x \cdot u \cdot e^{-x^2} = x \cdot e^{-x^2} \quad \Rightarrow$
   $u' = x$
4. $u' = x \quad \Rightarrow \quad u = \frac{1}{2}x^2 + C$
5. $y = u(x) \cdot e^{-x^2} \quad \Rightarrow \quad y = \left(\frac{1}{2}x^2 + C\right) \cdot e^{-x^2}$

BERNOULLI'S DIFFERENTIAL EQUATION.

As Bernoulli's differential equation,

$y' + p(x) \cdot y = q(x) \cdot y^a$

is referred to a differential equation of the form

As you can see, Bernoulli's differential equation resembles a linear non-homogeneous first-order differential equation in its structure. To determine whether a differential equation is a Bernoulli differential equation, the right-hand side involves the degree of y.

α at the level of y is both positive (α>0), negative (α<0), and a fractional number

$\left(a = \frac{1}{2} \quad \Rightarrow \quad y^{\frac{1}{2}} = \sqrt{y}\right)$ will be.

Bernoulli's equation can be given in different forms:

$r(x) \cdot y' + p(x) \cdot y = q(x) \cdot y^a$
$r(x) \cdot y' + y = q(x) \cdot y^a$
$y' + y = q(x) \cdot y^a$
$y' + p(x) \cdot y = y^a$

What is important is that the degree of y different than one is involved. When a>0, the solution y=0 is a particular solution of Bernoulli's equation.

Thus, the algorithm for solving Bernoulli's equation is as follows:

1. It is necessary to get rid of the y^a on the right. To do this, we divide both sides of the equation by y^a.
   $\dfrac{y'}{y^a} + p(x) \cdot y^{1-a} = q(x)$
2. It is necessary to get rid of y^{1-a}, for this should be defined as $y^{1-a} = z$.
3. $z' = (1-a)\dfrac{y'}{y^a} \quad \Rightarrow \quad y' = \dfrac{y^a}{1-a}z' \quad \Rightarrow \quad \dfrac{1}{1-a}z' + p(x) \cdot z = q(x)$

we arrive at a linear inhomogeneous 1st-order differential equation of the form We know the algorithm to solve it.

Example 3. $\sqrt{1-x^2} \cdot y' + y = arcsinx \cdot y^2, \qquad y(0) = -1$

1. You need to get rid of y on the right.

$$\frac{\sqrt{1-x^2} \cdot y'}{y^2} + \left(\frac{1}{y}\right) = arcsinx$$

2. It is necessary to get rid of y in the contoured adder, for this we perform a substitution.

$$z' = -\frac{y'}{y^2} \Rightarrow y' = -y^2 \cdot z' \Rightarrow -\sqrt{1-x^2} \cdot z' + z = arcsinx \Rightarrow$$

$$z' - \frac{1}{\sqrt{1-x^2}} z = -\frac{arcsinx}{\sqrt{1-x^2}}$$

As a result: we arrive at a linear inhomogeneous first-order differential equation from the Bernoulli differential equation. We know how to solve such equations.

CLERO DIFFERENTIAL EQUATION

Description. to the differential equation whose coefficients are linear with respect to x and y and are functions of

$$F(y') \cdot x + Q(y') \cdot y + R(y') = 0$$

It is called the LAGRANGE DIFFERENTIAL EQUATION.

The algorithm for solving this equation is as follows:

A variable is substituted to find a general solution.

The differential equation becomes:

$y = x \cdot f(p) + \varphi(p)$

In this $f(p) = -\frac{F(y')}{Q(y')}$, $\varphi(p) = -\frac{R(y')}{Q(y')}$

1) We differentiate this equation taking into account that

$$y' = p \Rightarrow dy = pdx .$$

$$dy = d(x \cdot f(p) + \varphi(p)) \Rightarrow pdx = f(p)dx + x \cdot f'(p)dp + \varphi'(p)dp$$

2) If the solution of this differential equation, which is linear with respect to x, is x=F(p,c), then the general solution of the Lagrangian differential equation is:

$$\begin{cases} x = F(p,c) \\ y = x \cdot f(p) + \varphi(p) = F(p,c) \cdot f(p) + \varphi(p) \end{cases}$$

Description. to the following differential equation, whose coefficients are linear with respect to x and y and are functions of

$y = x \cdot y' + \varphi(y')$

It is called CLERO DIFFERENTIAL EQUATION.

Clairo differential equation is a special case of Lagrange differential equation. The algorithm for solving this differential equation is as follows:

1) $y' = p \Rightarrow y = x \cdot p + \varphi(p)$

2) $y' = p \Rightarrow dy = pdx \Rightarrow dy = d\left(x \cdot p + \varphi(p)\right) \Rightarrow$

$y'dx = pdx + xdp + \varphi'(p)dp \Rightarrow pdx = pdx + xdp + \varphi'(p)dp$

We divide the last expression by dx

$$p = p + x\frac{dp}{dx} + \varphi'(p)\frac{dp}{dx} \Rightarrow (x + \varphi'(p))\frac{dp}{dx} = 0$$

3) $\begin{cases} x + \varphi'(p) = 0 \\ dp = 0 \end{cases} \Rightarrow$

First solution: $dp = 0 \Rightarrow p = C \Rightarrow y = C \cdot x + \varphi(C)$

Second solution: $\begin{cases} y = x \cdot p + \varphi(p) \\ x + \varphi'(p) = 0 \end{cases}$ is formed by solving a system of parametric equations.

The resulting second solution F(x,y)=0 does not contain an arbitrary constant and is not generated from the general solution by any value of C, so it is not a particular solution. Such solutions are special solutions (integral). Thus, the special solution of Clero's equation determines the line of inclination of the family of straight lines given by the general solution (integral), in other words, the attempt made to an arbitrary point of the special solution is also a solution of the differential equation.

Clairo's differential equation is often used to construct 2nd-order curves in analytical geometry. Geometrical problems defining a curve in terms of its experimental properties lead to the Clairot equation. This property is specific to the attempt and not to the point being attempted. Actually the equation of effort is:

$Y - y = y'(X - x)$ **yoki** $Y = y'X + (y - xy')$

It is determined by the relationship between any property of the experiment and:

$$\Phi(y - xy', y')$$

=0

If this equation is solved with respect to y=xy, exactly

$y = x \cdot y' + \varphi(y')$ We come to the Clero equation.

Example. $y = x \cdot y' + (y')^2$

1) $y' = p \Rightarrow y = x \cdot p + p^2$
2) $y' = p \Rightarrow dy = pdx \Rightarrow dy = d(x \cdot p + p^2)$

$y'dx = pdx + xdp + 2pdp \Rightarrow pdx = pdx + xdp + 2pdp$

We divide the last expression by dx

$$p = p + x\frac{dp}{dx} + 2p\frac{dp}{dx} \Rightarrow (x + 2p)\frac{dp}{dx} = 0$$

– this equation has two possible solutions.

3)

First solution:

$dp = 0 \Rightarrow p = C \Rightarrow y = C \cdot x + \varphi(C)$

The general integral (solution) of the Clairo equation forms a family of straight lines.

Solution 2: the solution is found in parametric form from the system of equations:

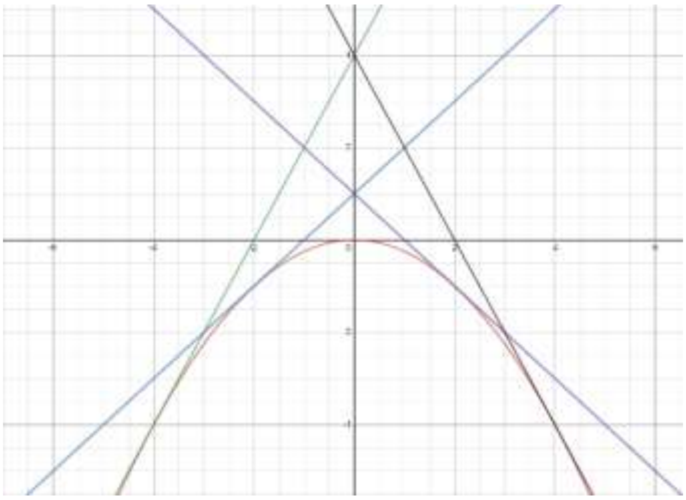
by finding p from this system

$$\begin{cases} y = x \cdot p + p^2 \\ x + 2p = 0 \end{cases} =$$

we pass to the second equation

$$p = -\frac{x}{2} \Rightarrow y = x \cdot \left(-\frac{x}{2}\right) + \left(-\frac{x}{2}\right)^2 = -\frac{x^2}{4} \Rightarrow y =$$

The second solution does not contain an arbitrary constant and is not derived from the general solution by any value of C, so it is not a particular solution. Such solutions are special solutions (integral).



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