Attempts of Central Asian Scholars to Prove Euclid's Fifth Postulate

Z.Sh. Kupaysinova
Assistant, Department of Higher Mathematics, Fergana Polytechnic Institute, Fergana, Uzbekistan

ABSTRACT
The article presents the work of Central Asian scientists who tried to prove the V postulate of Euclid.

Keywords: Fifth postulate, constant quantity, parallel straight lines.

Many Central Asian scientists tried to prove postulate V.

Ibn al-Haysam's work in the field of proving the fifth postulate.

Haysam, like many scientists, tried to prove Euclid's fifth postulate. His work in this field is presented in his work entitled "Additions to Euclid's Foundations". Haysam is considered the first scientist who studied Lambert's quadrilateral. In doing so, he examines a quadrilateral with 3 right angles and makes 3 hypotheses for the fourth angle:

– Sharp;
– Right;
– Blunt.

Haysam created many formulas in his work entitled "Measurement of Parabolic Bodies". Including the sum of squares, and the sum of cubes.

Ibn Sina's work in the field of proof of the fifth postulate.

Abu Ali Husayn Ibn Abdullah Ibn Sina (6.08.980-18.06.1037) is one of the scientists who tried to prove to postulate V. He mentioned the work he did to prove the V postulate in his work entitled "Letter".

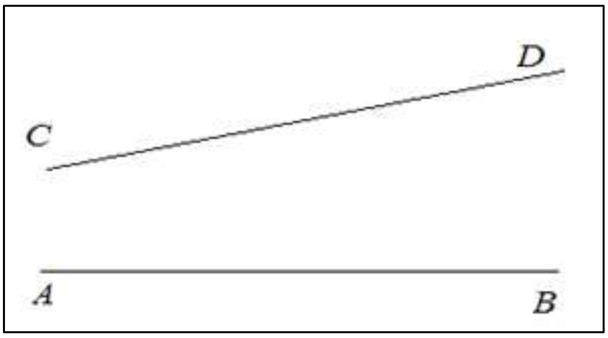

Figure 1.

Theorem: Different lines that are at some distance from each other can be such that the end of one of them overlaps with the other, if we continue them on this side, they intersect: if we continue them on the other side, they do not intersect, for example: AB and CD lines are like that.

Continuing this concept, Ibn Sina writes: "They were at such a distance from each other b can die so that the distances between their two ends are equal. Ibn Sina calls such two lines mutually parallel lines (Fig. 1) [1-9].

Therefore, Ibn Sina defines parallel lines as lines that are equidistant.

Euclid in the first book of his work "Principles" defines parallel straight lines as follows: "Such straight lines are mutually parallel, which, when they lie in the same plane and continue indefinitely on both sides, on one side and on the other they do not meet each other on either side" (Abu Ali Ibn Sina).

Comparing Euclid's and Ibn Sina's definitions of parallel straight lines, it can be said that Ibn Sina's definition is equivalent to the fifth postulate, while Euclid's definition is independent of the fifth postulate.

Ibn Sina then proves the following theorems:

1. If a line is drawn perpendicular to a line and it is continued perpendicularly to a second line parallel to this line, it will also be perpendicular to the second line.
2. If a straight line is transversal to two given parallel lines and intersects them, then the resulting corresponding angles and internal alternating angles are equal to each other, and the sum of the internal angleswill be equal to $2d$.
3. If a straight line intersects two other lines and the sum of the resulting two one-sided interior angles $2d$ equal to, these two lines are parallel to each other.

After that, theorem 4 is proved, which is Euclid's fifth postulate.

"If a line intersects two other lines and the sum of the two interior angles $2d$ is less than, then when these two lines are continued, they intersect on this side. For example, such straight lines AK and SL will be".

Proof: In this case, one line is perpendicular to the other line and therefore they intersect. Indeed, if one of them were not opposed to the other, they would be parallel to each other. If so, that is, if they are parallel, then the sum of the said angles would be equal to $2d$, which was proved earlier. For this reason AK and SL if the straight lines are continued, they intersect on the above-mentioned side.

So, Ibn Sina tries to prove the fifth postulate, based on his earlier definition of parallel lines. But it was said above that this definition is equivalent to the fifth postulate. Thus, Ibn Sina, along with many other mathematicians, tries to prove the fifth postulate as a theorem by distinguishing it from postulates. In proving the fifth postulate, he implicitly uses a sentence equivalent to this postulate. This cannot be proof of the theorem.

It should be noted that such attempts to prove the fifth postulate played a major role in preparing the ground for the creation of non-Euclidean geometry.

Works of Nasreddin Tusi in the field of proof of the fifth postulate.

First of all, Nasreddin Tusi notes that there can be three cases in the content of this postulate.

Case 1. A combination of both $2d$ one of the interior one-sided angles smaller than is a right angle;

Case 2. Both such angles are acute angles;

Case 3. One of these angles is an obtuse angle.

The above comments show that Nasriddin accepts and uses the following sentences in the process of proving the fifth postulate:

1. If two perpendiculars are passed to a straight line and they are intersected by a fourth straight line, the resulting corresponding angles and alternating angles are the sums of equal and equilateral angles $2d$ will be equal to
2. Straight lines lying in the same plane and perpendicular to a straight line intersect.
3. If one of the sides of an acute angle has equal sections and a perpendicular is passed from their ends to the other side of the acute angle, then this perpendicular also separates equal sections on the other side.
4. If a straight line enters an angle, it must leave it again, that is, if a straight line intersects one side of a triangle, this straight line must intersect another side of the triangle.

An examination of the foundations of geometry shows that each of these sentences is a strong (equivalent) sentence to Euclid's fifth postulate. Therefore, Nasreddin, similar to many mathematicians who conducted investigations in this field, for example, Omar Khayyam, in the process of proving the fifth postulate, uses sentences that are as strong as this postulate itself. Its proof will be only an attempt in this field. Because this postulate cannot be proved [10-21].

Thus, Nasriddin's investigations in the field of the theory of parallel straight lines described above played a great role in preparing the ground for the origin of non-Euclidean geometry.

Works of Omar Khayyam in the field of proof of the fifth postulate

Omar Khayyam made a great contribution to geometry. His work in the area of proving Euclid's postulate V is very famous.

In the work of Omar Khayyam entitled "Commentaries on difficult postulates of the book of Euclid" (1077). The V postulate problem is stated. In this work, Khayyam refers to the work of mathematicians who preceded him in the field of the theory of parallel straight lines - Heron, Evtokiy, al-Khazini, al-Shani, al-Nairizi, Ibn al-Haysam, and their points out that their proof of the V postulate is incomplete and criticizes that they do not focus on principles borrowed from the following philosopher (Aristotle).

1. Quantities are infinitely divisible, that is, they are composed of indivisibles.
2. A straight line can be continued indefinitely.
3. Any two intersecting straight lines move away from each other as they move beyond the tip of the intersection angle.
4. Two converging straight lines intersect, but their convergence cannot cause the two straight lines to move away from each other in the direction of converging.

Of two unequal finite quantities, the smaller one can be obtained so many times that it exceeds the larger one [19-23].

Here the fourth principle V is equivalent to a postulate.

Proclus (a Greek mathematician, who lived for V centuries) is very close to the proof. If Omar Khayyam did not intersect EH CD It is implicitly assumed in Proclus that the distance between straight lines does not change. His main mistake here is that Omar Khayyam, like his predecessors, is in the process of proof V using 4 principles equivalent to the postulate.

Omar Khayyam's main contribution to the theory of parallel straight lines is that, in the history of geometry, he was the first to reveal V and replaces the postulate with its equivalent 4-prinsp.

To prove that Euclid's postulate is a theorem, he looked at a right rectangle with two right angles at the bottom base, and if its two bottom angles are right angles, then the top two angles are right angles. came to the conclusion that it should be Omar Khayyam says, "Two straight lines perpendicular to one straight line cannot intersect on both sides of the straight line."

The Italian mathematician J. Saccheri, who was not aware of these works of Omar Khayyam, also dealt with postulate V and applied it to the rectangle. This rectangle was included in the basics of geometry under the name "Khaiyom-Sackeri quadrilateral".

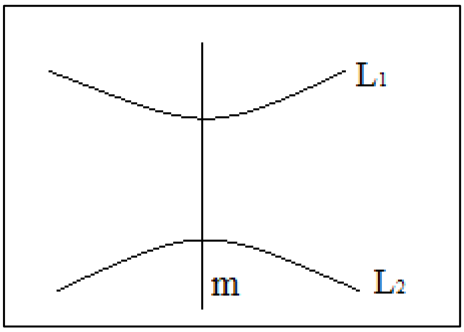

Figure 2.

If in Figure 2 A– right angle, B and if it is slightly smaller than a right angle, L_1 L_2 straight lines must intersect, even when intersecting m intersects the right side of the straight line. Many of Euclid's theorems, for example ("the angles at the base of an equilateral triangle are equal"), express extremely simple facts about the fifth postulate (Figure 2). In addition, the fifth postulate is more difficult to verify experimentally. If in the 2nd diagram AB distance is equal to 1 m, B and if the angle differs from the right angle by one second (in terms of angle), L_1 and L_2 straight lines m from 200 km, it is enough to say that it intersects at a distance of more than

Euclid in his work "Fundamentals" states postulate V "when two straight lines are cut by a third straight line, if the sum of the angles on one of its interior sides is less than two right angles, let them cross on that side. This definition of Euclid has caused doubts among mathematicians since ancient times because it is not very reasonable. They thought that it should be a theorem, not a postulate, and they mistakenly thought that it was included among the postulates, and tried to prove it.

References

1. N.D.Dadajonov, Yunusmetov R, Abdullaev A. Geometriya. 2-qism, Toshkent.«O'qituvchi», 1996 y.
2. A.Ortiqboev. Hayoldagi geometriya. Fizika, matematika va informatika. Jurnal 2 – son. Toshkent 2012 yil.
3. Abdurazakov, A. Shayev, N. Mirzamahmudova, N. Mahmudova Iqtisodchilar uchun matematika. 2021y.
4. Abdurazakov, A., Mirzamahmudova, N., & Maxmudova, N. (2021). Iqtisodiy masalalarni maple dastur tizimi yordamida yechish uslubiyoti. *Scientific progress*, *2*(7), 737-745.
5. kizi, N. D. B. (2022). On some problems with a mixed for a hyperbolic type equation with two lines of expression. *Central asian journal of mathematical theory and computer sciences*, 3(7), 1-6.
6. Yusupova, N. X., & Nomoanjonova, D. B. (2022). Innovative technologies and their significance. *Central asian journal of mathematical theory and computer sciences*, *3*(7), 11-16.
7. Kosimov, K., & Mamayusupov, J. (2019). Transitions melline integral of fractional integrodifferential operators. *Scientific Bulletin of Namangan State University*, *1*(1), 12-15.
8. Qo'Ziyev, S. S., & Mamayusupov, J. S. (2021). Umumiy o 'rta ta'lim maktablari uchun elektron darslik yaratishning pedagogik shartlari. *Oriental renaissance: Innovative, educational, natural and social sciences*, *1*(10), 447-453.
9. Abdurazakov, A., Mirzamahmudova, N., & Maxmudova, N. (2021). "Iqtisod" yo'nalishi mutaxassislarini tayyorlashda matematika fanini o'qitish uslubiyoti. *Scientific progress*, *2*(7), 728-736.
10. Расулов, Р., Сатторов, А., & Махкамова, Д. (2022). Вычисленние Квадрат Нормы Функционала Погрошности Улучшенных Квадратурных Формул В Пространстве. *Central asian journal of mathematical theory and computer sciences*, *3*(4), 114-122.
11. Rashidjon, R., & Sattorov, A. (2021). Optimal Quadrature Formulas with Derivatives in the Space. *Middle European Scientific Bulletin*, *18*, 233-241.
12. Акбаров, Д. Е., Кушматов, О. Э., Умаров, Ш. А., & Расулов, Р. Г. (2021). Исследования Вопросов Необходимых Условий Крипто Стойкости Алгоритмов Блочного Шифрования С Симметричным Ключом. *Central asian journal of mathematical theory and computer sciences*, *2*(11), 71-79.
13. Bozarov, B. I. (2021). An optimal quadrature formula in the Sobolev space. *Uzbek. Mat. Zh*, *65*(3), 46-59.
14. Bozarov, B. I. (2020). Optimal quadrature formulas with the trigonometric weight in Sobolev space. *Matematika Instituti Byulleteni Bulletin of the Institute of Mathematics Бюллетень Института*, (5), 1-11.
15. Далиев, Б. С. (2021). Оптимальный алгоритм решения линейных обобщенных интегральных уравнений Абеля. *Проблемы вычислительной и прикладной математики*, (5 (35)), 120-129.
16. Акбаров, Д. Е., Кушматов, О. Э., Умаров, Ш. А., & Далиев, Б. С. (2021). Исследование Вопросов Необходимых Условий Идеально Стойких Алгоритмов Шифрования. *Central asian journal of mathematical theory and computer sciences*, *2*(11), 65-70.
17. Алимжонова, Г. И. Қ., & Назарова, Г. А. (2022). Аналитик тафаккурни ривожлантиришнинг педагогик зарурати. *Scientific progress*, *3*(4), 187-196.
18. Қизи, Г. А. И. (2022). Техноэтик маданият тушунчасининг мазмун-моҳияти, уни ривожлантиришнинг педагогик зарурати. *Integration of science, education and practice. Scientific-methodical journal*, *3*(3), 329-333.
19. Shavkatjon o'g'li, T. B. (2022). Some integral equations for a multivariable function. *Web of Scientist: International Scientific Research Journal*, *3*(4), 160-163.

20. Муминова, Н. А. (2016). Применение тестовых заданий на уроках родного языка в начальных классах. *Журнал научных публикаций аспирантов и докторантов*, (5), 153-155.
21. Nazarova, G. (2021). Methods of directing economics to scientific research activities. *Current research journal of pedagogics (2767-3278)*, *2*(06), 90-95.
22. Fayzullayev, J. I., & Sattorov, A. M. (2021). Axborot va pedagogik texnologiyalar integratsiyasi asosida texnika oliy ta'lim muassasalari talabalarining kasbiy kompetentligini rivojlantirish. *Scientific progress*, *2*(7), 330-336.
23. Sattorov, A. M., & Qo'Ziyev, S. S. (2021). Matematika fani o'qituvchilarini tayyorlashda fanlararo integratsiyaning asoslari. *Scientific progress*, *2*(7), 322-329.